A Short Exercise to Assess the Consequences of Temporal and Spatial Aggregation on the Observed Spatial Interactions

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Abstract:
Since our observation of the regional economy depends on the scale of temporal/spatial units, even under the same underlying disaggregated level data generating process, we can encounter different neighborhood effects or spillovers. In this paper, the amount of spillover is defined by the forecast error variance decomposition (FEVD), and the direction of spillover is defined by the long-run sign of the cumulative impulse response function (CIRF). From an exercise using a constructed regional economic system, the size of spillover was found to decrease with spatial aggregation in a multi-level structure regional economic system. However, no monotonic trend was found in terms of the relative portion of positive/negative spillovers. In addition, the results from the real world data using different levels of aggregations, and the results drawn from the exercise on the constructed regional economic system are compared. From this comparison, a multi-level structure model which assumes the existence of higher level common factor affecting the regional units was found to concord with logical experiments conducted over the constructed regional economic system.

Key Words: Temporal Aggregation, Spatial Aggregation, Multi-level Structure, Spillover Effects

JEL classifications: R10, R12, C43, C63

1. Introduction

This paper aims to unravel how the observed spatial relationship changes as the frequency/scale of the temporal/spatial unit altered. Whether temporal or spatial, the level of aggregation always becomes a problem when analyzing spatial relationships. For example, in terms of the spatial dimension, most researchers seem to agree that more disaggregated data better reveals spatial dependencies. In terms of temporal dimension, researchers' opinions vary. Some argue that a longer time period allows spatial spillover effects to be propagated through the observed

spatial units, thus lower frequency data exhibit a larger amount of spillovers; however, other researchers provide opposing interpretations.

In this paper, spillover effects are defined using forecast error variance decomposition (FEVD) and cumulative impulse response function (CIRF). FEVD decomposes the variances of the regional behaviors into their sources for a given forecasting horizon. Thus, the "amount" of spillover can be defined as the portion of the neighborhood innovation relative to the regional variance. CIRF measures the cumulative effect of a regional shock on the future values of regional values. Thus, the "direction" of a spillover can be defined as the sign of the long-term cumulative response.

Although the conclusions are derived from an experiment using a simple artificial regional economic structure, this paper argues that we should expect to observe a smaller volume of spillovers with larger frequency/scale of temporal/spatial units. Also, when the disaggregated level regional economy is all competitive, i.e., the immediate next period neighborhood effect is negative for every regional unit, we should expect to observe more positive spillovers with odd numbers of temporal aggregations, but less positive spillovers with even numbers of temporal aggregations. However, when the regional economy is mixed with competitive and complementary relationships, there is no monotonic trend of the relative portion of positive spillovers over the level of aggregations.

This finding is more in concord with the assumption that there exists a common factor governing the behaviors of all of the regional units (multi-level structure). In our exercise on real world data, under the multi-level assumption, on average, we observe less spillover effects with larger scale of spatial or frequency of temporal units when we assume the existence of common factor. However, these patterns are not observed when we assume there is no common factor (single-level structure).

Over a century ago, W. F. Gosset (Student) also thought about this problem, and in his letter to Karl Pearson in December 1910, he made a similar conclusion: "Now in general the correlation weakens as the unit of time or space grows larger and I can't help thinking that it would be a great thing to work out the law according to which the correlation is likely to weaken with increase of unit" (Pearson, 1990). His notion of correlation was retranslated into FEVD and CIRF in the regional economic activity analysis context.

In this paper, a simple exercise was conducted using a restricted VAR(1) type of data generating process to see how the observed spatial relationship are altered as the level of aggregation of the observation changes. A more generalized version of the same exercise also can be done, but this simple exercise is believed to produce enough implications of the effect of the aggregation in the spatial analysis literature.

Additionally, as noted earlier, this paper incorporated a multi-level concept developed by Bai and Wang (2012) and Corrado and Fingleton (2011) in analyzing the spatial relationships. To

illustrate more, a multi-level structure of regional economic system assumes that there is are higher-level determinants that affect the behavior of spatial units. For example, the behavior of each of the U.S. states is determined by the state's own determinants as well as the national determinants such as federal-level monetary/fiscal policy, exchange rates, commodity prices and so on. Thus, in practice, the region common factor is the national level economic behavior, and in our model, it is an exogenous, or almost exogenous,[1] factor affecting the regional economic behavior. By introducing an hierarchical structure into spatial analysis, a multi-level analysis argues that the co-moving behaviors of spatial units are largely due to these higher-level determinant(s), whereas a single-level analysis argues that the higher-level structure is a summation of its sub-units, e.g. the U.S. is composed of the U.S. states, thus co-moving behaviors are largely due to the spillover effects. By comparing the forecast error variance decomposition and the impulse responses of regional observations using both concepts, this paper argues that a multi-level concept explains the real world data in a more realistic way.

Recent studies on the spatial or temporal aggregation problems are briefly introduced in the next section. In section 3, a multi-level and a single-level spatial analysis using the Bai and Wang (2012) formulation on the real world data are performed and the results are compared. In section 4, an artificial regional economic system is constructed to see how the observed spillovers change with the level of temporal/spatial aggregation, and section 5 provides some concluding remarks.

2. Literatures on Temporal and Spatial Aggregation

While the primary focus of this paper is on what we should expect to observe in terms of the spillover effects with different levels of data aggregation, most of the previous work has focused on other issues such as the biasedness or the efficiency loss of the estimated coefficients at the aggregated level when the true data generating process is defined at the micro level. Regarding the temporal aggregation problem, examples are just too many to introduce here since, depending on the interests of the researcher, each of the properties of time-series model, such as impulse response analysis, cointegration, unit root test and so on are dealt individually in many studies. To list a few related to this paper's interest, Brewer (1973), Wei (1981) and Weiss (1984) tackle the issues related to temporal aggregation in an empirical studies. Marcellino (1999) reviewing the literature on this issue showed that impulse response functions and forecast error variance decompositions, along with other properties such as Granger-causality and cointegration, change with the level of aggregation.

[1] Almost exogenous means regional shock does not have a significant impact on the region common economic behaviors. More practically, in the model structure, the coefficients associated with the effect from regional shock to the region common behavior should be close to zero so that the local impact on the global behavior decays very fast throughout time.

The spatial aggregation problem, or the modifiable areal unit problem (MAUP),[2] also has been an interest of spatial analysts for a long time since the pioneering work of Gehlke and Biehl (1934). The effects on standard regression estimators are dealt in Barker and Pesaran (1989), Okabe and Tagashira (1996), Tagashira and Okabe (2002) and Griffith *et al*. (2003). Their main findings are that "the GLS estimators of regression's parameters are BLUE with a sampling variance greater than that obtained using GLS on the original data" (Arbia and Petrarca, 2011). Arbia and Petrarca (2011) also explored the efficiency loss of the estimators in the presence of spatial dependency.

However, attempts to find a general "law" or some monotonic relationships, between the scale of grouping, or the level of aggregation, and the spillover effects, that can be defined in terms of forecasting error variance decomposition and the impulse response function, is very rare. There is some empirical work exploring the effect of the scale of unit effects on the correlation coefficients or Moran's I such as in Arbia (1989). Gehlke and Biehl (1934) explored the effect of the aggregation on the correlation coefficient between observations, and Smith (1938) explored the correlation coefficient with different plot sizes in an agricultural experiment. Also, Dusek (2004) showed how different geographical statistics varied with different levels of aggregations on Hungarian regional economic data. However, Gehlke and Biehl (1934)'s finding that the correlation coefficient for variables of absolute measurement increases when areal units are aggregated contiguously, while Dusek (2004)'s finding that more aggregated observations exhibit higher Moran's I are exactly the opposite of Gosset's idea that the correlation will weaken with the increase of scale.

On the contrary, in agricultural studies, Gosset's idea is supported in both theoretical and empirical works. For example, in Gelfand, *et al*. (2010), when the covariance between spatial units are defined in terms of area and distance, the inverse relationship between the aggregation level and the covariance are easy to assess. Following Gelfand, *et al*. (2010), suppose a stationary spatially continuous stochastic process $\mathrm{S(x)}$ with covariance function $\mathrm{cov\{S(x), S(y)\}} = \sigma^2\rho(u)$ where u is the distance between the locations x and y. The covariance between spatial averages of $\mathrm{S}(\cdot)$ over two regions A and B is:

$$\gamma(\mathrm{A}, \mathrm{B}) = (|\mathrm{A}| \times |B|)^{-1}\sigma^2 \int_A \int_B \rho(\|x - y\|)\, dx\, dy \qquad (1)$$

where $|\cdot|$ denotes area and $\|\cdot\|$ distance.

The above equation supports Gosset's statement since in a space where the correlations are decreasing with the distance, the correlation weakens with the larger scale of unit. For instance,

[2] MAUP refers to the spatial data problem associated with the scale of grouping and the zoning problem. In this paper, the spatial data problem is dealt only in terms of the scale of grouping, or the level of aggregation.

when the covariance is proportional to the inverse distance, then the aggregation of the four equal-sized spatial units will result in the half size value of the original covariance. Empirical study of the correlations defined by the distance between objects can be found in Whittle (1956, 1962) and MacCullagh and Clifford (2006).

However, in more general spatial economic analysis, where the spillovers are not defined by correlations, and negative spillover exists, this type of argument is not applicable. In this case, spatial dependency should be assessed using the concept of spillover that can be defined with, for example, forecast error variance decomposition or impulse response function, rather than correlation coefficient. In other words, the correlation coefficient measures the degree of the co-movement whereas the spillover effects measures the effects from the neighborhood.

This conceptual distinction is more straightforward in a multi-level structure regional economic system. For example, in a multi-level structure state-level economy, two independent counties can have high correlation coefficient if they are exposed to a same state-level shock, and having similar response functions in response to this shock. Thus, if an economy is a multi-level structure, regions with high correlation coefficient may not be highly correlated. That is, they are correlated to common factor rather than directly dependent to each other. On the contrary, of course, in a single-level structure economy, high-correlation coefficient implies big spillover effects, thus the concept of correlation coefficients and spillovers can be used interchangeably.

The researcher's belief on the existence of a common factor becomes more important in empirical analysis. For example, equating the co-movement to spillovers where the economy is a multi-level structure can exaggerate or bias the impact of local policy on its neighborhood regions. Likewise, assuming the common factor where there is no such thing can underestimate or bias the impact of local shock. However, in practice, we cannot distinguish whether the observed high correlations between regional units are due to the existence of common factor or the high dependency between regional units. This issue will be addressed in subsequent sections.

3. A Factor Analysis Exercise on the Real World Data

Although, we cannot distinguish whether the observed co-moving behaviors are due to the existence of common factor(s) or due to the high dependency between regional units; some differences can be observed between those two conflicting views if we analyze them while altering the scale of units. Using the approach of Bai and Wang (2012), a multi-level and a single-level dynamic factor analysis is conducted on selected regions. Chung and Hewings (2014) revealed that in a multi-level model, the portion of spillover effects decreases on average with the larger scale of the spatial or with higher frequency of temporal units, while this inverse relationship is not found in a single-level model.

3.1 Single and Multi-level Dynamic Factor Model Overview[3]

A single level dynamic factor model for R-regional units with a VAR(1) structure can be expressed as equations (2)~(4):

$$\begin{bmatrix} y_t^1 \\ \vdots \\ y_t^R \end{bmatrix} = \begin{bmatrix} \gamma^1(L) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \gamma^R(L) \end{bmatrix} \begin{bmatrix} f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} + \begin{bmatrix} u_t^1 \\ \vdots \\ u_t^R \end{bmatrix} \quad (2)$$

$$D^r(L)u_t^r = \varepsilon_t^r \quad \forall\, r \in \{1, \dots R\} \quad (3)$$

$$\begin{bmatrix} f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \dots & \varphi_{1R} \\ \vdots & \ddots & \vdots \\ \varphi_{R1} & \dots & \varphi_{RR} \end{bmatrix} \begin{bmatrix} f_{t-1}^1 \\ \vdots \\ f_{t-1}^R \end{bmatrix} + \begin{bmatrix} \eta_t^1 \\ \vdots \\ \eta_t^R \end{bmatrix} \quad (4)$$

where y_t^r is $p \times 1$ vector of endogenous region *r* observations,

f_t^r is unobservable fundamental forces that affect the dynamics of y_t^r, and

$E(\eta_t^r \mid f_{t-1}^1, \dots, f_{t-1}^R) = 0 \quad \forall\, t \in \{1, \dots, T\}\ and\ \forall\, r \in \{1, \dots, R\}$,

$E(u_t^r | f_t^1, \dots, f_t^R) = 0 \quad \forall\, t \in \{1, \dots, T\}\ and\ \forall\, r \in \{1, \dots, R\}$,

$E\left(u_t^{ri} u_t^{rj} \middle| f_t^1, \dots, f_t^R\right) = 0 \quad \forall\, i \neq j \in \{1, \dots, p\}, \forall\, t \in \{1, \dots, T\}\ and\ \forall\, r \in \{1, \dots, R\}$,

and $E\left(u_t^a u_t^b \middle| f_t^1, \dots, f_t^R\right) = 0 \quad \forall\, t \in \{1, \dots, T\}$ and $\forall\, a \neq b, r \in \{1, \dots, R\}$.

The observed variables of each region y_t^r is decomposed into two parts of shocks, regional dynamic factor, f_t^r, and idiosyncratic error, u_t^r. The spatio-temporal relationship of regional factor can be found in the coefficient matrix of state equation of dynamic factor, equation (4). From this coefficient, we can derive the forecast error variance decomposition (FEVD) and cumulative impulse response functions (CIRF), and assess the spatio-temporal dynamics of regional business cycles. Thus, in this sense, the "spillover" is larger if the neighborhood region's percentage portion of FEVD is larger, and is positive (negative) if the cumulative response to the impact from its neighbor is positive (negative).

A multi-level version of the above equations is shown in equations (5)~(7):

$$\begin{bmatrix} y_t^1 \\ \vdots \\ y_t^R \end{bmatrix} = \begin{bmatrix} \delta^1(L) & \gamma^1(L) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \delta^R(L) & 0 & \dots & \gamma^R(L) \end{bmatrix} \begin{bmatrix} g_t \\ f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} + \begin{bmatrix} u_t^1 \\ \vdots \\ u_t^R \end{bmatrix} \quad (5)$$

$$D^r(L)u_t^r = \varepsilon_t^r \quad \forall\, r \in \{1, \dots R\} \quad (6)$$

[3] A more detailed introduction of dynamic factor model can be found in Chung (2013), and the estimation scheme can be found in Bai and Wang (2012).

$$\begin{bmatrix} g_t \\ f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1\,(R+1)} \\ \vdots & \ddots & \vdots \\ \varphi_{(R+1)1} & \cdots & \varphi_{(R+1)(R+1)} \end{bmatrix} \begin{bmatrix} g_{t-1} \\ f_{t-1}^1 \\ \vdots \\ f_{t-1}^R \end{bmatrix} + \begin{bmatrix} \eta_t^g \\ \eta_t^1 \\ \vdots \\ \eta_t^R \end{bmatrix} \quad (7)$$

where y_t^r is $p \times 1$ vector of endogenous regional observations,
f_t^r is unobservable fundamental forces that affect the dynamics of y_t^r, and
g_t is unobservable fundamental force that affects the dynamics of $(y_t^1, \dots, y_t^R)$
$E(\eta_t^r \mid g_{t-1}, f_{t-1}^1, \dots, f_{t-1}^R) = 0 \quad \forall\, t \in \{1, \dots, T\}\ and\ \forall\, r \in \{g, 1, \dots, R\}$,
$E(u_t^r | g_t, f_t^1, \dots, f_t^R) = 0 \quad \forall\, t \in \{1, \dots, T\}\ and\ \forall\, r \in \{1, \dots, R\}$,
$E\left(u_t^{ri} u_t^{rj} \middle| g_t, f_t^1, \dots, f_t^R\right) = 0 \quad \forall\, i \neq j \in \{1, \dots, p\}, \forall\, t \in \{1, \dots, T\}\ and\ \forall\, r \in \{1, \dots, R\}$,
and
$E\left(u_t^a u_t^b \middle| f_t^1, \dots, f_t^R\right) = 0 \quad \forall\, t \in \{1, \dots, T\}$ and $\forall\, a \neq b, r \in \{1, \dots, R\}$.

The equations (5)~(7) are similar to the equation system (2)~(4), but now it assumes that there exists a global shock, g_t, which governs the behavior of all the regional units.
Using Gibbs Sampling Algorithm in WinBUGS[4] [5], FEVD and CIRF are estimated.

3.2 Exercises on the Selected Regions with Different Scale Units

A single-level dynamic factor model (equations 2~4) and a multi-level dynamic factor model (equations 5~7) are estimated on the selected regional employment series with different temporal frequencies and spatial scales: monthly frequency county level, group level, state level and regional division level data are analyzed, and the county level monthly, quarterly and biannual frequency data are also analyzed. At the county level, Peoria, Tazewell, McLean, Champaign and Vermilion Counties in Illinois were selected, since outside around those counties as a group, populations are very sparse, so those five counties are thought to have a natural common border. At the group level, those five counties are referred to as the "I74" group.[6] Other groups are the St.Louis Group, Quad City Group, Springfield Group and Chicago Group that are all located inside or close to the state of Illinois. Each group consists of 2~12 counties, and, like "I74" group, populations are very sparse outside the counties consisting each group. The detailed descriptions are provided in figure 1.

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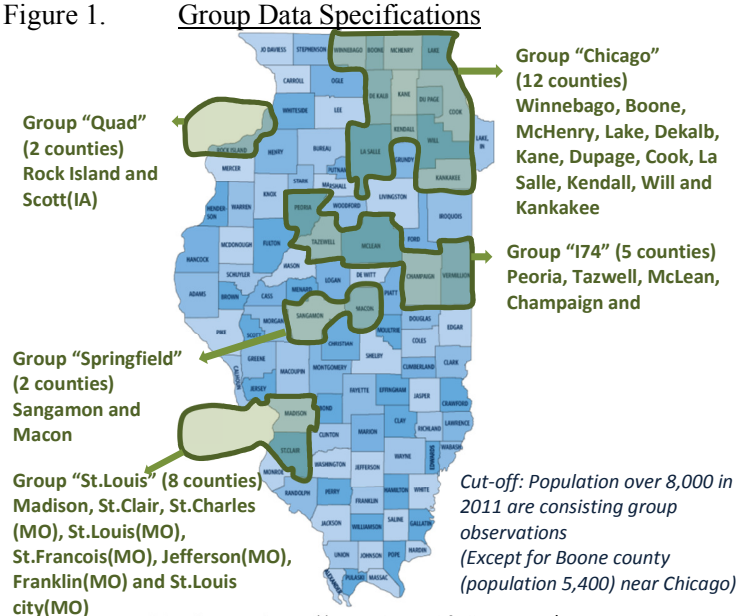
[4] Bayesian Inference Using Gibbs Sampling for Windows

[5] Only VAR(1) structure model estimation results are presented in section 3.2 since, according to the Deviations Information Criteria (DIC), any higher lag order does not outperform VAR(1) structure of equations introduced in section 3.1. Also, the Bayesian inference relies on priors, but for this exercise, uninformative priors were used. More detailed procedures can be found in Chung (2013).

[6] It is named after the interstate highway I-74, because those counties are located along this highway.

Figure 1. Group Data Specifications



Map Source: https://www.donatelifeillinois.org/wp-

At the state level, six Great Lake states, Illinois, Indiana, Michigan, Minnesota, Ohio and Wisconsin, were selected. The regional division level units are simply the four U.S. regional divisions (Northeast, Midwest, South and West).[7]. To compare the consequences of the use of different temporal frequencies of observations, county level monthly data were aggregated to generate quarterly and biannual series.

Using those data sets, a single-level dynamic factor model (equations 2~4) and a multi-level dynamic factor model (equations 5~7) were estimated. The FEVD results with different temporal scale derived from a multi-level model are presented in table 1[8].

Table 1. Temporal Aggregation using a Multi-level Structure Model
- Point Estimates of 1-year ahead Forecast Error Variance Decomposition

* Capital letters denote regions, for example, "G" denotes region common shock, and A: Peoria County, B: Tazewell County, C: McLean County, D: Champaign County, E: Vermilion County

County Level – monthly frequency

	A	B	C	D	E	G	Total		neighbor	own	common
A	23.4%	65.7%	2.8%	3.7%	3.1%	1.4%	100%		75.2%	23.4%	1.4%
B	18.3%	69.8%	4.1%	3.6%	2.6%	1.6%	100%		28.6%	69.8%	1.6%
C	15.2%	75.9%	1.2%	3.9%	3.3%	0.5%	100%		98.2%	1.2%	0.5%
D	32.3%	43.2%	16.3%	3.0%	1.0%	4.2%	100%		92.8%	3.0%	4.2%
E	28.4%	41.1%	13.2%	3.1%	3.9%	10.3%	100%		85.8%	3.9%	10.3%
								average	**76.1%**	20.3%	3.6%

[7] Except for the regional division level spatial units, the selection of the regional units at all other levels suffer borderline problem since there is a possibility that some relevant regional unit could have been omitted. However, further consideration of solving this borderline problem was not tried here since this section aims to sketch how the aggregation of spatial temporal unit affects the observed spillover effects in the real world data, and does not aims to exactly identify the regional economic system.

[8] The results for single-level model are available in Appendix 1. Appendices are available at www.real.illinois.edu/d-paper/14/AggAppendix.pdf

County Level – quarterly frequency

	A	B	C	D	E	G	Total		neighbor	own	common
A	4.3%	1.8%	10.0%	4.7%	1.9%	77.3%	100%		18.4%	4.3%	77.3%
B	2.4%	1.4%	5.8%	3.1%	15.9%	71.4%	100%		27.2%	1.4%	71.4%
C	0.9%	0.4%	21.6%	1.7%	58.6%	16.8%	100%		61.6%	21.6%	16.8%
D	1.8%	0.7%	22.5%	5.9%	55.3%	13.8%	100%		80.2%	5.9%	13.8%
E	1.5%	0.2%	34.6%	2.1%	60.6%	1.0%	100%		38.4%	60.6%	1.0%
								average	**45.1%**	18.8%	36.1%

County Level – biannual frequency

	A	B	C	D	E	G	Total		neighbor	own	common
A	0.2%	0.4%	1.0%	0.0%	0.0%	98.3%	100%		1.5%	0.2%	98.3%
B	0.7%	69.6%	4.4%	0.3%	2.4%	22.5%	100%		7.9%	69.6%	22.5%
C	0.0%	26.9%	1.7%	3.8%	0.0%	67.6%	100%		30.7%	1.7%	67.6%
D	0.6%	84.1%	3.1%	9.5%	2.3%	0.4%	100%		90.1%	9.5%	0.4%
E	2.9%	70.1%	4.5%	6.7%	8.2%	7.7%	100%		84.1%	8.2%	7.7%
								average	**42.9%**	17.8%	39.3%

The point estimates of 1-year ahead FEVD using a multi-level structure model shows that although the amount of spillover varies a lot depending on the individual regional units, on average, larger temporal scale observations generate less neighborhood portion of FEVD. Accordingly, as the temporal scale becomes larger, the portion from the innovation from the region own decreases, but the innovation from the region common factor increases. On the contrary, a single-level structure model shows no specific pattern related to the level of temporal aggregation, having the neighborhood portion of variance about 70% regardless of the temporal scale.

The FEVD results with different spatial scales exhibit a similar trend for the multi-level structure model. In table 2, the portion of the neighborhood innovation decreases with larger scale of spatial units on average, but the portion of the region common factor increases. In similar fashion to the case of temporal aggregation, a single-level structure model does not exhibit any specific trend with the level of the spatial aggregation.[9]

Table 2. Spatial Aggregation using a Multi-level Structure Model
- Point Estimates of 12-step ahead Forecast Error Variance Decomposition

* Capital letters denote regions, for example, "G" denotes region common shock.

[9] The results for single-level model are available in Appendix 2.

County Level

* A: Peoria County, B: Tazewell County, C: McLean County, D: Champaign County, E: Vermilion County

	A	B	C	D	E	G	Total		neighbor	own	common
A	23.4%	65.7%	2.8%	3.7%	3.1%	1.4%	100%		75.2%	23.4%	1.4%
B	18.3%	69.8%	4.1%	3.6%	2.6%	1.6%	100%		28.6%	69.8%	1.6%
C	15.2%	75.9%	1.2%	3.9%	3.3%	0.5%	100%		98.2%	1.2%	0.5%
D	32.3%	43.2%	16.3%	3.0%	1.0%	4.2%	100%		92.8%	3.0%	4.2%
E	28.4%	41.1%	13.2%	3.1%	3.9%	10.3%	100%		85.8%	3.9%	10.3%
								average	**76.1%**	20.3%	3.6%

Group Level

* A: St.Louis Group, B: Quad Cities Group, C: Springfield Group, D: Chicago Group, E: I74 Group

	A	B	C	D	E	G	Total		neighbor	own	common
A	3.1%	7.2%	65.0%	18.6%	0.5%	5.7%	100%		91.2%	3.1%	5.7%
B	1.8%	12.1%	64.0%	18.3%	0.5%	3.2%	100%		84.7%	12.1%	3.2%
C	1.9%	1.2%	84.9%	8.0%	0.2%	3.8%	100%		11.3%	84.9%	3.8%
D	1.4%	6.7%	62.2%	24.4%	1.0%	4.4%	100%		71.3%	24.4%	4.4%
E	2.0%	7.3%	59.5%	21.6%	1.4%	8.1%	100%		90.5%	1.4%	8.1%
								average	**69.8%**	25.2%	5.0%

State Level

* A: Illinois, B: Indiana, C: Michigan, D: Minnesota, E: Ohio, F: Wisconsin

	A	B	C	D	E	F	G	Total	neighbor	own	common
A	79.1%	2.3%	7.9%	6.2%	0.8%	0.2%	3.5%	100%	17.4%	79.1%	3.5%
B	57.5%	2.4%	29.2%	4.4%	0.4%	0.6%	5.5%	100%	92.1%	2.4%	5.5%
C	7.6%	0.2%	70.3%	10.3%	4.7%	3.1%	3.8%	100%	25.9%	70.3%	3.8%
D	4.8%	0.2%	6.4%	77.5%	3.3%	2.7%	5.2%	100%	17.3%	77.5%	5.2%
E	24.2%	1.4%	3.3%	2.3%	65.4%	0.1%	3.3%	100%	31.3%	65.4%	3.3%
F	8.4%	0.4%	12.4%	46.1%	11.3%	3.7%	17.6%	100%	78.7%	3.7%	17.6%
								average	**43.8%**	49.7%	6.5%

Regional Division Level

* A: Northeast Regional Division, B: Midwest Regional Division, C: South Regional Division, D: West Regional Division

	A	B	C	D	G	Total		neighbor	own	common
A	16.2%	0.2%	4.4%	0.3%	78.9%	100%		4.9%	16.2%	78.9%
B	0.7%	2.1%	0.2%	0.1%	96.9%	100%		1.0%	2.1%	96.9%
C	3.1%	0.3%	2.7%	0.8%	93.1%	100%		4.2%	2.7%	93.1%
D	8.0%	0.2%	2.5%	0.4%	88.9%	100%		10.7%	0.4%	88.9%
							average	**5.2%**	5.3%	89.5%

A regional economic performance attributable to neighborhood or region common factor is thought to be dependent on its geographical location or the relative position in the value chain, but the portion on average seems to exhibit some kind of monotonic trends related to the level of temporal/spatial aggregation in a multi-level structure model. If there really exists trends, then

it should be in concord with the thought experiment of a constructed regional economic system presented in the next section, and it is found that the real world results from the multi-level structure model are consistent in most aspects with the results drawn from the exercise on the constructed regional economic system.

Regarding the direction of the spillovers (CIRFs), no monotonic trend was found.[10] In the multi-level structure model, a positive spillover dominates in the most spatially aggregated observations (division level), but in the single-level structure model, a positive spillover dominates in the third-level spatial aggregation (state level). However, it is hard to say there is any trend existing related to the level of temporal/spatial aggregation (table 3).

Table 3. Numbers of the Signs of the Cumulative Responses to the Impulses from Neighboring Regions

Multi-level

sign of response	-	+	0	total	#+/#-
county	7	8	5	20	**1.1**
group	3	2	15	20	**0.7**
state	7	10	13	30	**1.4**
division	1	2	9	12	**2.0**
quarterly	4	4	12	20	**1.0**
biannual	4	6	10	20	**1.5**

Single-level

sign of response	-	+	0	total	#+/#-
county	5	8	7	20	**1.6**
group	4	6	10	20	**1.5**
state	5	21	4	30	**4.2**
division	4	8	0	12	**2.0**
quarterly	5	7	8	20	**1.4**
biannual	4	6	10	20	**1.5**

* Signs are decided according to the 90% confidence interval. (Thus, "0" means insignificant at 90% CI)
* For county, group, state and division observations, 12-step ahead CIRFs were used. For quarterly and biannual observations, 1-year ahead CIRFs were used.

In the next section, by constructing an artificial regional economic system, the consequences of the temporal/spatial aggregation over the amount and the direction of spillovers are compared with the results from this section's empirical findings.

[10] CIRF results are provided in appendix 3 (multi-level dynamic factor model) and in appendix 4 (single-level dynamic factor model).

4. Practice on Constructed Regional Economic System

4.1 Assumptions on the Constructed Regional Economic System

The aggregation scheme that we are using on our practice is average sampling[11], i.e., $Aggregated\ Observation\ at\ t = \sum Disaggregated\ Observations\ during\ t$. Also, the true data generating process is defined at the most disaggregated level.

The first assumption that the aggregation scheme should be average sampling is not applicable to most empirical analyses in its original form, but in many cases, aggregations are approximately average sampling. In the previous section, the dependent variable at the aggregated level is constructed from the aggregation of the disaggregated level data, log-transformed, and first-differenced. Thus, this example is not an average sampling per se. However, since the first-differenced value of log-transformed value is a first-order Taylor-expansion of the growth rate, and assuming that the initial status of the observations are approximately the same, the arithmetic average of the growth rate is almost the same as the first-differenced log-transformed observations.[12]

The second assumption that the true data generating process is defined at the most disaggregated level is necessary in order to conduct the experiments in this section. The reason is that if the true data generating process is defined at an aggregated level, we do not have to discuss what is going on at the disaggregated level because every significant interaction identified at the disaggregated level will all be spurious. Additionally, in a real world situation, spatial interactions are mostly vivid at the disaggregated level.[13]

In addition to those two restrictions, only a spatially stationary data generating process[14] is employed here, since a spatially non-stationary process is not within the scope of this paper.

The artificial regional economic system is consisted of 1,024 cities arranged in 32×32 rectangular country. Four cities comprise one county, thus there are 256 counties arranged in 16×16 panel. Likewise, four counties comprise one group (64 groups), four groups comprise one state (16 states), four states comprise one division (4 divisions). The graphical representation of the land structure is shown in figure 2.

---

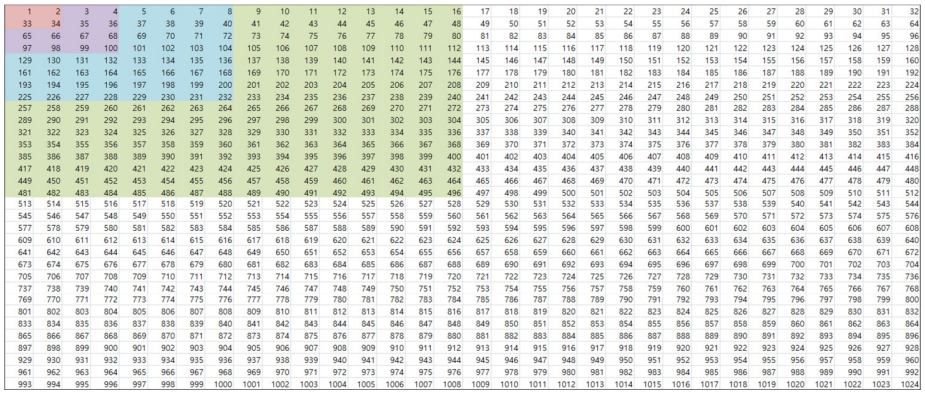
[11] If a time series is stock data, the aggregation of the time series can be a point-in-time sampling, for example, $Aggregated\ Observation\ at\ t = Last\ Disaggregated\ Observation\ during\ t$. For more detail, see Marcellino (1999).

[12] For example, $ln(a_t + b_t + c_t) - ln(a_{t-1} + b_{t-1} + c_{t-1}) \approx \frac{a_t - a_{t-1} + b_t - b_{t-1} + c_t - c_{t-1}}{a_{t-1} + b_{t-1} + c_{t-1}} \approx \frac{1}{3}(\frac{a_t - a_{t-1}}{a_{t-1}} + \frac{b_t - b_{t-1}}{a_{t-1}} + \frac{c_t - c_{t-1}}{a_{t-1}}) \approx \frac{1}{3}(\frac{a_t - a_{t-1}}{a_{t-1}} + \frac{b_t - b_{t-1}}{b_{t-1}} + \frac{c_t - c_{t-1}}{c_{t-1}}) \approx \frac{1}{3}\{(ln\, a_t - ln\, a_{t-1}) + (ln\, b_t - ln\, b_{t-1}) + (ln\, c_t - ln\, c_{t-1})\}$.

[13] In February 2013, Beverly 18, a movie theater in Champaign, IL, closed, and shortly after, Savoy 16, a neighborhood movie theater in Savoy, IL opened a new I-Max theater, which provides an example of a negative spatial spillover effect at the disaggregated level data.

[14] That is, for example, the root of $|I_R - A\, z| = 0$ from equation (8) fall outside the unit circle.

Figure 2. Constructed Regional Economy



The true data generating process is defined at city level as shown in equation (8):[15]

$$Y_t = \mathrm{A}Y_{t-1} + \varepsilon_t,\ \mathrm{t} = 1, \dots, \mathrm{T} \qquad (8)$$

where, for a single-level structure,

$$Y_t = (y_t^1, y_t^2, \dots\ y_t^{1024})',\ \mathrm{A} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,1024} \\ \vdots & \ddots & \vdots \\ a_{1024,1} & \cdots & a_{1024,1024} \end{pmatrix} \text{ and}$$

$$\varepsilon_t = (\varepsilon_t^1, \varepsilon_t^2, \dots\ \varepsilon_t^{1024}) \sim \mathrm{N}(0, \sigma^2 I_{1024}),\ \sigma^2 = 1,$$

and for a multi-level structure,

$$Y_t = (y_t^C, y_t^1, y_t^2, \dots\ y_t^{1024})',\ \mathrm{A} = \begin{pmatrix} a_{C,C} & a_{C,1} & \cdots & a_{C,1024} \\ a_{1,C} & a_{1,1} & \cdots & a_{1,1024} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1024,C} & a_{1024,1} & \cdots & a_{1024,1024} \end{pmatrix} \text{ and}$$

$$\varepsilon_t = (\varepsilon_t^C, \varepsilon_t^1, \varepsilon_t^2, \dots\ \varepsilon_t^{1024}) \sim \mathrm{N}(0, \sigma^2 I_{1024}),\ \sigma^2 = 1.$$

$\{y_t^r \mid r = 1, \dots, 1024\}$ are dependent variables representing the regional economic behaviors,

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[15] A more general version of this kind of structure can be expressed as equation (*):

$$Y_t = \sum_{i=1}^{p} A_i Y_{t-i} + \varepsilon_t,\ \ t = 1, \dots, T \qquad (*)$$

where $Y_t = (y_t^1, y_t^2, \dots\ y_t^R)'$ is an $R \times 1$ dependent variables,

$\{A_i | i = 1, .., p\}$ are $R \times R$ coefficient matrices,

$\mathrm{E}(\varepsilon_t) = 0, \mathrm{E}(\varepsilon_t \varepsilon_t') = \Sigma$ (a positive definite coviariance matrix), $\mathrm{E}(\varepsilon_t \varepsilon_{t'}') = 0\ \forall t \neq t'$

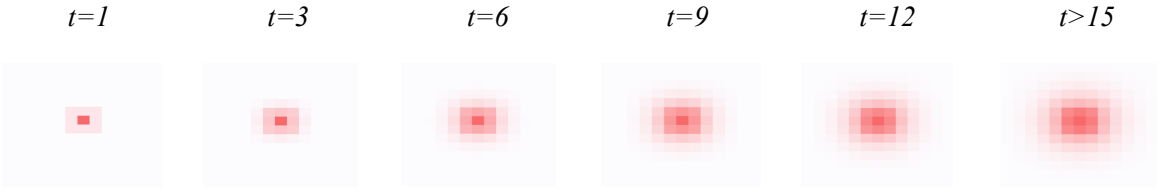
For stationarity, all roots of $\left| I_R - \sum_{i=1}^{p} A_i z^i \right| = 0$ fall outside the unit circle. Also, (*) can be expressed as VMA form as in equation (**):

$Y_t = \sum_{i=1}^{\infty} \Phi_i \varepsilon_{t-i}$, where $\Phi_i = \sum_{j=1}^{p} A_j \Phi_{i-j}, i = 1, 2, ...,$ and $\Phi_0 = I_R$ and $\Phi_i = 0\ \forall \mathrm{i} < 0$. Since in this general case where the error term structure is not diagonal, the time profile of the shock effects the FEVD and CIRF, thus in our case, the error term structure is set to be diagonal for simplicity.

whereas y_t^C is a region common factor. As noted earlier, the region common factor is little affected by local shocks by assumption, the elements, $\{a_{C,i} \mid i = 1, \dots, 1024\}$, are set to be zero in the coefficient matrix. One might claim that this exogeneity assumption is rather extreme because in reality, some spatial units or events can be powerful enough to affect the region common behavior. However, in the case where the region common factor is endogenous, the region common component is equivalent to just adding another regional unit in equation (8) with a single-level structure. Thus, by looking at the generated FEVD and CIRF results of the single-level structure model, we can easily conjecture that the results are drawn from an endogenous region common factor because the results should lie somewhere between multi-level structure model and single-level structure model.

The above data generating process defines how the constructed cities interact through time and space. In this economy, every regional interaction is determined by the coefficient matrix A such that a growth in region j at time $t-1$ will induce $a_{i,j}$ growth in region i at time t (neighborhood effect). Note that the above process is defined using demeaned variables, thus at the steady state, the growth rate of every region will grow at the historical average, and $Y_{steady\ state} = 0$. Thus, a typical local policy shock will exhibit a CIRF graph such as figure 3.[16]

Figure 3. A Typical CIRF of Spatially Stationary Process



For CIRF, since the region common factor is exogenous in a multi-level structure economy, we do not have to use a multi-level version of the model in analyzing the transmission of the regional shock through space. In other words, we can regard the single-level structure model as the residuals of the regional economic system net of the national factor, and then conduct the CIRF analysis. On the contrary, for FEVD, the existence of the region common shock sometimes alters the overall trend over the aggregation. Thus, FEVD results are presented for both multi-level and single-level structure economy, whereas CIRF results are presented for only single-level structure economy.

4.2 Derivation of Aggregated Level FEVD and CIRF

The derivation of the data generating process at the aggregated level is not very useful in our

[16] A graphical example of a CIRF of a spatially non-stationary process is provided in appendix 5.

analysis of spillover effects. Since the purpose of this paper is to see what we expect to observe at the aggregated level in terms of spillovers, and spillovers are defined by FEVD and CIRF in this paper, we can directly look at the aggregated form of FEVD and CIRF instead of deriving the aggregated level data generating process.

The derivation of the data generating process for the temporally aggregated observations is relatively easier than that for the spatially aggregated observations. In our case where there is only one lag dependent variable and i.i.d. error term with unit variance, the VAR(1) form can be preserved. For an $\mathrm{n}-$period aggregation, i.e., $Y_\tau = \sum_{i=0}^{n-1} Y_{t-i}$, equation (8) simply transforms into $Y_\tau = A^n Y_{\tau-1} + \epsilon_\tau$, where $\epsilon_\tau = \sum_{i=0}^{n-1}(A^i \sum_{j=0}^{n-1} \varepsilon_{t-i-j})$. However, in practice, since the error term, ϵ_τ, is the superposition of *n*-multivariate normal distributions, if we assume approximate normality on the error term of the aggregated form, then it suffers a problem that we are automatically assuming the effects of the innovations at all *n*-sub periods are the same within one time period at the aggregated level. In this case, the economic translation of FEVD or CIRF using the aggregated form of data generating process becomes different from the original disaggregated data generating process. In more detail, for the case of FEVD, an element $\theta_{rs}(h)$ in FEVD is defined as the proportion of the *h*-step ahead forecast error variance of region *r* that is accounted for by the innovations in region *s*. Since $\epsilon_\tau = \sum_{i=0}^{n-1}(A^i \sum_{j=0}^{n-1} \varepsilon_{t-i-j})$, innovations at sub periods *t-i* have different covariance structures.[17] Thus, when we derive FEVD using the aggregated form of the data generating process, we are ignoring these differences, and same logic applies to CIRF. For example, an innovation in January and one in March are treated as the same when we derive *h*-quarter ahead forecasting error variance decomposition with the assumption of the normality of ϵ_τ.

Similar problem occurs when we derive the data generating process for the spatially aggregated observations. Following Arbia and Petrarca (2011), suppose G is an $\mathrm{S} \times \mathrm{R}$ aggregation matrix where $\frac{\mathrm{R}}{\mathrm{S}} = \mathrm{n}$ (each aggregated level spatial unit contains the same n number of disaggregated level units).[18] Then, an aggregated form of the data generating process can be expressed as $Y_t^* = BY_{t-1}^* + \varepsilon_t^*$ where $Y_t^* = GY_t$, $B = GAG'(GG')^{-1}$ and $\varepsilon_t^* = G\varepsilon_t$. If we assume approximate normality, the aggregated level error term will be expressed as $\varepsilon_t^* \sim (0, n\sigma^2 I)$, as in Arbia and Petrarca (2011). However, as a matter of fact, ε_t^* is a superposition of multivariate normal distributions, thus suffering the same problem as in the temporal aggregation case. In other words, for example in our constructed regional economic system, FEVD or CIRF analysis with approximate normality assumption will treat a unit shock

[17] For example, $\varepsilon_{t-2} \sim N(0, \sigma^2(I + A + A^2)(I + A + A^2)^T)$.

[18] For example, when aggregating two units into one aggregated level unit (n=2) where there are four spatial units, $\mathrm{G} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$.

on peripheral region such as region #1 the same as a unit shock on the region located closer to the center such as region #34.[19]

Since our objective is to see what we will observe at the aggregated level, we do not have to numerically derive the aggregated form of the data generating process. Instead, we can simulate some coefficient structure and visualize FEVD and CIRF as the same fashion in section 3. In this manner, we can avoid the problem mentioned above.

In order to visualize what will happen in FEVD and CIRF, another assumption that the shocks at disaggregated level are distributed evenly across the initial aggregated period has been made.[20] Thus, the economic meaning of FEVD at the aggregated level, for example, an element $\theta_{rs}(h)$ in FEVD is defined as the proportion of the h-step ahead forecast error variance of region *r* that is accounted for by the same amount of innovations in region *s* at time $\mathrm{t} = 0, 1, \dots, \mathrm{n}-1 \ (\in \tau = 0)$. A similar logic also applies for CIRFs.

With the above assumption, since the disaggregated level FEVD is as in equation (9), the aggregated level FEVD can be derived as in equation (10):[21]

$$\theta_{rs}(h) = \frac{\sum_{l=0}^{h}(e_r' A^l e_s)^2}{\sum_{l=0}^{h}(e_r' A^l (A^l)' e_r)} \tag{9}$$

where e_r is a $\mathrm{R} \times 1$ selection vector (where the r^{th} element=1 with zeros elsewhere)

$$\theta_{rs}(H) = \frac{\sum_{m=0}^{n-1}\sum_{l=m}^{h+m}(e_r' A^l e_s)^2}{\sum_{m=0}^{n-1}\sum_{l=m}^{h+m}(e_r' A^l (A^l)' e_r)} \tag{10}$$

where $H \ni \{h, h+1, \dots, h+n-1\}$

For example, when aggregating a monthly data into a quarterly data, FEVD will be looking at 12th month for the monthly model, and 12~14 months for the quarterly model, as shown in figure 4. In this case, the one year ahead FEVD becomes a weighted average of 12~14 months ahead monthly level FEVD with three consecutive monthly shocks at the first three months.

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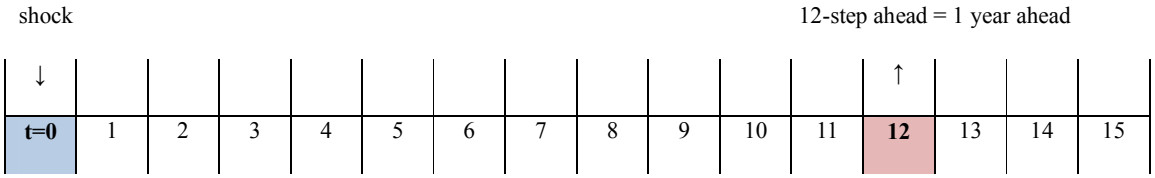
[19] More specifically, when a regional shock spills over to region sharing borders (rook contiguous), then region #1 spills over to regions #2 and #33 whereas region #34 spills over to regions #2, #33, #35 and #66.

[20] For example, aggregating at the quarterly interval, FEVD and CIRFs are calculated assuming that the same amounts of shocks are given for the first three months. We can also simulate and visualize the case that shocks are unevenly distributed across within an aggregated time period, but since there are infinitely many cases of uneven distributions, and since even distribution is representative, only evenly distributed case is visualized here.

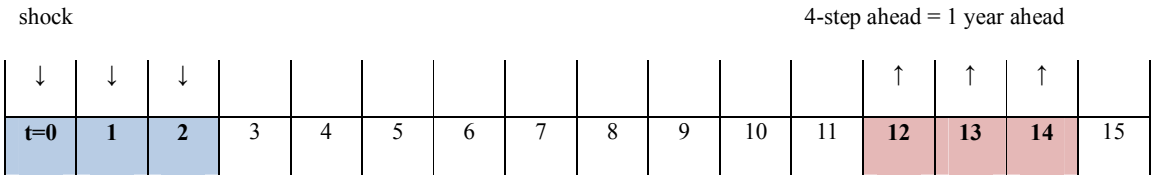
[21] For more general case, as in equation (*), FEVD can be derived as $\theta_{rs}(h) = \frac{\sum_{l=0}^{h}(e_r' \Phi_l P e_s)^2}{\sum_{l=0}^{h}(e_r' \Phi_l \Sigma \Phi_l' e_r)}$. Thus, a temporally aggregated version of FEVD should be $\theta_{rs}(H) = \frac{\sum_{m=0}^{n-1}\sum_{l=m}^{h+m}(e_r' \Phi_l P e_s)^2}{\sum_{m=0}^{n-1}\sum_{l=m}^{h+m}(e_r' \Phi_l \Sigma \Phi_l' e_r)}$.

Figure 4. Conceptual Comparison of One-year-ahead FEVD of Monthly and Quarterly Data

Monthly Data



Quarterly Data



CIRFs can be derived using the same logic.[22]

Similarly, from equation (9), a spatially aggregated version of FEVD can be expressed as equation (11):[23]

$$\theta_{RS}(h) = \frac{\sum_{l=0}^{h}(e_R' A^l e_S)^2}{\sum_{l=0}^{h}(e_R' A^l (A^l)' e_R)} \qquad (11)$$

where $R \; and \; S$ are aggregated spatial units, thus $e_R \; and \; e_S$ are $\mathrm{R} \times 1$ selection vectors where any r^{th} and s^{th} elements that belong to R and S respectively are ones and zeros elsewhere.

For the temporal aggregation, the disaggregated level (first level) observations are aggregated with 2 (second level) ~ 12 (twelfth level) periods of time. For the spatial aggregation, the

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[22] If there is a unit shock to region r, h-step ahead CIRF can be calculated as $\varphi_r(h) = \sum_{l=0}^{h} A^l e_r$ or in a more general case as in equation (*), $\varphi_r(h) = \sum_{l=0}^{h} \Phi_l P e_r$. Thus, a temporally aggregated version of CIRF can be expressed as $\varphi_r(H) = \sum_{m=0}^{n-1}\sum_{l=0}^{h+m} A^l e_r$, or from equation (*),$\varphi_r(H) = \sum_{m=0}^{n-1}\sum_{l=0}^{h+m} \Phi_l P e_r$. However, in this case, in order to see the response of unit shock on the aggregated spatial unit, the amount of shock should be $1/n$, i.e., the selection vector e_r is composed of $1/n$ for its r-th element, zeros elsewhere.

[23] Likewise before, more generalized version of the spatially aggregated version of FEVD can be expressed as $\theta_{RS}(h) = \frac{\sum_{l=0}^{h}(e_R' \Phi_l P e_S)^2}{\sum_{l=0}^{h}(e_R' \Phi_l \Sigma \Phi_l' e_R)}$. CIRFs can also be expressed as $\varphi_R(h) = \sum_{l=0}^{h} A^l e_R$ or for more general case (from equation (*)), $\varphi_R(h) = \sum_{l=0}^{h} \Phi_l P e_R$. In the case of CIRFs, the selection vectors also should be $\frac{1}{n}, (n \; is \; the \; number \; of \; units \; that \; belong \; to \; aggregated \; level \; of \; a \; spatial \; unit)$ for the elements that belong to R, zeros elsewhere.

disaggregated level (first level) observations are aggregated by four regional units at each level, up to the fifth level where the number of regional units is only four.

4.3 Change of the Neighborhood Portion with Aggregation – FEVD results

When we disaggregate the sources of the variances of regional activities into neighborhood and its own (and region common for the multi-level structure) innovations, on average, the neighborhood portion increases with more temporal aggregation, but it decreases with more spatial aggregation. However, in a multi-level structure regional economy, when the effect from the region common factor is relatively large, resembling the real world case, the neighborhood portion does not monotonically increase with the temporal aggregation: the neighborhood portion increased up to a certain level aggregation, and then it decreased afterwards.

The FEVDs with various values of coefficients are calculated. In equation (8), the autoregressive coefficient for region common factor is set to be 0.2 ($a_{C,C} = 0.2$), and the effect from the region common factor is set to be 0.15 ($\{a_{i,C} = 0.15 \mid i = 1, \dots, 1024\}$). The autoregressive coefficient of regions own is set to be 0.1 ($\{a_{i,i} = 0.1 \mid i = 1, \dots, 1024\}$). Each regional unit is assumed to be affected by its neighbors sharing common borders (rook contiguous), and various values of the effects are used, from 0.04~0.22 for the positive neighborhood effects, and -0.04~-0.22 for the negative neighborhood effects.[24]

The FEVDs of individual regional units vary a great deal depending on their locations; thus, it is not appropriate to try to find a specific pattern related to the change of the scale of units by looking at the individual level regional units.[25] However, when we average out our observations, we can see a clear pattern. The results for the temporal aggregations are presented in figure 5.[26]

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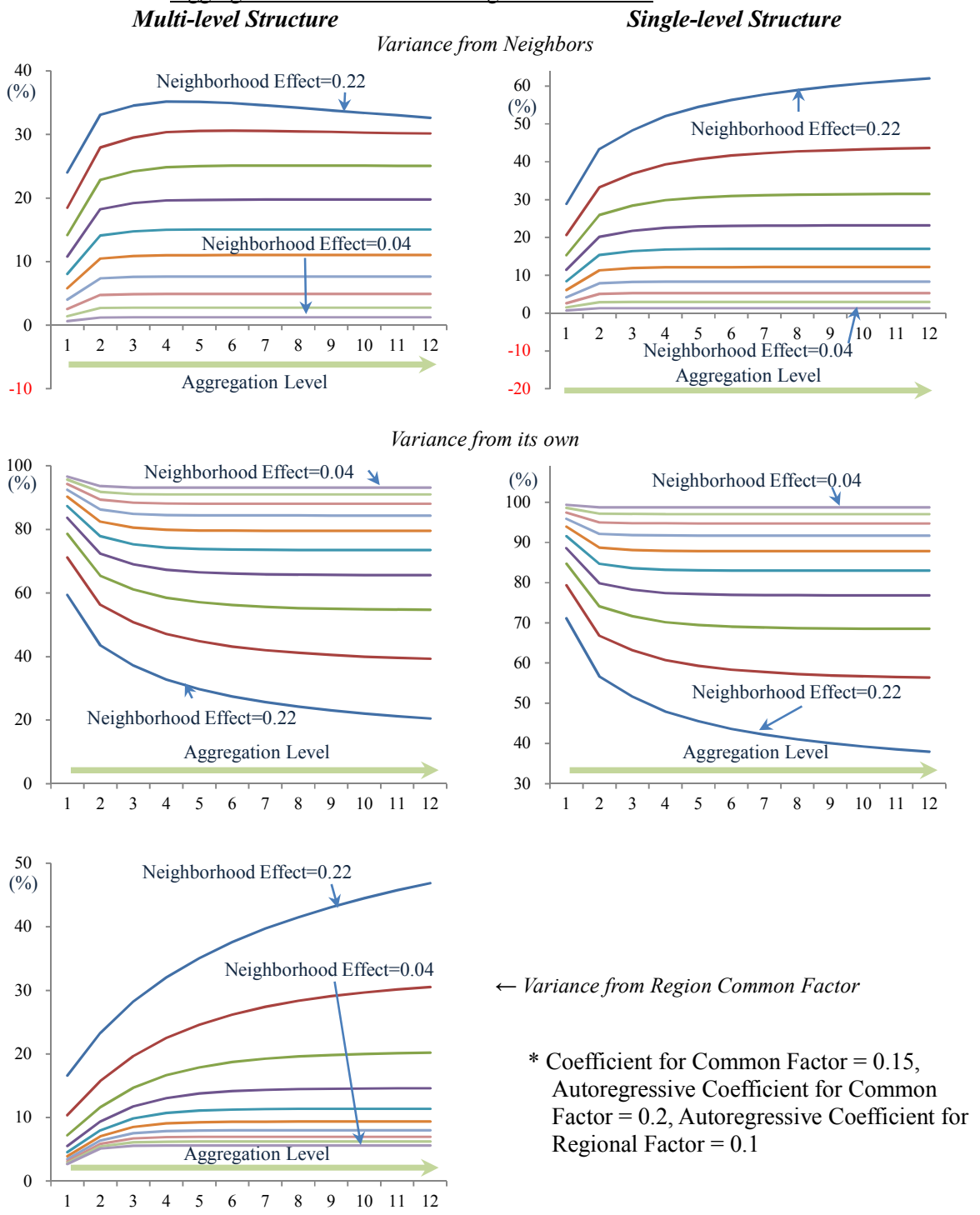
[24] In other words, the coefficients for those regions sharing borders are assigned with values ranging from -0.04~-0.22 & 0.04~0.22. For the neighborhood effect values bigger than 0.22 or less than -0.22 are not tried here because they are spatially non-stationary. Other values of autoregressive coefficient for region common factor, effect from the region common factor and autoregressive coefficient of regions own are also tried, but not presented here, because the implications drawn from the results are same.

[25] As it is already shown in the real world data example, the portion of neighborhood effect varies depending on the region. For our constructed regional economy, the variance of a region located at the border has larger portion of its own innovations.

[26] The results do not change much when the neighborhood coefficients are negative. Appendix 6 provides the results.

Figure 5. One-year-ahead Forecast Error Variance Decomposition with Different Temporal Aggregation Level - Positive Neighborhood Effect



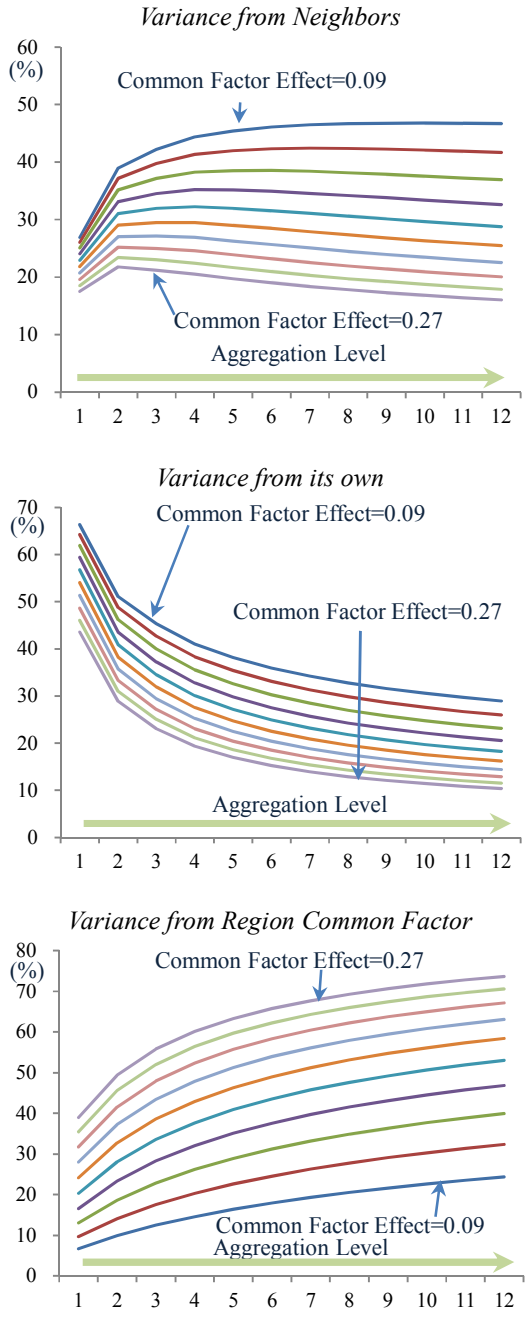
* Coefficient for Common Factor = 0.15, Autoregressive Coefficient for Common Factor = 0.2, Autoregressive Coefficient for Regional Factor = 0.1

Overall, both in multi-level structure economies and in single-level structure economies, regardless of whether the neighborhood effects are negative or positive, the portion of the neighborhood, or the amount of spillovers, increase with the level of aggregation, but the speed of increase decreases with the aggregation level. However, for the multi-level structure economy with a large neighborhood effect (in this case, 0.22), the spillover decreased after some level of aggregation (in this case, fourth level).

This difference in the trend of neighborhood portion, or the amount of spillover in the single-level economy and in the multi-level economy, is due to the existence of the region common component. In a single-level structure model, the spillover effects are propagating across multiple regional units and grow rapidly over time, whereas the autoregressive effect ($a_{i,i}$) remains in a single region, thus growing at a slower rate. Thus, the portion of the neighborhood innovation becomes larger with the temporal aggregations.[27] However, in a multi-level structure economy, since the region common innovation and the neighborhood innovations are both propagating across multiple regions, depending on their relative importance, the neighborhood portion decreases after a certain level of aggregation. For example, as shown in figure 6, when the effect from the common factor is 0.09 ($a_{i,C} = 0.09$), the neighborhood portion decreases after the eleventh level of aggregation, but when the effect from the common factor is 0.21 ($a_{i,C} = 0.21$), the neighborhood portion decreases after the third level of temporal aggregation. In sum, a larger value of the region common factor loading induced a more rapid decrease in the neighborhood portion. Conclusively, in a multi-level structure economy, the neighborhood portion of the variance can decrease or increase depending on the aggregation level and the relative importance of the region common factor, whereas in a single-level structure economy, the neighborhood portion of variance only increases with the level of temporal aggregation. Thus, at a more temporally aggregated observations, those researchers believing in the importance of the region common factor loadings will expect less spatial dependency, whereas those researchers not believing in the existence of region common factor will expect more spatial dependency.

[27] This phenomenon also appears when we assign the autoregressive coefficients larger value such as $a_{i,i} = 0.8$.

Figure 6. One-year-ahead FEVD with Different Temporal Aggregation Level - Different Common Factor Effects in Multi-level Economies



This trend over temporal aggregation concords with the average trends found in the multi-level estimation results from the real world data in the previous section. In the real world data, the portion of own factor decreased and that of common factor increased, generating the same results as those from our constructed regional economic system. Also, the portion of neighborhood innovation decreased in the real world data exercise, implying that the region common factor occupies a larger portion of the regional variances. On the contrary, for the single-level structure, it is hard to tell whether the constructed regional economic system and the real world exercise results are in agreement.

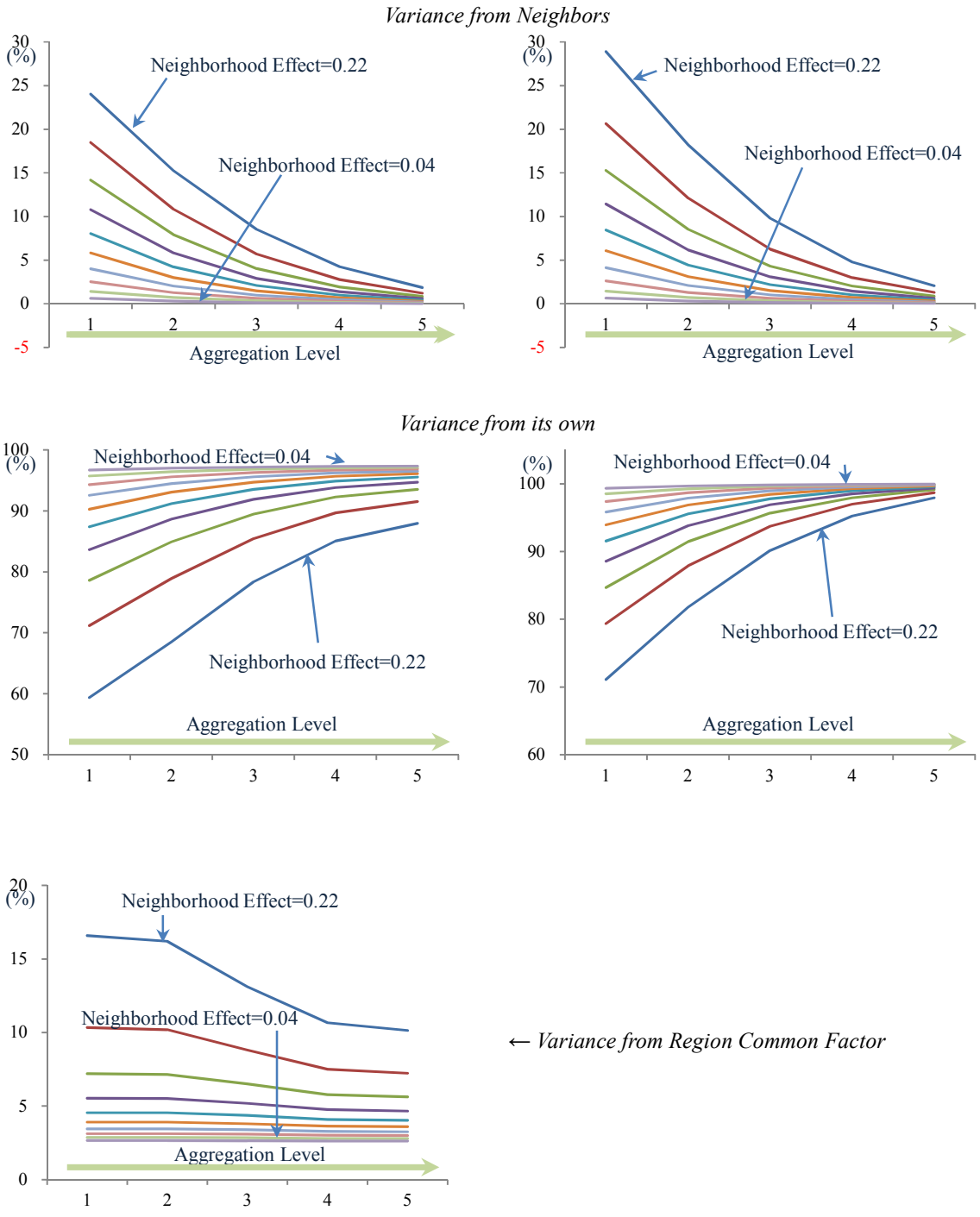
Contrary to the temporal aggregations, spatial aggregation shows a monotonic decrease of the neighborhood portion regardless of whether the regional economy is a multi-level structure or a single-level structure. As shown in figure 7, regardless of our choice of the values for the neighborhood effects, the amount of spillover decreases monotonically.[28]

* Neighborhood Effects are fixed at 0.22

[28] The results do not change much when the neighborhood coefficients are negative. Appendix 7 provides the results.

Figure 7. Twelve-step-ahead Forecast Error Variance Decomposition with Different Spatial Aggregation Level - Positive Neighborhood Effect

Multi-level Structure ***Single-level Structure***



This monotonic decrease with the level of spatial aggregation is mainly due to the fact that our regional economic system is spatially stationary. Since all the innovations are processed in a spatially stationary system, an effect of a local shock fades away with distance. Thus, a shock once regarded as a neighborhood innovation can become an own innovation at a more aggregated level. In other words, unlike the temporal aggregation exercise where the relative portions of sources of innovations depend on the speed of their propagation across regional units, the spatial aggregation exercise is comparable to watching the regional economic system with lenses with different focal lengths at the same time spot, as described in figure 8.

Figure 8. Example of Spatial CIRFs to a Local Shock with different levels of Aggregations

Observation at the Disaggregated Level	*Observation at the 2nd-level Aggregation*	*Observation at the 3rd-level Aggregation*	*Observation at the 4th-level Aggregation*
			

As shown in the above figure, at a disaggregated level, we see the cumulative response of a local shock at the most detailed precision, thus the responses around the innovation is categorized as neighborhood effect. However, at the 4th-level aggregation, those neighborhood effects are mostly trapped inside the aggregated spatial unit, thus the neighborhood effect almost disappears. Relating this idea with FEVDs, with the assumption that the innovations are equally generated across the regions, only those local innovations located near the borderline of the aggregated spatial unit can penetrate into the neighboring aggregated spatial units. In other words, as long as the regional economic system is spatially stationary, the spatial aggregation will result in less spillover effects.

4.4 Portion of Negative Spillovers with Aggregation – CIRF results[29]

Along with the amount of spillover effects, the sign of spillover effects is also assessed. It is

[29] CIRFs can be graphically visualized as a three-dimensional graph, as shown in figure 3 in previous section. To avoid the boundary location problem when the shock is imposed on the edge of the country, it is assumed that the shock is positioned in the middle of the country, i.e., unit #496 for city level CIRF and units #463, #464, #495 and #496 for county level CIRF.

found that when regional units have a negative immediate effect, i.e., neighbors are exhibit negative coefficients in equation (8), then odd numbers of temporal aggregation increase the portion of the positive cumulative responses, but even numbers of temporal aggregation decrease the portion of the positive cumulative responses. Regarding spatial aggregation, more aggregation reduced the portion of positive cumulative responses. However, when the regional economic system is mixed with positive and negative effects, the portion of cumulative responses is nonlinear with respect to the level of aggregation.

Even when the regional units are competing with their immediate neighbor, i.e., a positive unit shock produces negative impacts on the immediate neighbors, the next period response results in positive impacts on the immediate neighbor's immediate neighbor. This pattern continues throughout time; thus, when the impacts are propagated through the rook contiguous regional system, then the relative portion of negative cumulative response and that of positive cumulative response reverses with every another level of aggregation. This pattern is shown in the first column of table 4 and in the first row of figure 9. We can observe that with odd numbers of temporal aggregation, the number of positive CIRF increases, but with even numbers of temporal aggregation, the number of negative CIRF deceases.

Table 4.
Number of Positive CIRF / Number of Negative CIRF* with Different Temporal Aggregation Level

Neighborhood Effect Sharing:	Borders<0 Vertices=0	Borders=0 Vertices<0	Borders=-.05 Vertices=-.05	Borders=.05 Vertices=-.05	Borders=.05 Vertices=-.15	Borders=.15 Vertices=-.05	Borders<0 Vertices>0
disaggregated	0.8622	0.8622	0.6720	1.0250	1.0710	0.9184	1.0027
2-periods	1.1480	1.1480	1.1901	1.0413	0.9650	1.0613	1.0024
3-periods	0.8789	0.8789	0.7536	0.9855	1.0275	1.0360	1.0021
4-periods	1.1211	1.0000	1.0645	1.0000	0.9845	1.0813	1.0000
5-periods	0.9082	1.0000	0.8517	1.0277	1.0118	1.0521	1.0000
6-periods	1.0854	1.0000	1.0645	1.0000	0.9692	1.0813	1.0000
7-periods	0.9321	1.0000	0.9069	0.9961	1.0157	1.0687	1.0000
8-periods	1.0625	1.0000	1.0000	1.0000	1.0000	1.0480	1.0000
9-periods	0.9490	1.0000	0.9357	0.9655	1.0000	1.0687	1.0000
10-periods	1.0461	1.0000	0.9768	0.9922	1.0000	1.0480	1.0000
11-periods	0.9621	1.0000	0.9542	0.9845	1.0000	1.1201	1.0000
12-periods	1.0335	1.0000	0.9692	1.0000	1.0000	1.1157	1.0000

* 1-year head (assuming that the disaggregated level observations are monthly) CIRFs

Figure 9. # of positive CIRF / # of negative 1-year ahead CIRF in Temporal Aggregation Example

However, this monotonic trend cannot be found in other cases where regional units are mixed with competitive and complementary relationships, and even reversed in other cases (the third row of figure 9).

Similar argument applies to the spatial aggregation case as well. When the immediate neighborhood effects are all negative, then more spatial aggregation monotonically reduces the portion of the positive CIRFs. However, if the regional system is mixed with positive and negative effects, this monotonic trend no longer exists (table 5).

Table 5.Number of Positive CIRF / Number of Negative CIRF* with Different Spatial Aggregation Level

Neighborhood Effect Sharing:	Borders<0 Vertices=0	Borders=0 Vertices<0	Borders=-.05 Vertices=-.05	Borders=.05 Vertices=-.05	Borders=.05 Vertices=-.15	Borders=.15 Vertices=-.05	Borders<0 Vertices>0
City	1.1736	1.1736	1.4414	1.0032	0.9167	0.9531	1.0032
County	3.5417	0.0060	0.3203	0.4569	0.0060	1.6406	2.0179
Group	2.0833	0.0208	0.2250	0.2250	0.0208	1.4500	3.0833
State	1.5000	0.0667	0.1429	0.2308	0.0667	1.0000	1.6667
Division	1.0000	0.3333	0.3333	0.3333	0.3333	3.0000	1.0000

* 12-step ahead CIRFs

Conclusively, regarding CIRF, it is most likely that we cannot find any pattern regarding the numbers of positive/negative spillover effects.

5. Conclusion

Table 6 summarizes the results from the previous sections. In a single-level structure model, the FEVD results on different levels of temporal aggregations are opposite to Gosset's prediction that a larger scale of unit will reduce the spatial correlation. However, even though it could happen under certain conditions, such as large region common factor loadings on regional activities, a multi-level structure is in agreement with the prediction. Spatial aggregation results are also in in concordance with the prediction. If the region common factor plays the most important role in regional economic activities, as is the case of the regional employment series exercise in section 3, it can be concluded that we will observe less spillovers with larger scale of units.

Table 6. Summary Table

	Real World Data Exercise	*Exercise on the Constructed Regional Economy*
Multi-level Structure	**Temporal aggregation decreases the neighborhood portion of FEVD**	**When region common factor loading is large, neighborhood portion of FEVD decreases after a certain level of temporal aggregation.** Otherwise, temporal aggregation decreases the neighborhood portion of FEVD
	Spatial Aggregation decreases the neighborhood portion of FEVD	**Spatial Aggregation decreases the neighborhood portion of FEVD**
	No specific pattern found in terms of the relative portion of positive signs to the negative signs of CIRFs	No specific pattern found in terms of the relative portion of positive signs to the negative signs of CIRFs, if the regional relationship is mixed with positive and negative neighborhood effects.
	No specific pattern found in the neighborhood portion of FEVD in terms of temporal	Temporal aggregation increases the neighborhood portion of FEVD

	aggregation	
Single-level Structure	No specific pattern found in the neighborhood portion of FEVD in terms of spatial aggregation	Spatial Aggregation decreases the neighborhood portion of FEVD
	No specific pattern found in terms of the relative portion of positive signs to the negative signs of CIRFs	No specific pattern found in terms of the relative portion of positive signs to the negative signs of CIRFs, if the regional relationship is mixed with positive and negative neighborhood effects.

However, there are some limitations with our FEVD analysis on the constructed regional economic system. Most importantly, the neighborhood portion of total variance is decreasing with the level of spatial aggregation, but this is more or less due to the fact that the regional economic system is spatially stationary, and that the neighborhood effects are geographically constrained within neighboring units. In other words, since the effect of an innovation fades away more quickly with distance, and the spatial aggregation binds regional units located close to each other, the spillover parts of innovation should be reduced with larger scale of spatial units that results in increasing the distance from regional units. Thus, if the neighborhood effect is determined not by geographical closeness but by trade linkage, the spatial aggregation based on geographical location will not necessarily produce an inverse relationship between the neighborhood portion of FEVD and the level of spatial aggregation. Nevertheless, in many cases, we could expect smaller spillover effects with larger spatial scale because most human activities are physically constrained by their geographical locations.

Additional limitations relate to the relative sizes of regional units, the regional differences in the region common factor loadings, closed economy assumptions, and so forth, but the how the relaxation of these strong assumptions will change the conclusions of this paper remains for the future studies.

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