Bayesian estimation of labor demand by age: Theoretical consistency and an application to an input-output model

Kijin Kim and Geoffrey J. D. Hewings

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Bayesian estimation of labor demand by age: Theoretical consistency and an application to an input-output model

Kijin Kim[a]* and Geoffrey J.D. Hewings[a]

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This paper finds that a static labor demand model restricted with theoretical requirements yields empirically coherent wage elasticities of labor demand when the recent Census data are used. A Bayesian approach is used for more straightforward imposition of regularity conditions. The Bayesian model confirms elastic labor demand for youth workers, which is consistent with what past studies find. Comparison with other conventional methods suggests that monotonicity and concavity must be checked and addressed particularly in the case where one or more factor shares are so small that monotonicity is likely to be violated. Additionally, to explore the effects of changes in age structure on a regional economy, we integrate the estimated age-group-specific labor demand model into a regional input-output model. The new model suggests that *ceteris paribus* aging population attributes to lowering aggregate economic multipliers due to the rapidly growing number of elderly workers who earn less than younger workers.

Key words: Labor demand by age, Translog cost function, Bayesian SUR, Regularity conditions, Miyazawa's input-output model

JEL Classification: C11, D24, J23

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* Corresponding author. Email: kkim96@illinois.edu

[a] Regional Economic Applications Laboratory, University of Illinois, Urbana, Illinois, USA

1 Introduction

Economists often believe that if an economic theory-based model is applied to actual data, the resulting estimates would always satisfy theoretical properties. Yet, this belief is valid only if theoretical conditions are imposed during estimation. In practice, a number of studies on static demand models often exclude a validity check after estimation or proceed without referring to regularity conditions (O'Donnell and Coelli, 2005). Without evaluating theoretical requirements, it is poorly grounded to argue that the outcomes are intuitively correct or empirically consistent with past findings even if they are seemingly so. Therefore, the recent literature on static demand models strongly argues for the imposition of theoretical properties if necessary, following critical assessment (Sauer *et al.*, 2006).

This paper investigates theoretical and empirical consistencies of a static demand model. Particularly labor demand by age is investigated against the backdrop of aging population and an increasing awareness of its implications on labor markets. Examining the recent Census data, we find that a Bayesian labor demand model with regularity restrictions yields empirically coherent wage elasticities of labor demand. A Bayesian approach is implemented since regularity conditions can be more easily imposed than conventional constrained optimization approaches. The estimation results confirm elastic labor demand for youth workers aged 16-24 as past studies consistently find (Hamermesh and Grant, 1979). In addition, we find that labor demand for elderly workers aged 65 and over is elastic, little varying across sectors, as opposed to higher sectoral variability in labor demand elasticities for youth workers.

The labor demand model in this study is derived from the flexible translog labor cost function. The translog form is useful when no information is available on a functional form of a cost function because it approximates any arbitrary cost function. The model is constructed so that each industry has its own behavioral parameters along with four types of labor inputs (workers aged 16-24, 25-44, 45-64 and 65+). To ensure theoretical consistency, all regularity conditions of the cost function are thoroughly addressed. Among those conditions, homogeneity and symmetry can be easily imposed through parametric constrained estimation without any laborious procedure. Monotonicity and concavity, however, require special treatment because the constrained optimization often fail to converge due to the complexity of the non-linear constraints (O'Donnell and Coelli, 2005).

As an estimation strategy alternative to the maximum likelihood method, we adopt a Bayesian approach proposed by Griffiths *et al.* (2000). Its intuitive sampling nature facilitates the imposition of monotonicity and concavity at reference points. Under the Bayesian algorithm, sample parameters are repeatedly drawn from a certain density. If the candidate parameters satisfy regularity conditions at the reference points, they are accepted and used at the next draw. If not, new samples are drawn. Statistical inference is based on the empirical distribution of the accepted samples. It turns out that the Bayesian approach is superior to the non-Bayesian method in the sense that it yields more empirically plausible (i.e. elastic) labor demand elasticity for youth workers while prediction accuracy is maintained as good as the non-Bayesian counterpart.

After estimating the labor demand model by age group, we integrate the labor demand model into a regional input-output model. This attempt is to show that the integration can add to the representative agent-based input-output model new capability to conduct impact studies on heterogeneous agents. As an illustration, we examine economic impacts of changing age distribution on income of age group and sectoral output in Chicago. The new model implies that other things being equal, aging population may result in lower aggregate economic multipliers due to the rapidly growing number of elderly workers who earn less than younger workers.

This paper contributes to the literature on a static labor demand model in several ways. First, we present a representative example in which monotonicity is highly likely to be violated due to very small factor cost shares, in our case, for labor cost shares of youth and elderly workers.[1] Our empirical evidence suggests that monotonicity needs extra scrutiny especially when one or more factor cost shares are exceptionally small. Second, our model separately includes workers beyond the average retirement age[2]. In a number of studies on labor-labor substitution, older workers are generally those prior to retirement age and are often too broadly grouped together with other age-group workers in their 20s to 50s. Third, by using highly disaggregated geographic and industrial units of observations, our model reduces concerns about the aggregation problem since a model using aggregate data is subject to aggregation bias.[3]

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[1] Under a translog cost function, monotonicity implies nonnegative factor cost shares. In the presence of very small or large input cost shares, estimated shares are likely to deviate from the 0-1 range unless the range of predicted values is imposed *a priori*.

[2] Munnell (2011) calculates the recent average retirement age for men and women to be 64 and 62, respectively. She argues that the retirement age will continue to rise. The surveys in Hamermesh and Grant (1979) and Hamermesh (1996) cover studies on labor demand by age that had been published until the early 1990s. Among the papers in the surveys, Ferguson (1986) is the only study that includes workers aged 65 and over. We could not find any papers on labor demand for the elderly group henceforth. A most recent survey on demand for aggregate and heterogeneous (mostly by skill level) labor, including empirical studies released from 1980 to 2012, can be found in Lichter *et al.* (2014).

[3] For example, Lee *et al.* (1990) find statistically significant aggregation bias when a disaggregate employment model with 41 industries is compared with an aggregate employment model for the UK.

Our paper also adds to the recent efforts on regional model integration. Integrating different types of regional models has been actively embraced particularly in computable general equilibrium (CGE) models whose main objective is often policy simulation based on a representative agent assumption (Colombo, 2010). Similar applications also have been explored in a traditional input-output model and an econometric input-output model into which consumer and labor demand models are integrated (see, for example, Mongelli *et al.*, 2010, Kim *et al.*, 2015 and Maier *et al.*, 2015).

This paper is organized as follows. Section 2 presents recent features of labor force by age group. Section 3 describes a static model of labor demand and discusses theoretical properties of a cost function. Then, a Bayesian approach is described as an alternative to conventional methods. In section 4, data and exploratory analysis are presented. Section 5 shows estimation results for the Bayesian and non-Bayesian models, followed by an investigation of regularity conditions and labor demand elasticity estimates. Section 6 describes an application of the labor demand model to an input-output model. Section 7 concludes with major findings and implications.

2 Background: stylized facts on labor force by age[4]

Figure 1 presents some key stylized facts on job market conditions and labor characteristics by age group. We particularly focus on youth (aged 16-24) and elderly (65 and over) workers, and the remaining middle aged workers are divided into the 25-44 and 45-64 groups.

(a) *Labor force participation rates*: less than 20 percent of people in the oldest age group participate in labor market while the rest groups show much higher participation at 60-80 percent. The differences among age groups are also stark when it comes to changes in population and labor force. For example, the labor force of people 65 years and older grew 77 percent between 2001 and 2013, whereas its population grew only 34 percent. However, the population and labor force of the two middle groups grew at the same rate.

(b) *Unemployment rates*: unemployment rates tends to be lower with age while there exists a large gap in the unemployment rate (9-10 percentage points) between the youngest and the second-youngest age groups.

[4] Descriptive statistics are calculated from the 2000 and 2013 American Community Survey (ACS) microdata. Further details on the ACS are described in Section 4.

(c) *Class of workers*: the share of wage and salary workers in the private sector declines with age, whereas the share of the self-employed rises with age so that nearly one in five workers aged 65 and over is self-employed.

(d) *Sex*: among private wage and salary workers, female employees account for slightly less than half of all employees, not showing any large difference between age groups.

(e) *Education attainment*: the 25-44 age group employees shows the highest share of some college and higher education, followed by the 45-64 age group, the oldest group and the youngest group. Between 2000 and 2013, youth and elderly workers show larger improvement in education than the two middle groups in terms of college and higher education.

(f) *Work hours*: youth and elderly employees are more likely to be part-time workers (i.e. those who work less than 35 hours a week) than the other groups. For the period of 2001-2013, the elderly group is the only age group that shows an increase in the share of full-time workers.

(g) *Occupation*: the top five common jobs for elderly workers account for 19 percent of total occupations in 2013: retail salespersons, drivers, secretaries, cashiers, and managers. For the youngest group, the top five common jobs are cashiers, retail salespersons, waiters and waitresses, cooks, and customer service representatives and they account for 30 percent of all jobs.

(h) *Wages and salaries*: except for managers and retail supervisors, annual wages and salaries for the 10 most common occupations are lower than the US mean wage ($44,500) in 2013. Among the 10 occupations, cashier, waiters and waitresses, cooks are the lowest-paying jobs.

<< *Insert figure 1 here* >>

3 The Model

This section describes the theory of a static labor demand model and presents an estimation strategy. First, we discuss a labor demand model by age that is suitable for econometric estimation. Second, the implications of theoretical properties on estimation and results are reviewed. Third, after describing limitations of a parametric estimation method, a Bayesian approach is proposed as an alternative for the imposition of certain regularity conditions.

3.1 A translog labor cost function

In this subsection, we derive age-specific labor demand equations based on a translog labor cost function. To account for sector-specific firm behavior in demanding for labor by age, our labor demand model includes age-group-specific trends varying by sector.

We assume a twice-differentiable strictly quasi-concave production function with four types of aggregate inputs. Among the inputs, labor comprises *G* subtypes of workers of different age. By duality, a master cost function can be written:

$$C = C(P_{L_1}, \dots, P_{L_G}, P_K, P_E, P_M, Y) \tag{1}$$

where *L*, *K*, *E, M* and *Y* indicate labor, capital, energy, non-energy intermediate materials, and gross output, respectively; P_i is the price of factor *i* (*i=K, L, E, M*); P_{L_g} is the real wages for age group *g*. Assuming weakly separability between labor and the other factors, i.e. substitution between labor subgroups is independent of output and prices of the other input, equation (1) can be rewritten:[5]

$$C = C[P_L(w_1, \dots, w_G), P_K, P_E, P_M, Y] \tag{2}$$

where the price of a unit of labor P_L is assumed to be linearly homogeneous; $w_g \equiv P_{L_g}$ for $g = 1, \dots, G$.

A translog cost function (Christensen *et al.,* 1971) is chosen for the unit labor cost function with *G* types of labor $P_L(w_1, \dots, w_G)$ because it is a generalization of any arbitrary cost functions by a second-order approximation. It is also convenient for empirical estimation and interpretation due to the linearity in parameters in the derived factor shares equations. The translog unit labor cost function is given by:

$$\log(P_L) = \alpha_0 + \sum\nolimits_g \alpha_g \log(w_g) + \frac{1}{2}\sum\nolimits_g \sum\nolimits_h \beta_{gh} \log(w_g)\log(w_h) \tag{3}$$

This unit labor cost function is generally estimated by industry (see, for example, Jorgenson *et al.*, 2013 and Kratena *et al.*, 2013).

[5] In practice, many empirical studies on factor demand assume separability due to data availability (Atkinson and Manning, 1995). However, separability is essentially an empirical issue that requires statistical testing. If labor is not separable from other factors, the estimates of labor-labor substitution are biased when other factors are omitted in the model. Since this paper focuses on the regional level (i.e. the US states) where data on prices and quantities of other factors, especially capital among others, are usually not available, measurement errors due to constructing estimates for capital might be more problematic (Hamermesh and Grant, 1979). Furthermore, regional models are often developed upon a single-input (usually labor) assumption that inputs other than labor can be approximated by local employment (Glaeser *et al.* 1992; Bishop and Gripaios, 2010; Felipe and McCombie, 2012).

Based upon this form, a time trend, interactions with the time and group-specific wage, and region fixed effects are added to capture changes in the characteristics of labor over time and regional variation. Hence, the final specification is written as:

$$\log(W_t^r) = \alpha_0 + \mu^r + \theta t + \sum_g \alpha_g \log(w_{g,t}^r) + \frac{1}{2}\sum_g \sum_h \beta_{gh} \log(w_{g,t}^r) \log(w_{h,t}^r) + \sum_g \gamma_g \{\log(w_{g,t}^r)\} t \quad (4)$$

where the subscript *i* for industry is omitted for convenience; *r* is a region; *t* is time; *W* is the mean of annual wages and salaries that approximate the unit labor cost per year; μ^r is the region fixed effect. Applying Shepherd's lemma yields a set of *G* labor cost share equations as follows:

$$s_{g,t}^r = \frac{\partial \log(W_t^r)}{\partial \log(w_{g,t}^r)} = \alpha_g + \sum_h \beta_{gh} \log(w_{g,t}^r) + \gamma_g\, t, \quad g = 1, \ldots, G \quad (5)$$

The unit labor cost function takes into account the characteristics of labor by sector and age group as well as the cost structure by region, while the derived labor cost shares implies that industry behavior of labor demand depends on sector and workers' age. First, the common time trend in equation (4) approximates the *industry-specific* overall labor quality over time (analogous to using a time trend as a proxy for technology progress over time in production).[6] Labor quality may include knowledge, intelligence and strength of workers to which age and years of schooling contributes (Fuchs, 1964). Second, the region fixed effects μ^r account for *region-specific* cost differentials such as a fixed cost of labor varying by region. Third, the γ_g's in equation (5) represent *age-group-specific* characteristics - such as rising or falling labor group input share due to the aging of the population and an increase in labor force participation of the oldest group - holding the wage fixed.

It is worth noting that identification of the unit labor cost and labor cost shares are based on the assumption that labor supply is perfectly elastic so that changes in relative wages determine changes in labor demand. This assumption can be justified in studies with small units and we treat our unit of observations (i.e. state-specific 45 sectors) as "relatively small" enough to reduce concern about wages being exogenous. Similar identification assumption can be found, for example, in Slaughter (2001).

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[6] Although it is not explored here because of a relatively short time series data (13 years), a time varying trend, which can be estimated using the Kalman filter, might be a more sensible choice. (Jorgenson *et al.*, 2013)

With parameter estimates and predicted factor shares, partial own- and cross-price elasticities of labor demand for an age group, holding the wages of the other age-group workers constant, are given as follows:

$$\eta_{gg} = \frac{\beta_{gg}}{s_g} + s_g - 1 \quad \text{for } g = 1, \ldots, G$$

$$\eta_{gh} = \frac{\beta_{gh}}{s_g} + s_h \quad \text{for } g, h = 1, \ldots, G; g \neq h$$

Note that the labor demand elasticities here are *gross price elasticities* that measure substitution along the utilized labor isoquant holding the total labor input L (i.e., 'output' for the labor cost sub-model) constant. Another commonly used measure for labor demand elasticities is *net price elasticities* where output Y is held constant. Given L, for example, an increase in the wage of age group *g*, w_g, will lead to a decrease in demand for labor in the same group, L_g *(gross substitution)*. Following a resulting rise in the total price of labor P_L, aggregate labor L will decline and thus the L isoquant will shift inward *(net substitution)* at the new equilibrium.[7] Thus, the net price elasticities tend to be more negative than the gross price elasticities (Hamermesh, 1996).

3.2 Regularity conditions

A regularity check is necessary because a failure to comply with certain regularity conditions would result in biased elasticity estimates. Particularly to assess cost and production efficiencies for a sector or an individual firm, estimated cost and production functions must satisfy theoretical conditions. Otherwise, efficiency measures cannot be correctly interpreted since irregular shapes of these functions could result in over- or under-estimated efficiency measures (Sauer *et al.*, 2006; Henningsen and Henning, 2009). Among all the theoretical properties, monotonicity and concavity require special attention since these conditions are rather complex to implement and violation of the two conditions could result in theoretically and empirically inconsistent parameter estimates. In what follows, we briefly review requirements that a cost function must satisfy in theory.

As a result of the cost minimization, a cost function should be non-decreasing, linearly homogenous, concave and continuous in input prices (Varian, 1992). By Young's theorem, the twice continuously differentiable cost function requires a symmetric Hessian matrix as well. Homogeneity

[7] See Berndt and Wood (1979) for a geometric interpretation of differences between gross and net price elasticities.

in prices and the symmetry of the second-order derivative matrix can be imposed on equations (4) and (5) as

$$\sum_g \alpha_g = 1, \sum_h \beta_{gh} = 0, \sum_g \gamma_g = 0;$$
$$\beta_{gh} = \beta_{hg}, \quad g \neq h. \tag{6}$$

Monotonicity, i.e. non-decreasing in prices, requires non-negative labor cost shares in equation (5) since

$$\frac{\partial c_{L,t}}{\partial w^r_{g,t}} = \frac{c_{L,t}}{w_{g,t}} \frac{\partial \log(c_{L,t})}{\partial \log(w^r_{g,t})} = \frac{c_{L,t}}{w_{g,t}} \left(\alpha_g + \sum_h \beta_{gh} \log(w^r_{g,t}) + \gamma_g t\right) > 0.$$

Concavity is satisfied if the Hessian matrix of the cost function is negative semi-definite at the optimal point. Diewert and Wales (1987) prove that the negative semi-definiteness of the Hessian is assured if and only if given the nonnegative shares, the matrix *M* with the following entries is negative semi-definite:

$$m_{gh} = \beta_{gh} + s_g s_h - s_g \delta_{gh} \quad \text{for } g, h = 1, \ldots, G$$

where $M = \{m_{gh}\}$; β is a parameter in the cost function; *s* is a labor cost share; $\delta_{gh} = 1$ if $g = h$ and 0 otherwise. The eigenvalues of the M matrix are used to determine concavity because a matrix is negative semi-definite if and only if its largest eigenvalue is less than or equal to zero.

Each condition has important implications on estimation procedures and elasticity estimates. First, notice that due to homogeneity and symmetry, the number of parameters is reduced by the number of restrictions, i.e. $(G^2 + G)/2 + 2$. Second, if monotonicity is violated, negative signs of estimated cost shares will lead to seriously biased elasticity estimates in terms of signs. There is a high chance that monotonicity will be violated particularly when shares for one or more factors are very small relative to those for the rest of the factors. Third, concavity essentially means negative own-price elasticities, provided the shares are non-negative. Negative semi-definiteness requires the first-order principal minors of the M matrix, i.e. diagonal entries equivalent to own-price elasticities ($\beta_{gg}/s_g + s_g - 1 \text{ for } g = 1, \ldots, G$), to be non-positive.

3.3 Estimation: a Bayesian SUR model

After a brief review of conventional estimation methods and their limitations, we show that a Bayesian approach offers more convenient way of estimation to restrict the labor demand model with monotonicity and concavity.

We initially estimate the labor cost function and the share equations together with homogeneity and symmetry imposed, using the seemingly unrelated regression (SUR) model of Zellner (1962).[8] Joint estimation of the cost function and the share equations yield more efficient estimates than the OLS estimation of the cost function alone (Christensen and Greene, 1976). Additionally, joint estimation ensures that the cost function and the share equations are consistent with each other. For example, if the share equations in equation (5) are estimated alone, it is not possible to recover the region fixed effects and the time trend in the cost function (equation 4) by the integration of the share equations.

The maximum likelihood method (ML) does not allow for imposition of monotonicity or concavity (Griffiths *et al.*, 2000). Constrained maximization of the likelihood function is rather complex and the algorithms used for the optimization frequently have convergence problems (Henningsen and Henning, 2009). Furthermore, linear programming is apt for linear inequality constrains like monotonicity, but is not implementable with non-linear inequality constraints like concavity (O'Donnell and Coelli, 2005). A strand of recent literature on stochastic frontier analysis, whose main objective is to measure production/cost efficiency of firms, addresses the regularity problem using a multiple-step estimation procedure (Henningsen and Henning, 2009) or a Bayesian estimation (O'Donnell and Coelli, 2005; Griffin and Steel, 2007).

As an alternative to the ML method, following Griffiths *et al.,* (2000), we use a Bayesian SUR model to simultaneously estimate the translog unit labor cost function and the share equations with homogeneity, symmetry, monotonicity and concavity. Monotonicity is imposed at every data point (locally) whereas homogeneity and symmetry are restricted at any arbitrary point (globally). When imposed *globally*, it is known that concavity destroys the second-order flexibility of the translog function (Diewert and Wales, 1987). As a result, concavity is generally imposed only locally at a single or multiple reference points, which may result in concavity holding at many points, but still maintaining the flexibility (Ryan and Wales, 2000). Therefore, following Ryan and Wales (2000), we

[8] One of the share equations is dropped due to the singularity of the covariance matrix. In addition, we use the maximum likelihood (ML) method to ensure that estimates are invariant to the choice of the omitted equation.

impose concavity at a single point where labor demand elasticities are measured, i.e. a mean vector of predicted labor cost shares. Later, we check concavity *ex*-post for every data point.

To obtain a sequence of sample parameter vectors, the Metropolis-Hastings (MH) algorithm is used because it can be computationally more efficient in the Bayesian SUR model than other popular algorithms such as the Gibbs sampling (Griffiths *et al.*, 2000). The procedure of the MH algorithm is described below:

Step 1: Set initial values for a parameter vector $\boldsymbol{\lambda}^{(\mathbf{0})} = [\boldsymbol{\alpha}^{(\mathbf{0})}, \boldsymbol{\beta}^{(\mathbf{0})}, \boldsymbol{\gamma}^{(\mathbf{0})}, \boldsymbol{\mu}^{(\mathbf{0})}, \theta^{(0)}]'$ where $\boldsymbol{\alpha}$ is a vector of parameter α's and the same applies to $\boldsymbol{\beta}, \boldsymbol{\gamma}$ and $\boldsymbol{\mu}$; θ is a parameter on the time trend. The values are chosen so as to satisfy homogeneity, symmetry, monotonicity and concavity. Set $n = 1$.

Step 2: Set $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(\boldsymbol{n}-\mathbf{1})}$

Step 3: Draw a candidate $\tilde{\boldsymbol{\lambda}}$ from a *proposal* density $N(\boldsymbol{\lambda}, c\boldsymbol{\Omega})$ where c is a constant and $\boldsymbol{\Omega}$ is a variance-covariance matrix estimated from the ML method with homogeneity and symmetry imposed. [9]

Step 4: Evaluate monotonicity at every data points and concavity at the mean of fitted labor cost shares using $\tilde{\boldsymbol{\lambda}}$. If either monotonicity or concavity is violated, update $\boldsymbol{\lambda}^{(\boldsymbol{n})} = \boldsymbol{\lambda}$, set $n = n + 1$ and go to Step 2. Otherwise, proceed to Step 5.

Step 5: Calculate $\alpha = \min(\frac{f(\tilde{\boldsymbol{\lambda}}|\boldsymbol{y})}{f(\boldsymbol{\lambda}|\boldsymbol{y})}, 1)$ where $\boldsymbol{y}$ is a vector of observations on a dependent variable; $f(\boldsymbol{\lambda}|\boldsymbol{y})$ is the *marginal* posterior density of $\boldsymbol{\lambda}$.[10]

Step 6: Accept $\tilde{\boldsymbol{\lambda}}$ with probability α and set $\boldsymbol{\lambda}^{(\boldsymbol{n})} = \tilde{\boldsymbol{\lambda}}$. Set $n = n + 1$ and go to Step 2.

The constant *c* in Step 2 is determined by trial and error so that the acceptance rate for $\tilde{\boldsymbol{\lambda}}$ ranges from 0.1 to 0.4. Depending on convergence, 100,000 to 400,000 samples are drawn for each sector

[9] The objective of the Bayesian method is to obtain samples for statistical inference from a target (posterior) distribution. However, since a target density is often analytically intractable, the MH algorithm generates a sequence of samples from a proposal density instead. In the limit, these samples follow the target density. See Chib and Greenberg (1996) for more details on the MH algorithm.

[10] When conventional non-informative prior distributions (for example, the inverted-Wishart distribution for the variance-covariance matrix) are assumed, the marginal posterior density $f(\boldsymbol{\lambda}|\boldsymbol{y})$ is proportionate to the determinant of the variance-covariance matrix for the errors in the SUR model. See appendix A for more details on joint, conditional and marginal postrior density functions for Bayesian inference in the SUR model.

and the first 10,000, 20,000 or 30,000 samples are discarded for a burn-in. Then, every 100th, 200th or 300th observation is redrawn in the remaining samples (thinning).

4 Data

This section provides an overview of the data used for empirical estimation of labor demand model. The data show that aging population has contributed to the rising labor cost share for elderly workers over the last decade and that youth and elderly workers are concentrated on less physically demanding sectors.

4.1 The American Community Survey (ACS)

We use the American Community Survey (ACS) Public Use Microdata (PUMS) compiled by the Census Bureau because it is the most comprehensive publicly available data. In the ACS, an individual generally represents 100 people while in another popular survey, the March Current Population Survey (CPS) by the Bureau of Labor Statistics (BLS), a sample represents more than 1,000-1500 individuals.

Based on the 2000-2013 ACS PUMS, we aggregate the number of employees and mean annual pre-tax wages and salary per employee by state (excluding Alaska and Hawaii) and by sector for each survey year. [11] An individual is mostly a 1-in-100 random sample except for a 1-in-240 sample from 2001 to 2004. Table 1 presents 45 sectors reclassified from the 3-digit North American Industry Classification System (NAICS). An employed person is grouped by their age: 1) a youth worker aged 16-24 who participates the labor market at an early stage; 2) a worker aged 25-44 as the most actively working group and 3) a worker aged 45-64 who is at around the peak of their career and subsequently preparing for retirement; and 4) an elderly worker aged 65 and over who continue working or is reattached to the labor market after retirement. The final samples used for analysis only include private wage and salary workers with non-zero labor income: Armed Forces, state, local and federal government employees, and self-employed workers are excluded.[12]

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[11] The data can be downloaded from the IPUMS USA, the Minnesota Population Center, University of Minnesota (https://usa.ipums.org/usa/; Ruggles *et al.*, 2010). According to the Employment Cost Trends (ECT) compiled by the BLS, wages and salaries make up around 70 percent of employee compensation costs and the remaining 30 percent is comprised of benefits such as health insurance, paid leave, legally required benefits, retirement and savings, and etc. However, neither the ACS nor the ECT provides comprehensive benefits data by worker's age.

[12] Self-employment in the ACS includes both the unincorporated (a dominant type) and incorporated self-employed while the CPS treats the incorporated self-employed as wage and salary workers.

<< *Insert table 1 here* >>

4.2 Characteristics of labor cost by age

Figure 2 shows that the labor cost shares for the 45-64 and 65+ age groups have been constantly rising. The share of labor costs for the two oldest groups rose to 50% in 2013, from 39% in 2000. This rise is attributed to the increases in employment and wages for the two groups. First, as baby boomers age, employment for the 45-64 and 65+ workers increased 40% and 70% since 2000 to reach 38.1 and 4.5 million, respectively, in 2013. However, employment for the rest younger workers declined 1% over the same period. Second, the oldest workers' real labor income, in particular, shows a large gain of 41% between 2000 and 2013. [13] Annual wage for the 45-65 group rose 3% while wages for the 25-44 and 16-24 groups fell 14% and 4%, respectively.

The rapidly rising wage for the elderly workers can be characterized, as seen in Section 2, by "a rise in labor force participation of high-skilled full-time workers aged 65 and over." The youth workers, on the other hand, experienced falling wage as a result of a rising share of part-time workers, combined with a decline in labor force participation possibly to pursue higher education.

<< *Insert figure 2 here* >>

Figure 3 shows that the youngest and oldest age groups tend to work predominantly in service sectors.[14] It also indicates that they are less likely to work in physically demanding industries such as construction and manufacturing than the middle age groups. Food services show the highest employment share for the youngest group (45%) while membership organizations and private household services have the largest employment share for the oldest group (11%).

<< *Insert figure 3 here* >>

5 Results

In this section, three sets of results are presented. First, parameter estimates are presented focusing on an implication from methodological differences between the models. Second, a complete assessment of monotonicity and concavity is provided for all sectors. Third, we report labor demand elasticity estimates and offer a simple simulation exercise to measure the effects of relative wage

[13] In the CPS, median inflation-adjusted weekly earnings for wage and salary workers show similar trends over the 2000-2013 periods.

[14] See appendix B for labor cost shares, employment and wages by sector for all age groups.

changes on employment. Throughout this section, homogeneity and symmetry are globally imposed so that these properties hold at *any* input prices. For the Bayesian models, monotonicity is imposed at every data point while concavity is restricted only at the mean of predicted labor cost shares where wage elasticities of labor demand are evaluated.

5.1 Parameter estimates[15]

Table 2 presents non-Bayesian and Bayesian parameter estimates for the translog cost function and share equations. For illustration, we choose membership organizations and household services (sector 45) with the highest employment share for elderly workers.

The results show that including share equations and imposing monotonicity generally incur considerable changes in parameter estimates. Examining from column 1 through 6 in table 2, we find that the first large changes in parameter estimates occur when the shares are included in the estimation. Once the share equations are present, estimates stay little changed regardless of whether the cost function is added (column 2 to 4). The second large changes occur when monotonicity is imposed. It does not seem that imposing concavity in addition to monotonicity causes changes in estimates to any great extent.

We find that imposing theoretical requirements does not incur significantly large losses of prediction errors in the Bayesian models. To compare *ex post* prediction performances between the Bayesian and non-Bayesian models, mean absolute errors (MAEs) are calculated. The Bayesian SUR with monotonicity and concavity (column 6) generally shows only a little larger MAEs for predicted values of the cost and shares than the SUR model with no restriction (column 3). Meanwhile, in the Bayesian model with all theoretical requirements, every data point meets monotonicity and only 11 percent of total observations violate concavity while 2 percent violates monotonicity, and 57 percent fail to comply with concavity in the SUR model. Similar patterns are also found in the other sectors.

For sector 45, the non-Bayesian OLS and SUR models clearly violate monotonicity and concavity. Out of 685 observations, 8.2 percent violates monotonicity in the cost-only OLS model (column 1) and 1.5 percent in the SUR models (column 2 & 3), mostly occurring in the fitted labor cost shares for youth and elderly workers. As for concavity, 49 percent of observations violate concavity in the

[15] State fixed effects were initially explored, but majority of sectors showed a fair amount of insignificant state fixed effects. Thus, region fixed effects were scaled down to the four Census regions, i.e. Northeast, Midwest, South and West. Time dummy variables accounting for the recent financial crisis in the US (2008 and 2009) did not significantly change the results, and thus they were not included in the final specification.

share-only SUR model and 57 percent in the cost-share joint SUR model. Particularly, all data points predicted by the cost function alone violate concavity. The Bayesian SUR model with no restriction but homogeneity and symmetry (column 4) essentially features the same estimates and the same number of observations that violate regularity conditions as the non-Bayesian SUR model (column 3).

Among the non-Bayesian models, the evaluation of regularity conditions and goodness-of-fit justifies the need for *simultaneous* estimation of a cost function and factor share equations, as discussed in section 3.3. First, for sector 45, when the share equations are estimated together with the cost function, the percentage of observations in violation of monotonicity and concavity significantly declines compared to the cost-only model. Second, the cost-share joint SUR model yields the best goodness-of-fit according to the Bayesian information criterion (BIC). These two findings are also true for majority of industries.

<< *Insert table 2 here* >>

5.2 Evaluation of monotonicity and concavity

In figure 4, monotonicity and concavity are evaluated at *every* data point for the cost-share joint models by sector. As a benchmark model, we present the SUR model without *a priori* monotonicity and concavity conditions in the form of bar graphs. Note that when the two conditions are not imposed, 95 percent of total samples meet monotonicity while concavity holds only in 30 percent of observations. It is commonly found in the literature on technology that concavity is more often violated than monotonicity (Barnett, 2002).[16] Also recall that concavity is satisfied conditionally on monotonicity.

Figure 4 shows that imposing monotonicity results in an improvement in concavity to a great extent. Overall, when only monotonicity is imposed at every data point, the share of concavity-satisfying samples increases to 69 percent, up from 30 percent in the SUR model. Furthermore, imposing concavity on top of monotonicity makes extra 10 percent of samples satisfy concavity so that 79 percent of samples comply with concavity in the fully restricted Bayesian model.

[16] Barnett (2002) further explains that since monotonicity is less often violated, researchers commonly impose only curvature in practice.

Figure 4 confirms that the imposition of concavity at a single point does improve concavity at other data points, as Ryan and Wales (2000) find.[17] When concavity is imposed only at the mean shares, the overall share of concavity-satisfying samples increases to 79 percent from 69 percent. However, represented by the distance between a square marker and a bar in the graph, the degree to which concavity improves varies considerably by sector. For example, sector 38 (health care) is one of the few sectors that shows a great improvement in concavity while concavity imposition at a single point has modest or little effects on other points in many of the remaining sectors.

It is worth noticing that the primary and secondary sectors are more likely to satisfy concavity than the tertiary sectors in our most preferred Bayesian model with all restrictions. In other words, cost frontiers inferred from observed wages and employment in the agricultural and manufacturing sectors are more theoretically well-behaved than those in the service sectors.

<< *Insert figure 4 here* >>

5.3 Labor demand elasticity estimates

Figure 5 presents the distributions of own-price labor demand elasticities for all sectors by estimation method. Wage elasticities of labor demand are evaluated at the mean of predicted labor cost shares. Complete sets of own- and cross-price elasticities for the fully restricted Bayesian model are reported in appendix C.

Elasticity comparison by method shows that negativity of own-price elasticities are guaranteed only if monotonicity and concavity are satisfied. Considering the fact that many empirical studies find these two conditions frequently violated, unrestricted models are likely to generate elasticity estimates that lack not only theoretical consistency but also empirical feasibility. As panel (a) shows, we cannot exclude the possibility of numerous large positive own-price elasticities without the imposition of the theoretical conditions.

Examining elasticity estimates reveals that the Bayesian model with all theoretical requirements in panel (f) predicts *elastic* labor demand for youth and elderly workers. More specifically, labor demand for elderly workers is the most elastic, a median of -0.71, with small variation across sectors. Labor demand for youth workers is the second most elastic, -0.60, but with much larger variation by sector.

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[17] An empirical example in Ryan and Wales (2000) shows that choosing one concavity-restricted point could make all points satisfy concavity. Our finding suggests that the choice of a restriction point affects the degree to which concavity holds at other points.

For the remaining two mid-aged groups, labor demand elasticities are similar, -0.14 for the 25-44 group and -0.13 for the 45-64 group with smaller variation across sectors.

Elastic labor demand for youth and elderly workers estimated from the fully restricted Bayesian model is more empirically and theoretically coherent than the other approaches. Both non-Bayesian and Bayesian models consistently estimate elastic labor demand for the oldest group compared to other age groups. Particularly elastic labor demand for youth workers containing teenage workers has long been supported in past empirical studies despite no consensus for other age groups (Hamermesh and Grant, 1979). However, some of the models with no restrictions yield *inelastic* elasticity estimates for youth workers.

One can naturally ask why many past studies on labor demand for youth workers neglected regularity conditions other than homogeneity and symmetry. Given that the fact that labor cost share for youth workers are relatively large in 1980s through the early 2000s, showing a downward trend from 15 percent to 9 percent[18], we can suspect that monotonicity, in particular, was likely to be satisfied in labor demand studies using the data for those periods. Furthermore, smooth time series data with highly aggregated sectors might have reduced the probability of violating monotonicity and concavity.

In figure 6, we characterize own-price labor demand elasticities for youth and elderly workers by sector. The scatter plot shows that labor demand for youth and elderly workers tend to be elastic in the service sectors where employment for these groups is concentrated. By contrast, labor demand for the same groups is inelastic in the more physically demanding sectors such as construction and manufacturing.

<< *Insert figure 5 and figure* 6 *here* >>

Figure 7 shows that all age-group employee pairs except for the youth-elderly pair are substitutes, i.e. positive cross-price elasticity.[19] According to the estimates, for example, wage subsidies for hiring applicants aged 65 and over, say, equivalent to the amount of 10 percent of market wage, would incentivize private employers to hire more of the age-group workers by 7 percent (a median of

[18] These figures are based on the aggregate employment and wage at the US level in the CPS data.

[19] We are measuring the effects of input price on quantity demanded: two inputs are *p*-substitutes if $\eta_{gh} = \frac{\partial \log X_g}{\partial \log w_h} > 0$; *p*-compliments, otherwise. By contrast, *q*-substitute ($\epsilon_{gh} < 0$) or *q*-compliment ($\epsilon_{gh} > 0$) are based on the cross-demand elasticity of factor price ($\epsilon_{gh} = \frac{\partial \log w_g}{\partial \log X_g}$). In the case of three or more inputs, equal signs for η_{gh} and ϵ_{gh} are not guaranteed (Hamermesh, 1996).

η_{44}'s), resulting in an increase in the employment of the youngest workers by 4 percent (η_{14}), while the 25-44 and 45-64 age-group workers would be substituted with the 65+ age-group workers by 3 percent (η_{24}) and 2 percent (η_{34}), respectively.

To comprehensively evaluate the employment effects of wage decline in each age group, a simple simulation exercise is conducted in figure 8 by taking into account own- and cross-wage elasticities of labor demand. Each box plot represents a distribution of employment changes for 45 sectors in response to negative wage shock by 10 percent. The simulation shows that real wage declines for the youngest and oldest workers lead to a net positive growth in total employment, resulting from a larger contribution from own-price labor demand than from cross-price demand while wage reduction for the two middle age groups induces job losses in total.

<< *Insert figure 7 and figure 8 here* >>

6 An application to a regional input-output model

Following an investigation in section 5.3 on the impact of relative wages changes on employment, a question that naturally arises centers on the economy-wide impact of distributional changes in the heterogeneity of labor (or households more broadly). For an empirical exploration to this question, we modify Miyazawa's extended input-output framework (Miyazawa, 1968) to account for heterogeneity in age of consumers and workers at a regional level. Miyazawa's approach provides a simple yet very useful framework that facilitates analysis of endogenous, heterogeneous households once consumption and income data disaggregated by household characteristics become accessible. In this section, we evaluate the sensitivity of a Chicago economy to changes in age structure, represented by economic multipliers, following a description of the Miyazawa's model and the data used.

6.1 Miyazawa's extended input-output model

The input-output model in Miyazawa (1968) is originally constructed for three regions in Japan where the household sector in each region is endogenous. The Miyazawa's approach is "the most parsimonious" extended input-output formulation in that an extension of multiple household sectors is based solely on an input-output table rather than a social accounting matrix (SAM) (Hewings *et al.*, 2001). As such, Pyatt (2001) claims that the Miyazawa multipliers should be interpreted as *factorial* income multipliers involved with wage and salary payments in an input-output table as distinguished from *institutional* income multipliers based on a SAM. The Miyazawa system is specified as the following:

$$\begin{bmatrix} x_{n\times 1} \\ \cdots \\ y_{q\times 1} \end{bmatrix} = \begin{bmatrix} A_{n\times n} & \vdots & C_{n\times q} \\ \cdots & \cdots & \cdots \\ V_{q\times n} & \vdots & 0_{q\times q} \end{bmatrix} \begin{bmatrix} x_{n\times 1} \\ \cdots \\ y_{q\times 1} \end{bmatrix} + \begin{bmatrix} f^*_{n\times 1} \\ \cdots \\ g_{q\times 1} \end{bmatrix} \tag{7}$$

where n is the number of sectors; q is the number of household groups; x is a vector of output; y is a vector of total income; A is a direct requirement coefficient matrix; V is a labor income coefficient matrix; C is a consumption coefficient matrix; f^* is a vector of exogenous final demand; g is a vector of exogenous income.

We can easily show that solving equation (7) for x and y yields

$$\begin{bmatrix} x \\ \cdots \\ y \end{bmatrix} = \begin{bmatrix} B(I + CKVB) & \vdots & BCK \\ \cdots & \cdots & \cdots \\ KVB & \vdots & K \end{bmatrix} \begin{bmatrix} f^* \\ \cdots \\ g \end{bmatrix} \tag{8}$$

where B is a traditional Leontief inverse matrix, i.e. $B = (I - A)^{-1}$; $K = (I - L)^{-1}$ for $L = VBC$. The K matrix is the "interrelational income multiplier" matrix, as Miyazawa defines, which indicates how much income in one group is generated by a unit of income increase in the other group. The matrix of "multi-sector income multipliers" KVB indicates how much income in one group is generated by a unit of final demand increase in one sector.

6.2 Data construction for the Miyazawa analysis

The Miyazawa multipliers in equation (8) consist of the coefficient matrices of direct requirement (A), labor income (V) and consumption (C). The A matrix can be directly derived from the input-output table for Chicago (the 2009 base year).[20] To obtain the V and C matrices disaggregated by age group, we use age-specific labor and consumer demand models since only aggregate labor income and consumption by sector are available in the Chicago input-output table. In the remainder of this subsection, we elaborate on the procedures of disaggregating the V and C matrices by age.

To disaggregate total employee compensation in each sector by age group, we use the age-group specific labor cost share equations for Illinois estimated in the preceding sections.[21] The original

[20] The Chicago region in this study includes seven counties in Illinois: Cook, Du Page, Kane, Kendall, Lake, McHenry, and Will. The input-output table for Chicago is constructed by aggregating those county-specific input-output tables from IMPLAN. Sectors in the IMPLAN input-output tables are recategorized to match with 45 sectors in table 1. Employee compensation includes wages and salaries, benefits and non-cash compensation.

[21] Population and employment in the Chicago region account for 70 percent of total population and employment in

1×45 employee compensation vector from the input-output table is transformed into a 4×45 matrix where the (*i,j*)th entry shows compensation paid to workers in different age group *i* in sector *j*. In the final *V* matrix, labor income by age group is expressed as a share of output for each sector.

To estimate the consumption coefficient matrix *C*, we first disaggregate the original 45×1 column vector of household consumption into a 45×4 matrix. The (*i,j*)th entry of the 45×4 matrix represents consumption of households in age group *j* on good *i*. Following Kim *et al.* (2015), the almost ideal demand system (AIDS; Deaton and Muellbauer, 1980) model is used to estimate age-group-specific consumer demand for Chicago.[22] Next, each entry in the 45×4 sector-by-age-group matrix is divided by the column sum to represent the consumption share of total expenditure. Finally, a consumption coefficient matrix *C* is generated by multiplying each column of shares by average propensity to consume of the corresponding age group, i.e. the ratio of total expenditure to total income.[23] Hence, each entry in the C matrix indicates the consumption share of *total income* (*y*).

One might argue that a bias could occur due to a unit mismatch between an individual worker as a labor income earner and a household as a consumer. Unfortunately, data on expenditure by individual family members are not available for the Chicago region in the Consumer Expenditure Survey (CES), the data on which the consumer demand model is based. A bias occurs when two or more labor income earners in a household are in different age brackets. However, considering that age brackets used in this paper are wide and that age difference between a head of family and his/her spouse is relatively small in many cases, the bias from the unit mismatch would not be large.

6.3 Comparative statics: the effects of aging population

This subsection presents the Miyazawa multipliers and assesses how aging population would affect these multipliers in 2020 compared to 2009. To identify the effects of age distribution changes alone, we assume that production technology, the relative prices of goods, the relative wages of workers in different age groups, and in- and out-migration rates in Chicago do not change from the base year 2009 and thereafter. Therefore, the changes in the labor income coefficient matrix *V* can be

Illinois. Since the estimated labor cost share equations are only state-specific, we assume that the estimates for Illinois are good approximates for the Chicago region.

[22] More specifically, Kim *et al.* (2015) estimate the AIDS model for five nondurable goods and services using the data from the Consumer Expenditure Survey (CES). The five types of expenditures are then disaggregated into consumption in 45 sectors via a bridge matrix. Durable goods consumption is allocated across age groups, proportional to the number of households in each group.

[23] Average propensity to consume by age group is calculated from the 2009 CES for the US. It is worth mentioning that average propensity to consume significantly varies by age group: 1.11 for the under-25 group, 0.77 for the 25-44 group, 0.73 for the 45-64 group, and 0.93 for the 65+ group.

attributed to rising or falling employment shares for age-group workers represented by the age-group-specific linear time trends γ's in the share equations in equation 5. To calculate the consumption coefficient matrix *C* of 2020, we use the baseline forecasts for aggregate personal income and the number of households by age from the extended regional econometric input-output model for Chicago in Kim *et al.* (2015).

The interrelational income multipliers *K* in table 3 show in a column direction that given a labor income shock, the 25-44 and 45-64 groups are expected to receive much larger induced income than the youth and elderly groups. In 2009, for example, a $1 increase of wage and salary income in the oldest group induces 5 cents in the youngest group, 36 cents in the two middle age groups, and 4 cents in the oldest group. This is simply due to the fact that the two middle age groups account for the largest employment shares.

A row direction indicates income inducement generated by a $1 increase of wage and salary income in all groups (one should be subtracted for the principal diagonal elements). Higher induced income generated by the youngest and oldest groups is due to higher propensities to consume for these two groups, characterized by "earn less and spend more."

Table 3 also shows that aging population increases induced income that the 45-64 and 65+ groups receive in 2020 while the other younger groups experience a decline in induced income. This can be explained by the population projection that expects a large positive growth in the population of the 45-64 and 65+ groups.[24] It is, however, important to note that the Miyazawa analysis suggests that with aging population, the entire local economy could suffer from a decline in additional income generated by an income shock.

<< *Insert table 3 here* >>

The multi-sector income multipliers *KVB* in table 4 show that sectors with higher employment share for a specific age group (see figure 3) tend to generate higher income inducement for the age group. For example, among the eight aggregate sectors, a $1 direct demand impact from the service sector generates the highest induced income for the youngest and oldest group. It is the construction sector that generates the highest income inducement for the middle age groups.

[24] According to the Census Bureau, the Illinois population aged 45-64 and 65+ is projected to increase 14% and 61%, respectively, between 2000 and 2030 while the total population is expected to grow only 8% over the same period.

Comparing multi-sector income multipliers between 2009 and 2020 suggests that increasing employment shares for older workers result in higher multipliers for the 45-64 and 65+ groups and smaller multipliers for the 16-24 and 25-44 groups. Recall that the linear time trends in labor share equations vary by age group and by sector. Therefore, the degrees to which multipliers in each cell change depend on the corresponding time trend estimates that represent changes in age-specific employment by sector. The construction sector, for example, shows the largest decline (-0.71 percent) in total income inducement from 2009 to 2020 since construction employment for young workers are predicted to fall more rapidly than employment of young workers in other sectors.

<< *Insert table 4 here* >>

In table 5, output multipliers are compared among sectors when a household sector is treated as either exogenous or endogenous. Type I multipliers are the column sums of the Leontief inverse $B = (I - A)^{-1}$ while type II multipliers are the column sums of $B(I + CKVB)$. Type II multipliers of 2009 imply that a $1 increase in total household consumption generate $1.511 of indirect and induced income on average, whereas type I multipliers shows a dollar increase in demand generates only $0.563. Note that output multipliers show larger declines than multi-sector income multipliers in percentage terms. These findings for the output multipliers continue the "hollowing-out" trend noted by Hewings *et al.* (1998) that was attributed to the increasing spatial fragmentation of production in the US economy.

<< *Insert table 5 here* >>

7 Conclusions

In this paper, we estimate wage elasticities of labor demand by age using a Bayesian SUR model. This approach is relevant for a wide spectrum of demand analysis since it facilitates the imposition of regularity conditions implied by economic theory. When applied to the ACS data, the Bayesian approach shows that the labor demand for youth workers is elastic. This finding is empirically consistent with past empirical studies that highlight elastic labor demand for youth workers. Labor demand for the elderly workers is also found to be elastic with smaller sectoral variation, relative to large variation in wage elasticities of labor demand for youth workers across sectors.

Additionally, we present an application of the labor demand model used together with the consumer demand model proposed by Kim *et al.* (2015) to the Miyazawa extended input-output model. As an illustration, the effects of changing age structure on the Chicago economy are evaluated. The results suggest that *ceteris paribus* aging population attributes to lowering aggregate economic multipliers of a regional economy mainly because the number of elderly workers who earn less labor income than younger groups is expected to grow more rapidly.

This paper provides a good example where empirical consistency can be acquired by strengthening theoretical coherence without significantly incurring additional costs such as loss of prediction accuracy. Additional implications of main findings in this paper are as follows. Monotonicity and concavity must be checked and addressed particularly in the case where one or more factor shares are so small that monotonicity is in doubt. Moreover, it is desirable for a static factor demand model with a translog cost function to simultaneously estimate a cost function and factor shares. The share equations alone do not contain enough information to recover the corresponding cost structure.

One policy implication is that a labor policy that intends to influence the price of labor for youth workers needs to be differentiated by sector, while a labor policy targeting the oldest group's wages is expected to produce similar degrees of changes in labor demand across sectors. In addition, a simulation suggests that the effectiveness of wage policy in terms of total job creation varies depending on a target age group when own- and cross-wage elasticities of labor demand are taken into account.

An interesting extension to this study for future research is to include not only wage and salary but also benefits in the input prices since the employer's cost of providing retirement benefits and health insurance is much higher for older workers than for younger workers (Munnell and Sass, 2008). In addition, the inclusion of institutional income (factor income plus non wage and salary income) might alter the results. Further, embedding the results in a full econometric input-output model would provide important insights into the way how changes in economic structure, demographic structure and the interactions between income generation and consumption affect forecasts, compared to those using a single representative household.

References

Atkinson, J., & Manning, N. (1995). A survey of international energy elasticities. In T. Barker, P. Ekins, & N. Johnstone (Eds.), *Global Warming and Energy Demand*. New York, NY: Routledge.

Barnett, W. A. (2002). Tastes and technology: Curvature is not sufficient for regularity. *Journal of Econometrics*, *108*(1), 199–202.

Berndt, E., & Wood, D. (1979). Engineering and Econometric Interpretations of Energe-capital complementarity. *The American Economic Review*, 342–354.

Bishop, P., & Gripaios, P. (2010). Spatial Externalities, Relatedness and Sector Employment Growth in Great Britain. *Regional Studies*, *44*(4), 443–454.

Chib, S., & Greenberg, E. (1996). Markov Chain Monte Carlo Simulation Methods in Econometrics. *Econometric Theory*, *12*(3), 409–431.

Christensen, L. R., & Greene, W. H. (1976). Economies of scale in US electric power generation. *The Journal of Political Economy*, *84*(4), 655–676.

Christensen, L. R., Jorgenson, D. W., & Lau, L. J. (1971). Conjugate Duality and the Transcendental Logarithmic Production Function. *Econometrica*, *39*, 255–256.

Colombo, G. (2010). Linking CGE and microsimulation models: A comparison of different approaches. *International Journal of Microsimulation*, *3*(1), 72–91.

Deaton, A., & Muellbauer, J. (1980). An Almost Ideal Demand System. *The American Economic Review*, *70*(3), 312–326.

Diewert, A. W. E., & Wales, T. J. (1987). Flexible Functional Forms and Global Curvature Conditions. *Econometrica*, *55*(1), 43–68.

Felipe, J., & McCombie, J. (2012). Problems with regional production functions and estimates of agglomeration economies: A caveat emptor for regional scientists. *Levy Economics Institute of Bard College Working Paper*, (725).

Ferguson, B. S. (1986). Labour force substitution and the effects of an ageing population. *Applied Economics*, *18*, 901–913.

Fuchs, V. R. (1964). Quality of Labor. *Productivity Trends in the Goods and Service Sectors, 1929-61: A Preliminary Survey*, 23–33.

Glaeser, E. L., Kallal, H. D., Scheinkman, J. A., & Shleifer, A. (1992). Growth in cities. *Journal of Political Economy*, *100*(6), 1126–1152.

Griffin, J. E., & Steel, M. F. J. (2007). Bayesian stochastic frontier analysis using WinBUGS. *Journal of Productivity Analysis*, *27*(3), 163–176.

Griffiths, W. E. (2003). Bayesian inference in the seemingly unrelated regressions model. In D. E. A. Giles (Ed.), *Computer-Aided Econometrics*. New York, NY: Marcel Dekker, Inc.

Griffiths, W. E., O'Donnell, C. J., & Cruz, A. T. (2000). Imposing regularity conditions on a system of cost and factor share equations. *Australian Journal of Agricultureal and Resources Economics*, *44*(1), 107–127.

Hamermesh, D. S. (1996). *Labor demand*. Princeton, New Jersey: Princeton University Press.

Hamermesh, D. S., & Grant, J. (1979). Econometric studies of labor-labor substitution and their implications for policy. *Journal of Human Resources*, *14*(4), 518–542.

Henningsen, A., & Henning, C. H. C. a. (2009). Imposing regional monotonicity on translog stochastic production frontiers with a simple three-step procedure. *Journal of Productivity Analysis*, *32*(3), 217–229.

Hewings, G. J. D., Okuyama, Y., & Sonis, M. (2001). Economic interdependence within the Chicago metropolitan area: A Miyazawa analysis. *Journal of Regional Science*, *41*(2), 195–217.

Hewings, G. J. D., Sonis, M., Guo, J., Israilevich, P. R., & Schindler, G. R. (1998). The hollowing-out process in the Chicago economy, 1975-2011. *Geographical Analysis*, *30*(3), 217–233.

Jorgenson, D. W., Jin, H., Slesnick, D. T., & Wilcoxen, P. J. (2013). An econometric approach to general equilibrium modeling. In *Handbook of Computable General Equilibrium Modeling 1* (pp. 1133–1212).

Kim, K., Kratena, K., & Hewings, G. J. D. (2015). The extended econometric input–output model with heterogeneous household demand system. *Economic Systems Research*, *27*(2), 257–285.

Kratena, K., Streicher, G., Temurshoev, U., Amores, A. F., Arto, I., Mongelli, I., … Andreoni, V. (2013). FIDELIO 1: Fully Interregional Dynamic Econometric Long-term Input-Output Model for the EU27.

Lee, K., Pesaran, M. H., & Pierse, R. G. (1990). Aggregation bias in labour demand equations for the UK economy. In T. Barker & M. H. Pesaran (Eds.), *Disaggregation in econometric modelling* (pp. 113–145). London, New York: Routledge.

Lichter, A., Siegloch, S., & Peichl, A. (2014). The own-wage elasticity of labor demand: a meta-regression analysis. *ZEW-Centre for European Economic Research Discussion Paper*, (14-016).

Maier, T., Monnig, A., & Zika, G. (2015). Labour demand in Germany by industrial sector, occupational field and qualification until 2025 – model calculations using the IAB / INFORGE model, *27*(1), 19–42.

Miyazawa, K. (1968). Input-Output Analysis and Interrelational Income Multiplier as a Matrix. *Hitotsubashi Journal of Economics*, *8*(2).

Mongelli, I., Neuwahl, F., & Rueda-Cantuche, J. M. (2010). Integrating a household demand system in the input–output framework. Methodological aspects and modelling implications. *Economic Systems Research*, *22*(3), 201–222.

Munnell, A. H. (2011). What is the average retirement age? *Center for Retirement Research*.

Munnell, A. H., & Sass, S. A. (2008). *Working Longer: The Solution to the Retirement Income Challenge*. Washington, D.C.: Brookings Institution Press.

O'Donnell, C. J., & Coelli, T. J. (2005). A Bayesian approach to imposing curvature on distance functions. *Journal of Econometrics*, *126*(2), 493–523.

Pyatt, G. (2001). Some Early Multiplier Models of the Relationship between Income Distribution and Production Structure. *Economic Systems Research*, *13*(February 2015), 139–163. doi:10.1080/09537320120052434

Ruggles, S., Alexander, T. J., Genadek, K., Goeken, R., Schroeder, M. B., & Sobek, M. (2010). Integrated Public Use Microdata Series: Version 5.0 [Machine-readable database]. Minneapolis: University of Minnesota.

Ryan, D. L., & Wales, T. J. (2000). Imposing local concavity in the translog and generalized Leontief cost functions. *Economics Letters*, *67*, 253–260.

Sauer, J., Frohberg, K., & Hockmann, H. (2006). Stochastic efficiency measurement: the curse of theoretical consistency. *Journal of Applied Economics*, *IX*(1), 139–165.

Slaughter, M. J. (2001). International trade and labor-demand elasticities. *Journal of International Economics*, *54*, 27–56.

Varian, H. R. (1992). *Microeconomic analysis* (3rd ed.). New York, NY: W. W. Norton & Company, Inc.

Zellner, A. (1962). An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias. *Journal of the American Statistical Association*, *57*(298), 348–368.

Appendix A. Likelihood function, prior and posterior distributions in the Bayesian SUR model

This appendix is to explain the specifications of model and distributions used in this study for Bayesian inference in the seemingly unrelated regressions (SUR) model. Further details can be found in Griffiths *et al.* (2000) and Griffiths (2003). The SUR model with *M* equations using a total of *T* observations for estimation is given by

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim N(\boldsymbol{0}, \boldsymbol{\Sigma_M} \otimes \boldsymbol{I_T})$$

$$\begin{bmatrix} \mathbf{y_1} \\ \mathbf{y_2} \\ \vdots \\ \mathbf{y_M} \end{bmatrix} = \begin{bmatrix} \mathbf{X_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X_M} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_\mathbf{M} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon_1} \\ \boldsymbol{\varepsilon_2} \\ \vdots \\ \boldsymbol{\varepsilon_M} \end{bmatrix}$$

where $\boldsymbol{y}$ is an $\mathrm{MT} \times 1$ vector of dependent variables; $\boldsymbol{X}$ is an $\mathrm{MT} \times \mathrm{K}$ matrix of explanatory variables where $\mathrm{K} = \sum_{i=1}^{M} k_i$;$\boldsymbol{\beta}$ is a $\mathrm{K} \times 1$ coefficient vector; $\boldsymbol{\varepsilon}$ is an $\mathrm{MT} \times 1$ vector of contemporaneously correlated random errors (i.e. $E[\varepsilon_{it}\varepsilon_{js}] = \sigma_{ij}$ if $t = s$ and 0 otherwise where $i, j = 1, \cdots, M;\ t, s = 1, \cdots, T$).

Under this specification, a likelihood function for $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ can be specified as

$$L(\boldsymbol{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}) = (2\pi)^{-MT/2} |\boldsymbol{\Sigma}|^{-T/2} \exp\{-0.5(\boldsymbol{y} - \boldsymbol{X\beta})'(\boldsymbol{\Sigma^{-1}} \otimes \boldsymbol{I_T})(\boldsymbol{y} - \boldsymbol{X\beta})\}.$$

The likelihood can be rewritten as

$$L(\boldsymbol{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}) = (2\pi)^{-MT/2} |\boldsymbol{\Sigma}|^{-T/2} \exp\{-0.5\mathrm{tr}(A\boldsymbol{\Sigma^{-1}})\}$$

where A is an $\mathrm{M} \times \mathrm{M}$ matrix with *(i,j)*th element $a_{ij} = (\boldsymbol{y_i} - \boldsymbol{X_i\beta_i})'(\boldsymbol{y_j} - \boldsymbol{X_j\beta_j})$.

A conventional noninformative joint prior for $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ is given by

$$p(\boldsymbol{\beta}, \boldsymbol{\Sigma}) = p(\boldsymbol{\beta})p(\boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-(M+1)/2}.$$

Hence, the Bayes' theorem yields the joint posterior density for $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$:

$$\begin{aligned} f(\boldsymbol{\beta}, \boldsymbol{\Sigma}|\boldsymbol{y}) &\propto L(\boldsymbol{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma})p(\boldsymbol{\beta}, \boldsymbol{\Sigma}) \\ &\propto |\boldsymbol{\Sigma}|^{-(T+M+1)/2} \exp\{-0.5\mathrm{tr}(A\boldsymbol{\Sigma^{-1}})\}. \end{aligned}$$

A *target* density, the marginal posterior density $f(\boldsymbol{\beta}|\boldsymbol{y})$ from which we want to draw samples for inference, can be written as

$$f(\boldsymbol{\beta}|\boldsymbol{y}) = \int f(\boldsymbol{\beta}, \boldsymbol{\Sigma}|\boldsymbol{y}) d\boldsymbol{\Sigma} \propto |A|^{-T/2}.$$

In the Metropolis-Hastings algorithms, candidates for $\boldsymbol{\beta}$, say $\widetilde{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\beta}}$, are actually sampled from a *proposal* density (for example, a multivariate normal distribution) whose limiting invariant distribution is the target distribution, and then if $\widetilde{\boldsymbol{\beta}}$ satisfies the regularity conditions, we accept $\widetilde{\boldsymbol{\beta}}$ with probability of $\alpha = \min(\frac{f(\widetilde{\boldsymbol{\beta}}|\boldsymbol{y})}{f(\widehat{\boldsymbol{\beta}}|\boldsymbol{y})} = \frac{|\tilde{A}|^{-T/2}}{|\hat{A}|^{-T/2}}, 1)$ where $\tilde{A} = [\tilde{a}_{ij}]_{\mathrm{M}\times\mathrm{M}} = (\boldsymbol{y_i} - \boldsymbol{X_i}\widetilde{\boldsymbol{\beta}}_{\boldsymbol{i}})'(\boldsymbol{y_j} - \boldsymbol{X_j}\widetilde{\boldsymbol{\beta}}_{\boldsymbol{j}})$.

Appendix B. Labor cost shares by age group by sector: 2000-2013 average

Sector	Labor cost shares (%)				Employment (thou. person)				Annual wages (thou. $2009)			
	16-24	25-44	45-64	65+	16-24	25-44	45-64	65+	16-24	25-44	45-64	65+
1 Livestock & Ot	**11.1**	48.3	36.1	4.5	176	406	250	37	12.7	24.1	29.2	24.8
2 Agri., Forestr	8.3	47.9	40.4	3.4	22	68	45	5	15.0	28.5	36.1	25.7
3 Mining	5.1	45.2	47.7	2.1	54	266	218	11	32.5	59.1	76.0	63.8
4 Utilities	2.3	39.2	**56.9**	1.6	40	308	366	14	27.8	61.2	74.9	54.5
5 Construction	7.3	**54.5**	36.4	1.8	966	3722	1933	113	20.3	39.1	50.3	42.6
6 Food & Kindred	5.5	46.8	45.8	1.9	170	646	511	29	16.1	35.7	44.2	32.7
7 Tobacco Prod.	3.5	46.8	47.7	2.0	12	67	53	3	20.8	50.2	65.2	43.2
8 Apparel & Text	4.2	45.5	47.1	3.1	52	301	268	21	16.8	31.4	36.6	31.2
9 Leather & Leat	4.6	50.8	39.4	**5.1**	2	8	6	1	14.7	38.0	41.3	37.5
10 Lumber & Wood	6.9	48.0	42.8	2.2	45	171	126	9	18.9	34.4	41.5	31.9
11 Paper & Allie	2.6	41.7	**53.7**	2.1	22	164	173	8	21.5	47.3	57.5	50.0
12 Printing & Pu	3.6	51.5	42.7	2.3	140	742	545	46	18.2	49.7	56.1	35.3
13 Petroleum & C	2.0	39.6	**56.4**	2.0	6	52	57	2	29.1	70.3	91.7	75.4
14 Chemicals & A	1.8	46.3	50.2	1.7	61	542	472	21	22.5	63.2	78.6	58.3
15 Rubber & Misc	4.1	46.2	47.4	2.3	45	243	204	11	18.8	39.5	48.2	43.1
16 Stone, Clay,	3.9	44.7	48.8	2.7	30	174	153	9	20.7	40.9	50.7	45.8
17 Primary Metal	3.3	41.8	52.6	2.3	32	212	218	10	23.5	45.1	54.9	51.1
18 Fabricated Me	4.5	44.4	48.0	3.0	109	555	496	35	20.8	40.1	48.4	43.5
19 Industrial Ma	3.3	43.5	50.6	2.6	85	527	510	30	22.5	47.9	57.7	50.2
20 Computer & ot	2.0	48.8	47.4	1.8	103	894	771	39	23.8	65.7	74.1	57.7
21 Transp. Equip	2.8	42.0	**53.2**	2.1	139	967	961	41	24.1	52.4	66.9	60.5
22 Furniture & R	6.2	49.6	41.8	2.4	47	214	153	10	18.8	33.1	39.0	32.8
23 Miscellaneous	3.3	48.1	46.3	2.3	88	491	405	27	18.6	49.0	57.2	42.0
24 Wholesale	3.6	49.2	44.5	2.7	375	1982	1489	130	18.9	48.6	58.5	40.1
25 Retail	**10.5**	49.7	36.7	3.1	3856	5938	4103	614	11.5	35.2	37.6	21.2
26 Air Transp.	2.0	43.8	52.6	1.7	25	213	191	9	18.8	49.0	65.5	42.6
27 Railroad Tran	4.5	48.3	45.4	1.8	152	614	459	28	16.4	43.7	55.0	35.0
28 Water Transp.	3.7	41.1	50.9	4.4	3	12	12	2	20.7	53.4	70.6	41.6
29 Truck Transp.	4.0	48.3	45.1	2.5	159	972	787	59	19.5	38.3	44.2	32.8
30 Transit & Gro	2.5	39.7	51.9	**5.8**	18	149	177	33	15.6	28.9	31.8	18.7
31 Pipeline Tran	3.3	37.7	**56.3**	2.7	1	8	9	1	36.7	65.5	86.5	66.1
32 Information	3.4	**55.2**	40.1	1.3	144	859	521	25	22.1	60.7	72.7	49.4
33 Motion Pictur	5.9	**59.1**	32.8	2.2	66	117	48	6	11.3	63.5	86.6	51.2
34 Finance & Ins	3.4	53.5	40.9	2.2	602	3279	2148	147	22.2	64.3	75.1	57.8
35 Real Estate	4.2	46.7	43.8	**5.2**	222	874	713	132	16.1	44.8	51.5	33.3
36 Professional	4.0	**54.5**	38.9	2.6	1287	5757	3358	328	18.1	54.7	67.0	45.3
37 Educational S	4.3	42.4	48.4	**4.8**	622	1612	1431	160	9.5	36.1	46.4	41.0
38 Health Care	3.7	47.4	45.9	3.0	1178	5888	4781	411	16.6	42.4	50.6	38.2
39 Social Assist	8.7	47.6	39.8	3.9	355	895	645	91	11.0	23.9	27.6	19.6
40 Arts, Enterta	**10.5**	51.2	34.7	3.6	516	737	466	92	10.0	34.3	36.7	19.6
41 Accommodation	8.9	50.2	37.8	3.1	252	627	423	50	12.4	28.1	31.4	21.6
42 Food Serv.	**23.6**	53.5	21.7	1.3	3257	2905	1029	96	8.8	22.3	25.5	16.2
43 Repair & Main	9.3	**54.1**	34.8	1.9	244	673	374	32	16.5	34.8	40.4	25.7
44 Personal & La	**10.6**	52.4	33.3	3.7	257	632	364	60	12.1	24.2	26.8	18.2
45 Membership Or	3.6	38.4	51.1	**7.0**	262	919	1048	249	10.8	32.8	38.3	22.1
Average	5.5	49.6	42.2	2.7	362	1053	744	73	14.0	43.2	52.0	33.7

Note: Figures in bold represent the five sectors with the highest shares given an age-group.
Source: Author's calculation based on the 2000-2013 ACS

Appendix C. Price-elasticities of labor demand by age group: Bayesian SUR estimates evaluated at fitted mean shares with monotonicity and concavity imposed[1)]

	Labor demand elasticity of															
	16-24 group				25-44 group				45-64 group				65+ group			
	w.r.t. Δwage of				w.r.t. Δwage of				w.r.t. Δwage of				w.r.t. Δwage of			
	16-24	25-44	45-64	65+	16-24	25-44	45-64	65+	16-24	25-44	45-64	65+	16-24	25-44	45-64	65+
	η_{11}	η_{12}	η_{13}	η_{14}	η_{21}	η_{22}	η_{23}	η_{24}	η_{31}	η_{32}	η_{33}	η_{34}	η_{41}	η_{42}	η_{43}	η_{44}
1 Livestock & Ot	-0.434	0.329	0.139	-0.034	0.085	-0.146	0.022	0.038	0.048	0.029	-0.131	0.054	-0.078	0.342	0.365	-0.629
2 Agri., Forestr	-0.621	0.183	0.443	-0.005	0.037	-0.143	0.047	0.058	0.107	0.056	-0.177	0.015	-0.011	0.595	0.127	-0.711
3 Mining	-0.690	0.628	0.023	0.040	0.074	-0.164	0.070	0.020	0.003	0.066	-0.088	0.019	0.078	0.338	0.341	-0.757
4 Utilities	-0.755	0.023	0.868	-0.136	0.001	-0.098	0.072	0.025	0.037	0.048	-0.098	0.013	-0.181	0.514	0.409	-0.742
5 Construction	-0.497	0.634	-0.096	-0.041	0.098	-0.336	0.219	0.019	-0.022	0.320	-0.317	0.019	-0.178	0.520	0.351	-0.693
6 Food & Kindred	-0.630	0.415	0.255	-0.039	0.058	-0.239	0.157	0.024	0.037	0.161	-0.219	0.021	-0.108	0.477	0.411	-0.779
7 Tobacco Prod.	-0.757	0.833	0.054	-0.130	0.077	-0.343	0.187	0.079	0.005	0.196	-0.194	-0.007	-0.151	0.991	-0.084	-0.756
8 Apparel & Text	-0.727	0.365	0.388	-0.026	0.041	-0.108	0.009	0.058	0.040	0.009	-0.076	0.027	-0.028	0.555	0.286	-0.813
9 Leather & Leat	-0.651	0.414	0.264	-0.027	0.057	-0.125	0.015	0.054	0.045	0.019	-0.125	0.061	-0.026	0.381	0.348	-0.703
10 Lumber&Wood	-0.524	0.344	0.192	-0.012	0.053	-0.187	0.100	0.033	0.033	0.110	-0.159	0.017	-0.028	0.487	0.230	-0.690
11 Paper & Allie	-0.572	0.358	0.374	-0.161	0.026	-0.232	0.168	0.038	0.021	0.129	-0.164	0.015	-0.188	0.608	0.309	-0.729
12 Printing & Pu	-0.799	0.584	0.290	-0.075	0.053	-0.120	0.055	0.011	0.029	0.061	-0.133	0.043	-0.124	0.207	0.700	-0.782
13 Petroleum & C	-0.784	0.115	0.794	-0.125	0.008	-0.032	0.025	-0.001	0.044	0.020	-0.130	0.066	-0.081	-0.005	0.786	-0.700
14 Chemicals & A	-0.600	0.859	-0.144	-0.115	0.047	-0.099	0.040	0.012	-0.007	0.037	-0.054	0.024	-0.144	0.272	0.595	-0.723
15 Rubber & Misc	-0.489	0.533	0.100	-0.144	0.053	-0.221	0.137	0.031	0.009	0.126	-0.157	0.021	-0.197	0.424	0.320	-0.547
16 Stone, Clay,	-0.646	0.430	0.237	-0.022	0.047	-0.231	0.172	0.012	0.024	0.159	-0.222	0.040	-0.030	0.155	0.539	-0.665
17 Primary Metal	-0.650	0.478	0.274	-0.102	0.046	-0.286	0.198	0.042	0.022	0.163	-0.202	0.018	-0.131	0.560	0.284	-0.713
18 Fabricated Me	-0.760	0.659	0.189	-0.088	0.081	-0.140	0.050	0.009	0.022	0.046	-0.118	0.051	-0.139	0.112	0.697	-0.671
19 Industrial Ma	-0.671	0.410	0.319	-0.058	0.037	-0.125	0.085	0.003	0.026	0.076	-0.146	0.044	-0.078	0.042	0.745	-0.709
20 Computer & ot	-0.524	0.280	0.281	-0.036	0.017	-0.224	0.216	-0.010	0.016	0.204	-0.266	0.046	-0.043	-0.186	0.950	-0.721
21 Transp. Equip	-0.731	0.713	0.155	-0.136	0.057	-0.258	0.195	0.005	0.010	0.160	-0.214	0.044	-0.188	0.093	0.926	-0.831
22 Furniture & R	-0.621	0.463	0.233	-0.075	0.066	-0.428	0.361	0.000	0.042	0.454	-0.579	0.083	-0.160	-0.001	0.979	-0.818
23 Miscellaneous	-0.284	0.113	0.145	0.027	0.009	-0.071	0.048	0.013	0.014	0.053	-0.087	0.020	0.041	0.249	0.339	-0.630
24 Wholesale	-0.053	0.119	0.038	-0.104	0.010	-0.061	0.011	0.040	0.003	0.012	-0.024	0.009	-0.155	0.727	0.158	-0.730
25 Retail	-0.027	0.128	-0.088	-0.012	0.029	-0.231	0.169	0.034	-0.026	0.219	-0.190	-0.003	-0.041	0.498	-0.036	-0.421
26 Air Transp.	-0.537	0.510	0.165	-0.138	0.033	-0.102	0.057	0.012	0.009	0.047	-0.086	0.030	-0.160	0.225	0.652	-0.716
27 Railroad Tran	-0.675	0.182	0.492	0.001	0.018	-0.134	0.094	0.022	0.050	0.097	-0.153	0.006	0.002	0.533	0.148	-0.683
28 Water Transp.	-0.706	0.247	0.416	0.043	0.038	-0.174	0.080	0.056	0.054	0.069	-0.148	0.024	0.040	0.343	0.175	-0.559
29 Truck Transp.	-0.628	0.266	0.479	-0.117	0.024	-0.056	-0.004	0.036	0.044	-0.004	-0.065	0.025	-0.169	0.584	0.389	-0.805
30 Transit & Gro	-0.707	0.170	0.607	-0.069	0.017	-0.100	0.042	0.041	0.045	0.030	-0.144	0.069	-0.034	0.200	0.462	-0.628
31 Pipeline Tran	-0.252	-0.007	0.119	0.140	-0.001	-0.097	0.071	0.027	0.017	0.056	-0.098	0.025	0.216	0.231	0.268	-0.715
32 Information	-0.810	0.494	0.305	0.011	0.039	-0.062	0.003	0.020	0.031	0.004	-0.044	0.009	0.026	0.603	0.203	-0.832
33 Motion Pictur	-0.303	0.319	-0.009	-0.007	0.082	-0.284	0.155	0.047	-0.004	0.244	-0.272	0.031	-0.019	0.507	0.215	-0.702
34 Finance & Ins	-0.230	-0.005	0.234	0.002	0.000	-0.012	0.004	0.008	0.022	0.005	-0.054	0.027	0.003	0.168	0.473	-0.644
35 Real Estate	-0.579	0.795	-0.095	-0.121	0.091	-0.203	0.026	0.085	-0.011	0.027	-0.027	0.012	-0.106	0.652	0.085	-0.631
36 Professional	-0.020	0.054	0.015	-0.049	0.005	-0.072	0.054	0.012	0.002	0.074	-0.100	0.024	-0.095	0.254	0.359	-0.519
37 Educational S	-0.120	0.123	0.029	-0.032	0.014	-0.127	0.092	0.021	0.003	0.079	-0.132	0.051	-0.034	0.196	0.548	-0.710
38 Health Care	-0.652	0.447	0.194	0.011	0.050	-0.245	0.114	0.081	0.016	0.083	-0.103	0.004	0.006	0.376	0.024	-0.405
39 Social Assist	-0.130	0.104	0.030	-0.003	0.022	-0.082	0.018	0.042	0.008	0.023	-0.050	0.020	-0.009	0.511	0.195	-0.698
40 Arts, Enterta	-0.058	0.012	0.080	-0.034	0.003	-0.046	0.003	0.040	0.029	0.004	-0.071	0.038	-0.106	0.504	0.326	-0.724
41 Accomm.	-0.067	0.164	-0.038	-0.058	0.039	-0.152	0.068	0.046	-0.012	0.092	-0.100	0.021	-0.181	0.597	0.203	-0.619
42 Food Serv.	-0.019	0.033	-0.004	-0.010	0.017	-0.141	0.100	0.024	-0.005	0.248	-0.238	-0.005	-0.187	0.847	-0.070	-0.589
43 Repair & Main	-0.420	0.562	-0.044	-0.098	0.115	-0.225	0.062	0.049	-0.014	0.096	-0.092	0.010	-0.480	1.158	0.156	-0.834
44 Personal & La	-0.046	0.085	0.010	-0.049	0.020	-0.095	0.023	0.051	0.004	0.038	-0.076	0.034	-0.141	0.615	0.255	-0.729
45 Membership Or	-0.635	0.446	0.187	0.001	0.047	-0.243	0.124	0.072	0.014	0.089	-0.100	-0.002	0.001	0.373	-0.016	-0.357
Mean	-0.491	0.342	0.198	-0.049	0.041	-0.162	0.089	0.032	0.020	0.096	-0.143	0.027	-0.084	0.410	0.358	-0.675
Median	-0.600	0.344	0.187	-0.039	0.039	-0.141	0.070	0.031	0.017	0.069	-0.130	0.024	-0.081	0.424	0.326	-0.709

Notes: 1) Monotonicity are imposed at all data points and concavity is imposed at a single point, i.e., mean labor cost shares; 2) η_{gh} = %Δ(labor demand of age group *g*)/%Δ(wage of age group *h*); 3) Shaded cells represent own-price elasticities.

Table 1. Sector description

1 Livestock & other agri. prod.	16 Stone, clay, & glass prod.	31 Pipeline trans.
2 Agriculture, forestry & fisheries	17 Primary metals prod.	32 Information
3 Mining	18 Fabricated metal prod.	33 Motion picture & sound recording
4 Utilities	19 Industrial machinery & equip.	34 Finance & insurance
5 Construction	20 Computer & other electric prod.	35 Real estate
6 Food & kindred prod.	21 Trans. equip. manuf.	36 Professional & management serv.
7 Tobacco prod.	22 Furniture & related product	37 Educational serv.
8 Apparel & textile prod.	23 Misc. manuf.	38 Health care
9 Leather & leather prod.	24 Wholesale trade	39 Social assistance
10 Lumber & wood prod.	25 Retail trade	40 Arts, entertainment, & recreation
11 Paper & allied prod.	26 Air trans.	41 Accommodation serv.
12 Printing & publishing	27 Railroad trans. & trans. serv.	42 Food serv.
13 Petroleum & coal prod.	28 Water trans.	43 Repair & maintenance
14 Chemicals & allied prod.	29 Truck trans. & warehousing	44 Personal & laundry serv.
15 Rubber & misc. plastics prod.	30 Transit & ground passenger trans.	45 Membership org. & households serv.

Notes: Resources 1-3; Construction 5; Non-durables 6-9 & 11-15; Durables 10 & 16-23; TCU (transportations, communications, and utilities) 4 & 26-32; Trade 24-25; FIRE (finance, insurance, and real estate) 34-35; Services 33 & 36-45

Table 2. Parameter estimates for membership organizations & households services (sector 45)

	OLS[1]	SUR[1]		Bayesian SUR[1]		
	No monotonicity or concavity	No monotonicity or concavity		No restriction	Monotonicity[2]	Monotonicity & concavity[2,3]
	Cost only (1)	Share only (2)	Cost & share (3)	Cost & share (4)	Cost & share (5)	Cost & share (6)
α_0	-0.0503 (0.015)	-	-0.0123 (0.003)	-0.0121 (0.003)	-0.0206 (0.004)	-0.0228 (0.003)
α_1	0.0538 (0.012)	0.0736 (0.002)	0.0721 (0.002)	0.0715 (0.002)	0.0568 (0.002)	0.0561 (0.002)
α_2	0.4966 (0.027)	0.4307 (0.007)	0.4347 (0.006)	0.4353 (0.006)	0.4473 (0.006)	0.4477 (0.006)
α_3	0.4127 (0.032)	0.4095 (0.007)	0.4119 (0.006)	0.4121 (0.006)	0.4178 (0.006)	0.4195 (0.006)
α_4	0.0369 (0.018)	0.0862 (0.003)	0.0813 (0.003)	0.0810 (0.003)	0.0781 (0.003)	0.0766 (0.003)
β_{11}	0.0428 (0.005)	0.0299 (0.001)	0.0308 (0.001)	0.0308 (0.001)	0.0127 (0.000)	0.0127 (0.000)
β_{12}	0.0240 (0.014)	-0.0098 (0.003)	-0.0118 (0.003)	-0.0120 (0.003)	0.0020 (0.002)	0.0030 (0.002)
β_{13}	-0.0386 (0.015)	-0.0178 (0.003)	-0.0142 (0.003)	-0.0137 (0.003)	-0.0124 (0.002)	-0.0129 (0.002)
β_{14}	-0.0283 (0.008)	-0.0023 (0.001)	-0.0049 (0.001)	-0.0051 (0.001)	-0.0023 (0.001)	-0.0028 (0.001)
β_{22}	0.0231 (0.045)	0.1530 (0.013)	0.1498 (0.012)	0.1498 (0.012)	0.1389 (0.012)	0.1430 (0.013)
β_{23}	-0.0442 (0.048)	-0.1412 (0.012)	-0.1377 (0.011)	-0.1378 (0.011)	-0.1420 (0.011)	-0.1461 (0.012)
β_{24}	-0.0030 (0.018)	-0.0020 (0.004)	-0.0003 (0.004)	0.0000 (0.004)	0.0011 (0.004)	0.0001 (0.005)
β_{33}	0.0951 (0.060)	0.2074 (0.013)	0.1991 (0.012)	0.1987 (0.012)	0.1946 (0.012)	0.1976 (0.013)
β_{34}	-0.0124 (0.022)	-0.0484 (0.004)	-0.0472 (0.004)	-0.0472 (0.004)	-0.0402 (0.004)	-0.0385 (0.004)
β_{44}	0.0437 (0.014)	0.0527 (0.003)	0.0524 (0.003)	0.0523 (0.002)	0.0413 (0.001)	0.0411 (0.001)
θ	0.0008 (0.001)	-	-0.0005 (0.000)	-0.0005 (0.000)	-0.0009 (0.000)	-0.0008 (0.000)
γ_1	0.0020 (0.001)	0.0002 (0.000)	0.0002 (0.000)	0.0002 (0.000)	-0.0002 (0.000)	-0.0002 (0.000)
γ_2	-0.0117 (0.002)	-0.0065 (0.001)	-0.0073 (0.001)	-0.0074 (0.001)	-0.0069 (0.001)	-0.0067 (0.001)
γ_3	0.0078 (0.002)	0.0044 (0.001)	0.0049 (0.001)	0.0050 (0.001)	0.0049 (0.001)	0.0046 (0.001)
γ_4	0.0019 (0.001)	0.0020 (0.000)	0.0021 (0.000)	0.0021 (0.000)	0.0023 (0.000)	0.0023 (0.000)
Region FE[4]	Yes	-	Yes	Yes	Yes	Yes
Observations	685	685	685	685	685	685
Violating conc. @ a single pt.	Yes	No	No	No	No	No
%violating mono.	8.2	1.5	1.5	1.5	0.0	0.0
%violating conc. @ all pts.	100.0	48.6	56.5	56.5	14.9	11.2
Loglikelihood	1317.7	4445.8	5909.6	-	-	-
BIC[5]	-2524.4	-8813.302	-11708.21	-	-	-
MAE[6] (cost)	0.0248	-	0.0256	0.0256	0.0265	0.0265
MAE[6] (avg. of shares)	0.0351	0.0286	0.0287	0.0287	0.0293	0.0292
(All sectors)						
Observations	22216	22216	22216	22216	22216	22216
%violating mono.	8.3	5.5	4.7	4.7	0.0	0.0
%violating conc. @ all pts.	84.6	68.8	69.8	69.8	30.9	20.6
Loglikelihood (mean)	-1643.0	-5839.8	-7928.9	-	-	-
MAE[6] (cost; mean)	0.0315	-	0.0333	0.0333	0.0340	0.0342
MAE[6] (shares; mean)	0.0444	0.0353	0.0356	0.0356	0.0364	0.0376

Notes: 1) Homogeneity and symmetry imposed; 2) Monotonicity are imposed at all data points and concavity is imposed at the mean of predicted labor cost shares; 3) Concavity is satisfied conditionally on monotonicity; 4) Four Census regions: West, Midwest, Northeast and South; 5) Bayesian Information Criterion = -2×loglikelihood + log(#obs.) × #parameters; 6) Mean Absolute Error; 7) Figures in parentheses are standard errors for OLS and SUR, and standard deviations of the Metropolis-Hastings samples for the Bayesian SUR.

Table 3. The effects of age distribution changes on *interrelational* income multipliers (K matrix)

	Age group of income origin				
	16-24	25-44	45-64	65+	Total
Age group of income receipt: 2009					
16-24	1.056	0.037	0.036	0.047	1.175
25-44	0.409	1.281	0.274	0.360	2.325
45-64	0.385	0.268	1.265	0.361	2.279
65+	0.038	0.027	0.028	1.040	1.133
Total	1.888	1.614	1.602	1.808	6.912
Age group of income receipt: 2020					
16-24	1.044	0.029	0.028	0.037	1.138
25-44	0.348	1.238	0.232	0.303	2.122
45-64	0.445	0.308	1.304	0.413	2.470
65+	0.049	0.035	0.035	1.052	1.171
Total	1.886	1.611	1.599	1.804	6.901
Changes in indirect & induced impacts (%) : 2020-2009					
16-24	-21.5	-21.6	-21.5	-20.5	-21.3
25-44	-14.9	-15.1	-15.3	-15.9	-15.3
45-64	15.6	14.9	14.6	14.3	14.9
65+	29.3	28.8	28.7	28.0	28.7
Total	-0.20	-0.43	-0.49	-0.49	-0.39

Notes: 1) The [*i,j*]th entry represents a direct increase of $1 in income to group *j* leads to *k* cents in income payments to group *i*; 2) It is assumed that technology and relative prices of goods and labor groups do not change from 2009 on.

Table 4. The effects of age distribution changes on multi-sector income multiplier (KVB matrix)

	Sector of final demand origin								
	Resource	Const.	Non-dur.	Dur.	TCU	Trade	FIRE	Services	Total
Age group of income receipt: 2009									
16-24	0.037	0.062	0.042	0.046	0.042	0.055	0.026	0.069	0.377
25-44	0.220	0.486	0.378	0.404	0.370	0.409	0.279	0.452	2.997
45-64	0.214	0.419	0.385	0.416	0.393	0.389	0.241	0.419	2.877
65+	0.018	0.033	0.032	0.033	0.031	0.034	0.020	0.044	0.245
Total	0.489	0.999	0.837	0.898	0.836	0.887	0.566	0.983	6.496
Age group of income receipt: 2020									
16-24	0.028	0.037	0.033	0.034	0.032	0.044	0.015	0.056	0.279
25-44	0.187	0.411	0.309	0.326	0.309	0.337	0.239	0.393	2.510
45-64	0.250	0.502	0.452	0.492	0.454	0.460	0.282	0.476	3.368
65+	0.022	0.042	0.041	0.043	0.039	0.043	0.028	0.055	0.313
Total	0.487	0.992	0.834	0.895	0.834	0.884	0.564	0.980	6.470
Differences (%): 2020-2009									
16-24	-23.2	-40.0	-21.5	-25.5	-24.0	-20.2	-39.0	-19.0	-26.0
25-44	-15.0	-15.3	-18.3	-19.2	-16.7	-17.4	-14.6	-13.0	-16.2
45-64	16.7	20.1	17.5	18.5	15.6	18.1	17.4	14.0	17.3
65+	16.7	24.0	24.2	27.3	24.0	25.8	30.9	23.4	24.6
Total	-0.53	-0.71	-0.31	-0.36	-0.34	-0.34	-0.45	-0.33	-0.42

Notes: 1) The [*i,j*]th entry in the matrix represents the total (direct, indirect and induced) income for group *i* resulting from a dollar increase in consumption in sector *j*; 2) It is assumed that technology and relative prices of goods and labor groups do not change from 2009 on; 3) TCU = transportation, communications, and utilities; FIRE = finance, insurance, and real estate

Table 5. The effects of age distribution changes on *output* multipliers[1,2)]

	Sector of final demand origin								
	Res.	Const.	Non-dur.	Dur.	TCU	Trade	FIRE	Serv.	Avg.
Type I:									
Direct & indirect (2009)[1)]	1.427	1.587	1.862	1.691	1.624	1.329	1.483	1.506	1.563
Type II:									
Direct, indirect & induced (2009) [2)]	2.002	2.754	2.833	2.734	2.594	2.366	2.140	2.660	2.511
Type II:									
Direct, indirect & induced (2020) [2)]	1.994	2.731	2.824	2.722	2.585	2.356	2.132	2.650	2.499
Changes in indirect & induced impacts (%): 2020-2009	-0.82	-1.30	-0.49	-0.65	-0.61	-0.78	-0.75	-0.61	-0.75

Notes: 1) Column sums of $\mathrm{B} = (\mathrm{I} - \mathrm{A})^{-1}$; 2) Column sums of $\mathrm{B(I + CKVB)}$; 3) It is assumed that technology and relative prices of goods and labor groups do not change from 2009 on; 3) TCU = transportation, communications, and utilities; FIRE = finance, insurance, and real estate

Figure 1. Characteristics of workers by age group

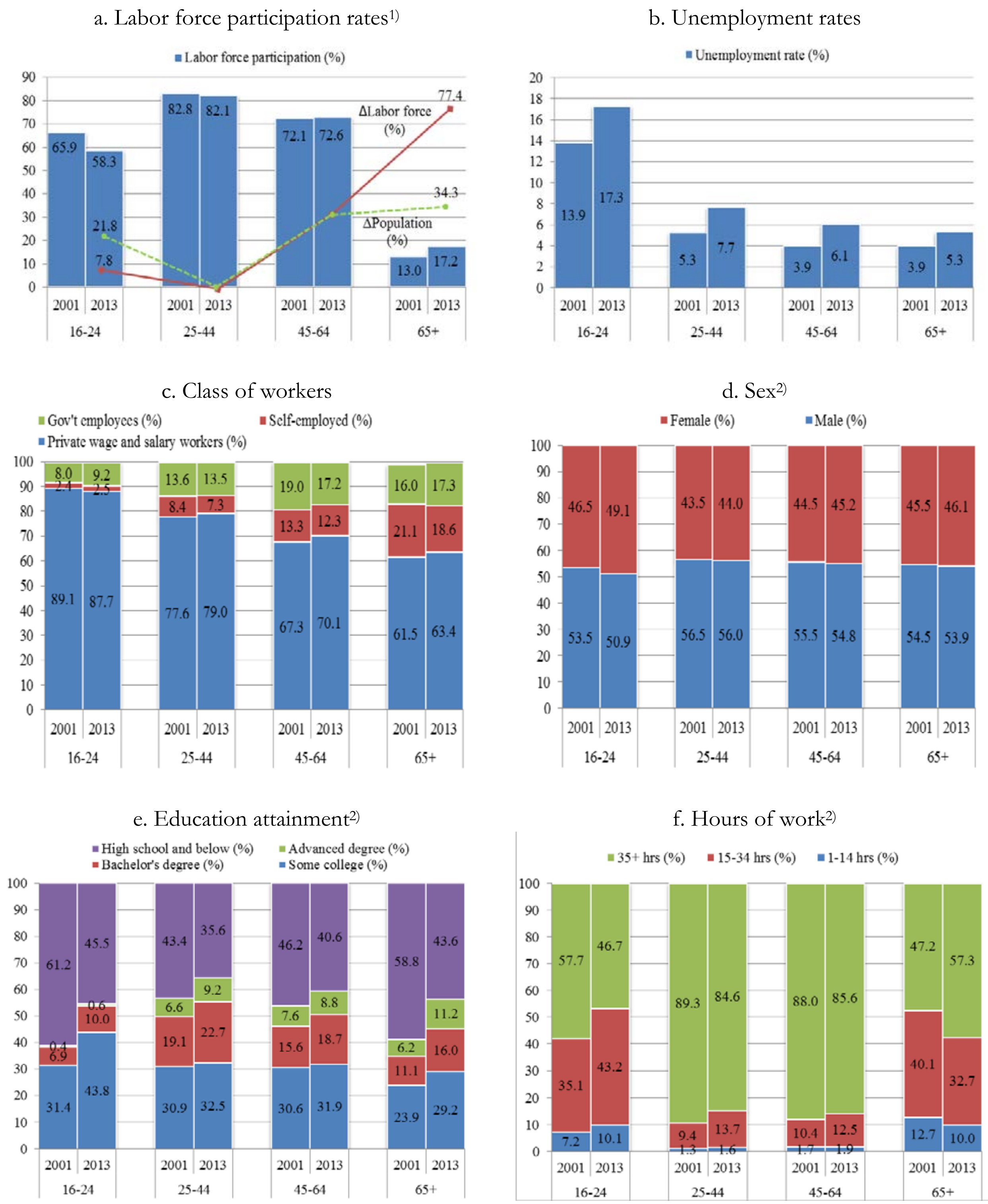
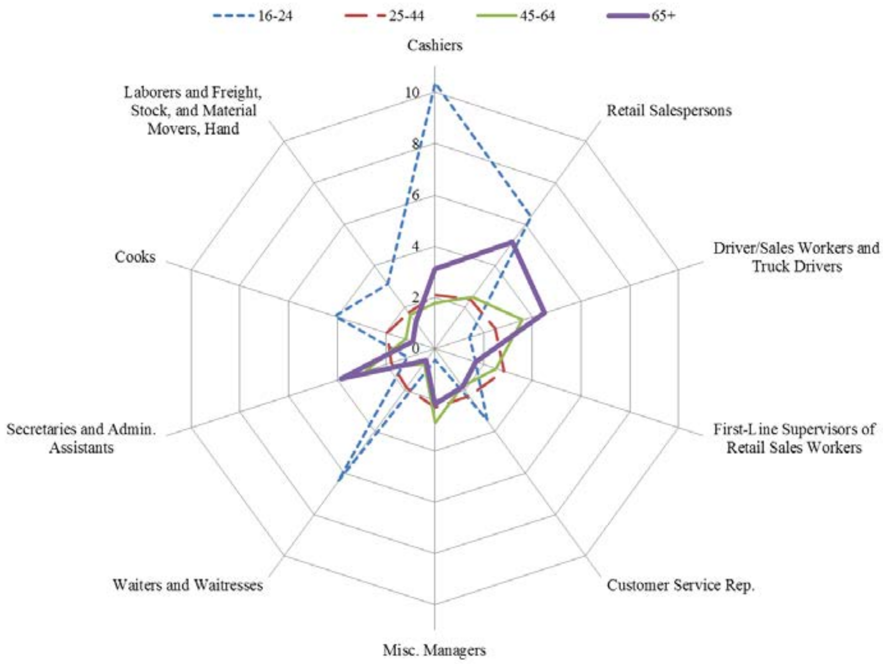
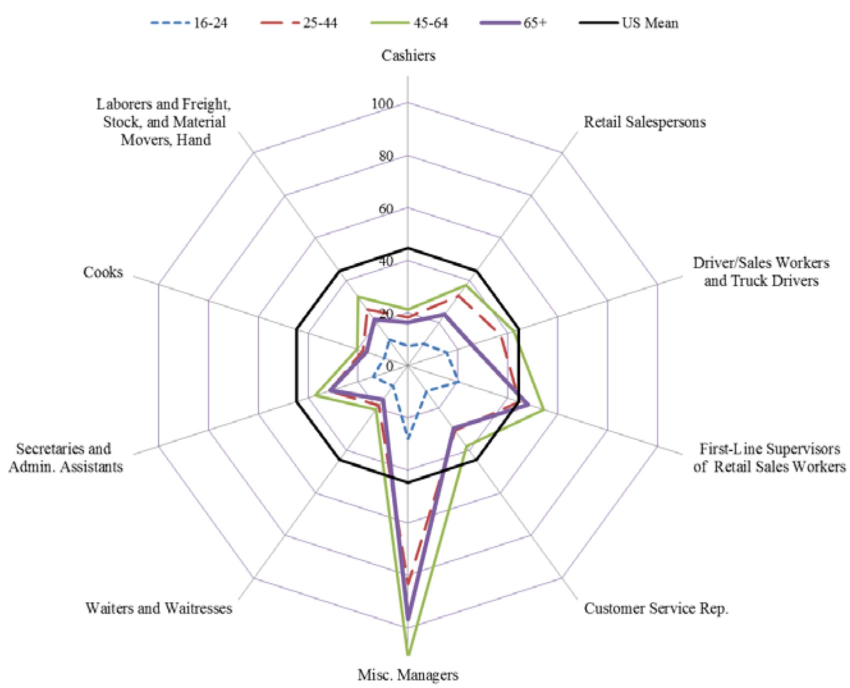


Figure 1. Continued

g. The 10 most common occupations (2013):
Occupation shares (in percent)



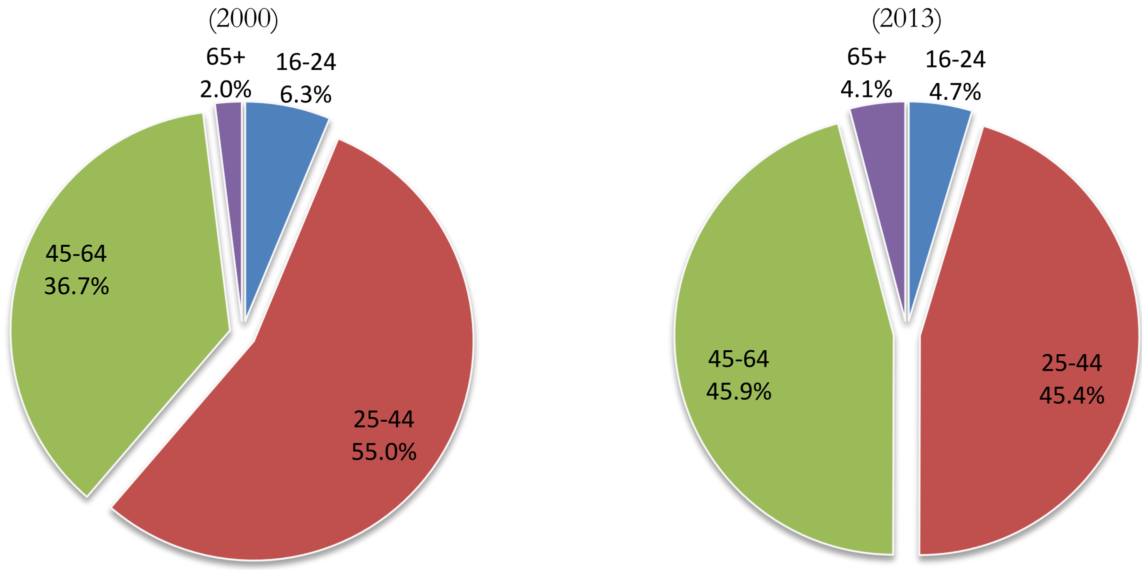
h. The 10 most common occupations (2013):
Annual wages and salaries (in the current thousand dollars)



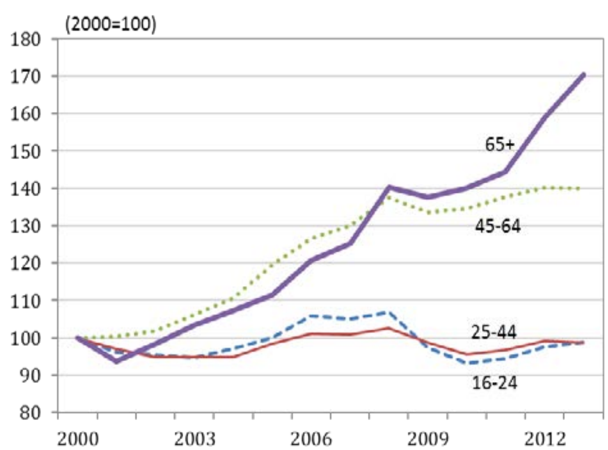
Notes: 1) Line graphs indicate the rate of changes during 2001-2013 for the corresponding age group; 2) Among private wage and salary workers.
Source: Authors' calculation based the 2001 and 2013 American Community Survey (ACS)

Figure 2. Labor costs, employment and wages by age in the US (2000-2013)

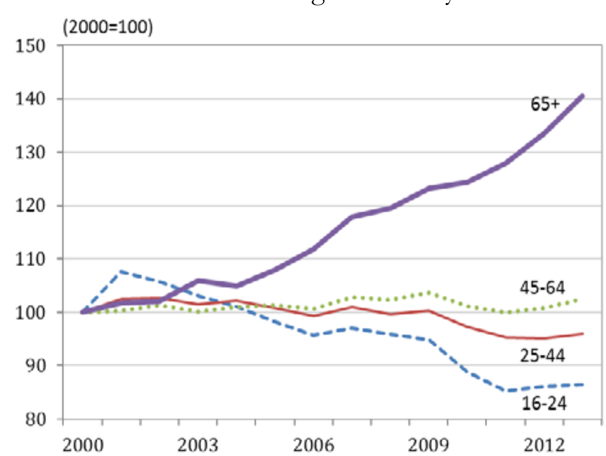
a. Labor costs



b. Employment

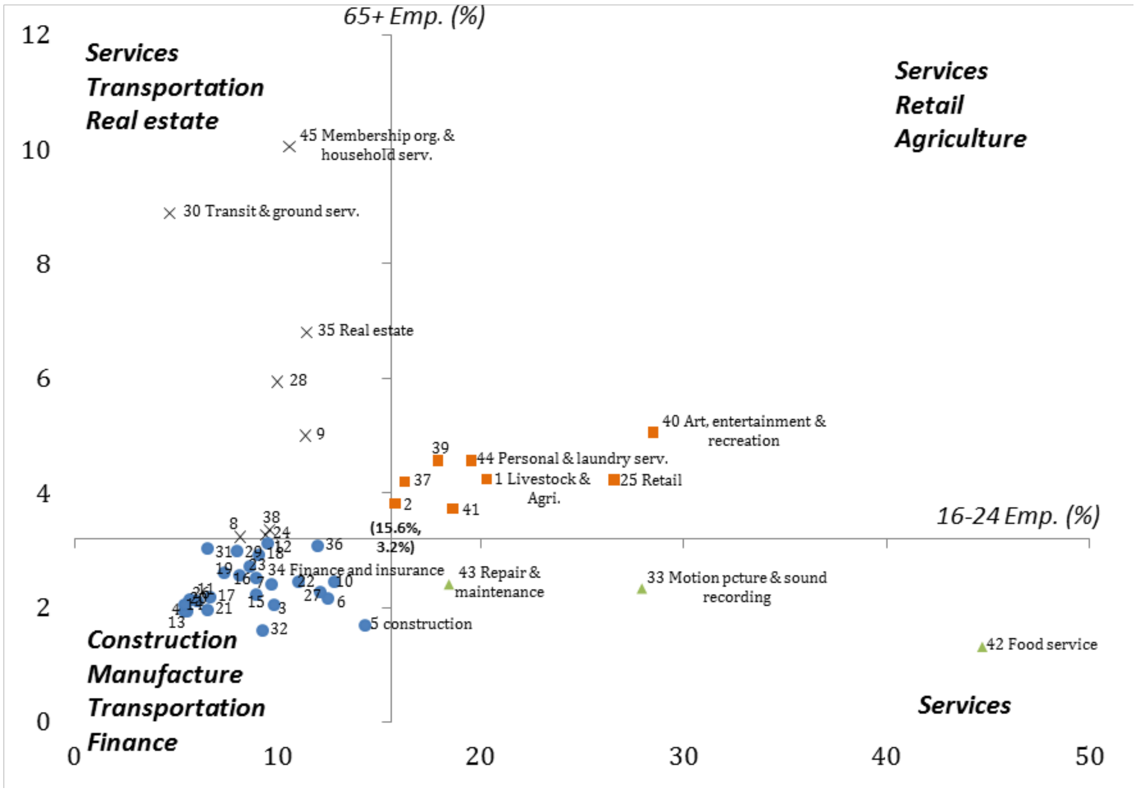


c. Real wage and salary



Note: Self-employed, Armed Forces and government employees are excluded.
Source: Authors' calculation based on the 2000-2013 ACS

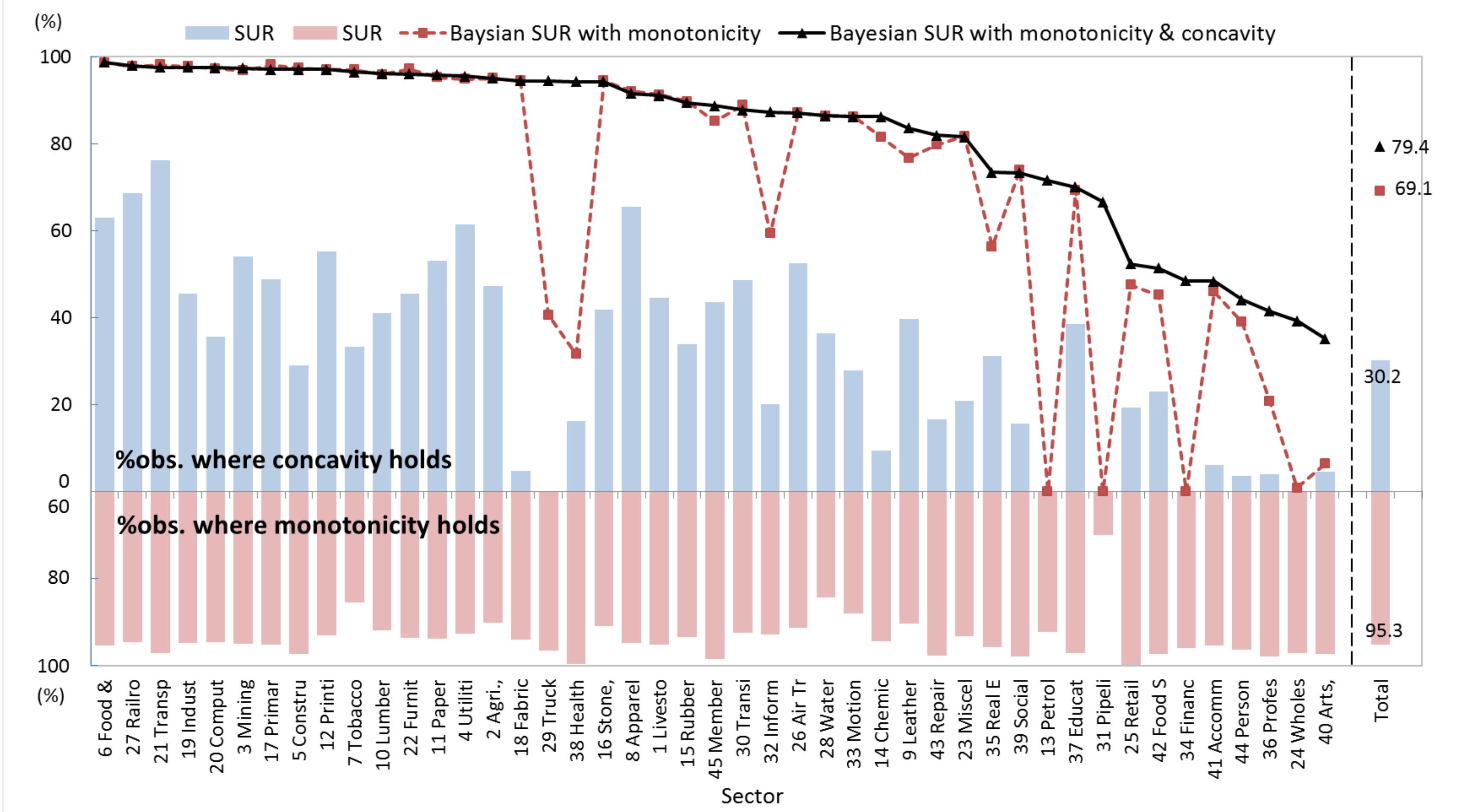
Figure 3. Employment shares by sector for the youngest and oldest age-group employees: 2000-2013 average



Notes: 1) The origin represents mean shares; 2) The bold fonts represent aggregate sectors for those appearing most in the corresponding quadrants; 3) Each symbol is specific to each quadrant.

Source: Authors' calculations based on the 2000-2013 ACS

Figure 4. Monotonicity and concavity by sector: percentage of observations where these properties hold[1)]

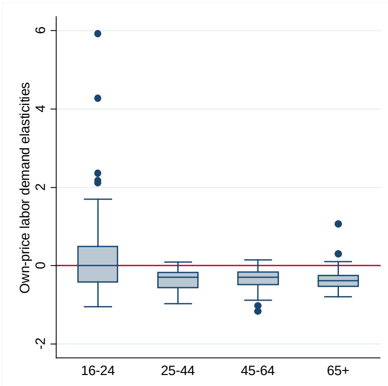


Notes: 1) A cost function and labor cost shares are simultaneously estimated; homogeneity and symmetry are globally satisfied; 2) Concavity is imposed at a single reference point, i.e., means shares of predicted labor cost shares; 3) Concavity is satisfied conditionally on monotonicity; 4) Sectors are sorted in a descending order of the proportion of concavity-satisfying samples in the Bayesian SUR model with monotonicity and concavity imposed; 5) Total number of observations is 22,216.

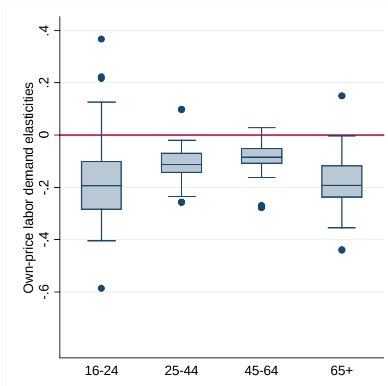
Figure 5. Distributions of own-price labor demand elasticities for 45 sectors by estimation method: evaluated at mean predicted labor cost shares[1)]

(Non-Bayesian models)

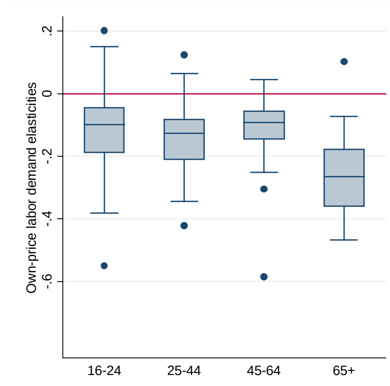
a. Cost only;
no restrictions[2)]



b. Share only ;
no restrictions[2)]

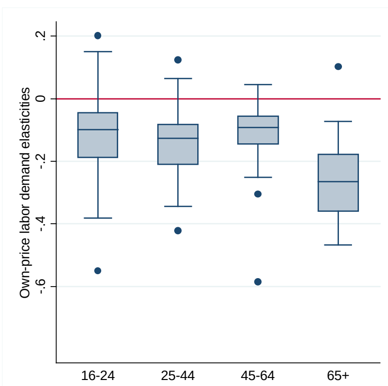


c. Cost & Share ;
no restrictions[2)]

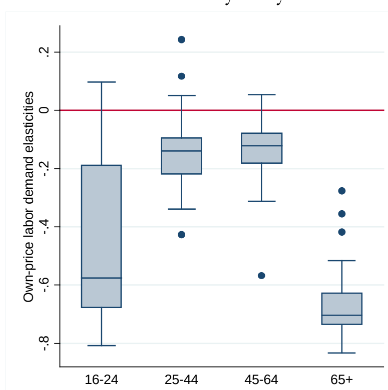


(Bayesian models)

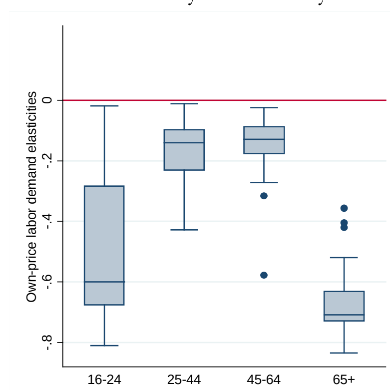
d. Cost & Share;
no restrictions[2)]



e. Cost & Share;
monotonicity only[3)]

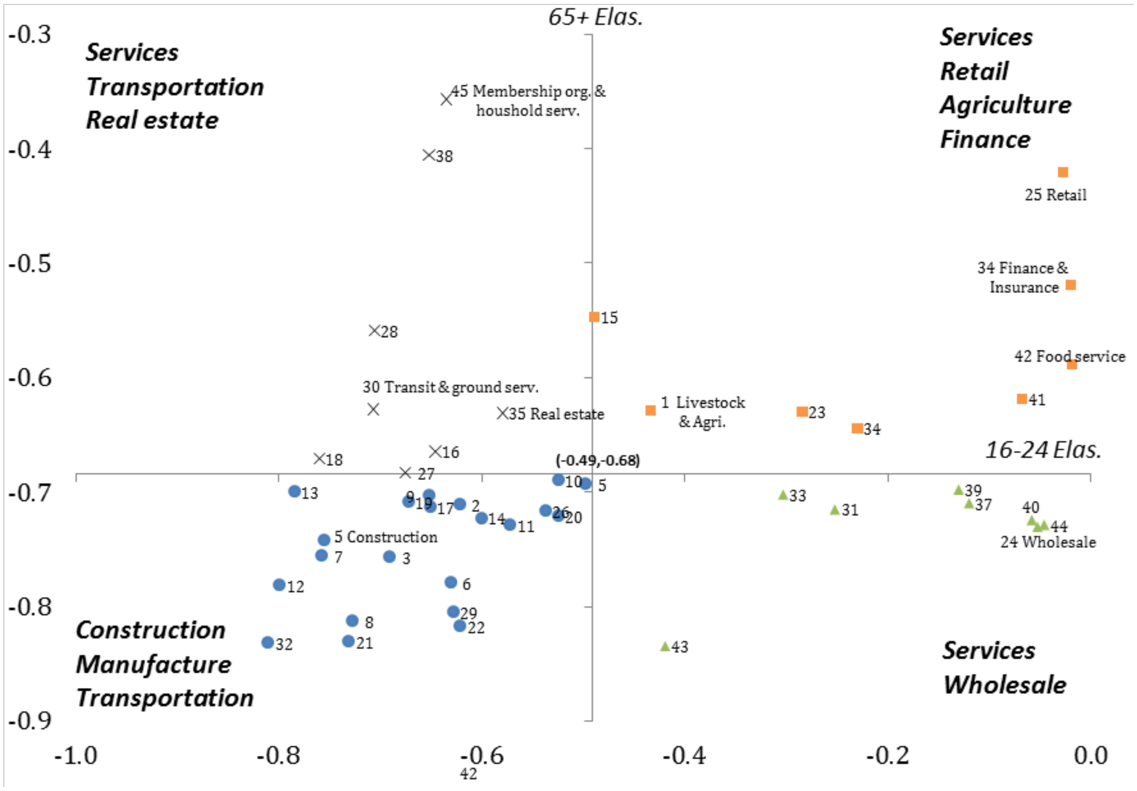


f. Cost & Share;
monotonicity & concavity[3)]



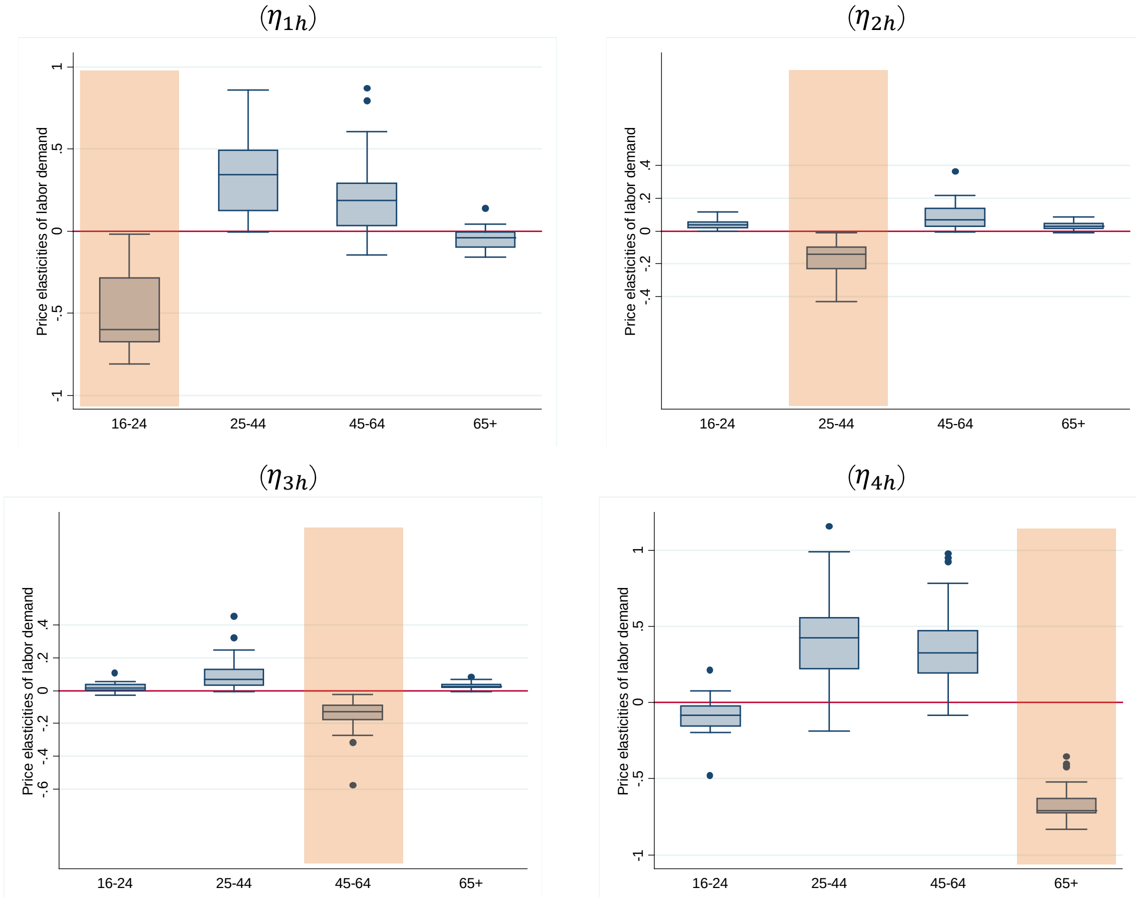
Notes: 1) Homogeneity and symmetry are globally imposed; 2) One very large positive number in the 16-24 group is intentionally omitted for easier comparisons; 3) Monotonicity is imposed at all data points and concavity is imposed at the mean of predicted labor cost shares

Figure 6. Own-price labor elasticities by sector for the youngest and oldest age-group employees: evaluated at fitted mean shares



Notes: 1) The origin represents mean of elasticities; 2) The bold fonts represent aggregate sectors for those appearing most in the corresponding quadrants; 3) Each symbol is specific to each quadrant.

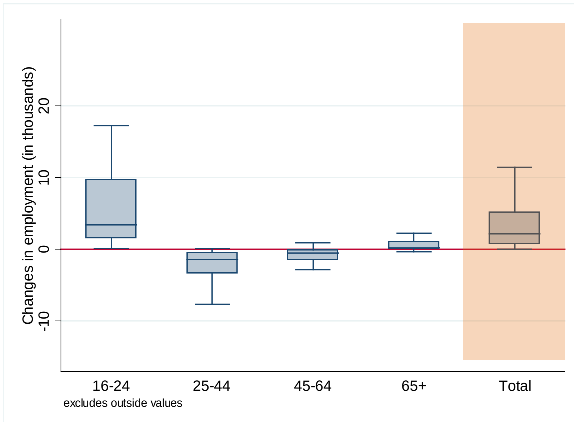
Figure 7. Distributions of cross-price labor demand elasticities for 45 sectors: the Bayesian SUR evaluated at fitted mean shares with monotonicity and concavity imposed



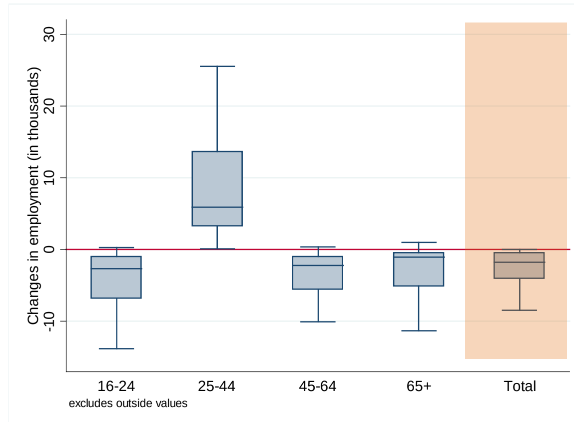
Notes: 1) Homogeneity and symmetry are globally imposed; 2) Monotonicity are imposed at all data points and concavity is imposed at a single point, i.e., mean labor cost shares; 3) Shaded areas are own-price elasticities and the rests are cross-price elasticities; 4) η_{gh} = %Δ(labor demand of age group *g*)/%Δ(wage of age group *h*)

Figure 8. The effects of wage decline on employment in 45 sectors: a 10-percent wage decline in each age-group

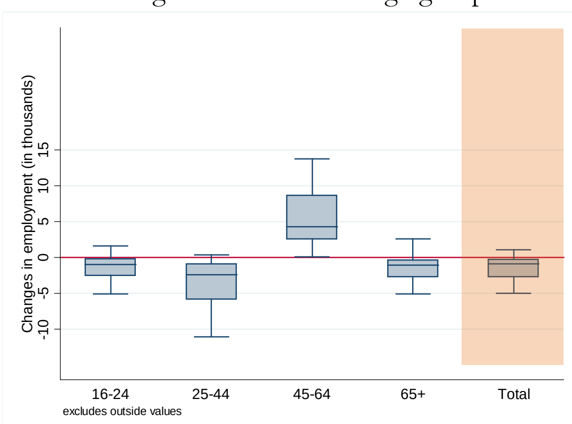
a. Wage decline in 16-24 age-group



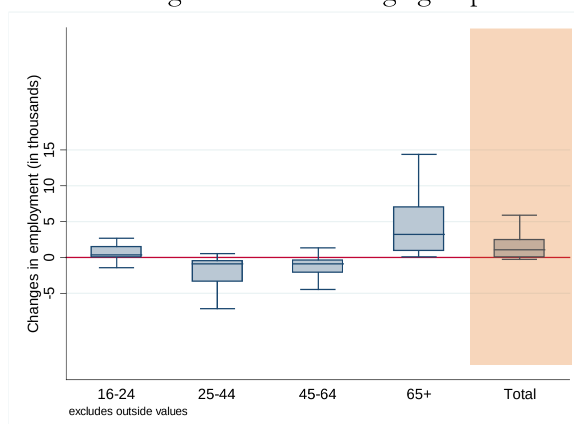
b. Wage decline in 25-44 age-group



c. Wage decline in 45-64 age-group



d. Wage decline in 65+ age-group



Notes: 1) Elasticities are calculated from the Bayesian SUR estimates with monotonicity and concavity imposed; 2) Calculation of changes in employment is based on the 2013 figures.