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## Competitive and Complementary Relationship between Regional Economies: A Study of the Great Lake States

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# Competitive and Complementary Relationship between Regional Economies: A Study of the Great Lake States

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**Abstract:** This paper uses a multi-level dynamic factor model suggested by Bai and Wang (2012) to identify the spatio-temporal dynamics of regional business cycles, focusing on six Great Lakes states,<sup>1</sup> Illinois, Indiana, Michigan, Minnesota, Ohio and Wisconsin. The identification scheme suggested by Bai and Wang (2012) enables separate identification of the shock common to the Great Lakes region and the individual shock to each region as well as to assess the interactions between those shocks. The model is estimated using the Gibbs Sampling algorithm. Four monthly time-series variables for each state for the period of January 2003 to September 2012 were organized to implement a dual layer structure dynamic factor model for the Great Lakes region. The main advantage of using this multi-level structure model is that it is possible to assess the effect of a shock originating in one particular region on the other regions separately from the region common shock. In contrast, a single level structural model does not separate the region common shock from the region specific shock. By separating the common shock and the individual shocks, this estimation strategy prevents the possible misunderstanding or overestimation of regional interdependency induced from the co-movements of regional business cycles. Since each region is exposed to the region common shock, the degree of co-movement of each region's business cycle is strong, possibly exaggerating or biasing the effect of region specific shocks. The simulation results show that incorporating the multi-level structure in a regional dynamic factor model significantly alters the regional interdependency relationship induced from the single level structure model. The variance decomposition shows that much of the region specific business activities can be explained by the region common shock, and the impulse response function shows that the effect of region specific shock is exaggerated or biased in the single level structure model.

**Key Words:** Spatial Dependency, Spillover, Multi-level Analysis, Regional Business Cycle

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<sup>1</sup> Individual "region" denotes each state in Great Lake Region, and "region common" denotes Great Lake States.

## **1. Introduction**

After a hiatus of several decades, attention is being drawn again to the identification and impact of regional business cycles. Earlier work on a sample of Midwest economies (Park and Hewings, 2012) revealed the difficulties of assessing the contributions of national level impacts from the idiosyncratic effects within each sub-national (state) economy. Further, while the earlier work examined business cycle coordination among the Midwest states, it did not attempt to estimate the spillover effects of changes in one state on their neighbors. In a situation where many macroeconomic variables are available and many regional units are present, there are two important questions related to the identification of the regional business cycles. One question addresses whether the overall economic situation of a specific region is improving or worsening, because, in some cases, some of the macroeconomic variables have improved while others have not. A principal components analysis or Stock and Watson (1989)'s dynamic factor model can answer this question by reducing the dimension of the macroeconomic variables of interest. In particular, Stock and Watson's (1989) type of dynamic factor model enables identification of the temporal evolution of an overall unobserved state of a sub-national economy.

A second, critical question focuses on the identification of the sources of the shocks on regional economies and the spatial interaction of those shocks in the case where there are multiple regional units under consideration. Answering this question will require decomposition of the regional economic indicators into the sources of the shocks and addition of a spatial dependency structure into the dynamic factor model.

Although, the study of (1) the temporal evolution of regional business cycles, (2) the interdependency between regional economies, and (3) the identification of the sources of the shocks of regional economies are closely related, the methodologies combining all those perspectives appear to be limited. For example, the study of the temporal evolution of regional business cycle typically involves the development of a dynamic factor model, the study of interdependency between regional economies usually uses Spatial Autoregressive models, and the identification of the sources of the shocks of regional economies usually uses principal component method. Some integration is provided in the development of Spatial VAR or Spatial Panel analysis in that the approach combines (1) and (2).

Even though it is not directly related to the study of regional economy, an application of Bernanke, *et al.* (2005) factor augmented VAR (FAVAR) model adds interdependency structure to the analysis of the regional economy, thus combines (1) and (2). In the usual Stock and Watson (2002)'s dynamic factor model of multiple regions, adding non-zero off diagonal elements in the coefficient matrix of the state equation of dynamic factor can capture the interdependency structure of the regional economies. The description is provided in section 2.

However, the direct application of FAVAR to the analysis of a multi-regional economic system should be considered carefully, since the individual regions might be exposed to economic forces that are common to all of the regions. This necessitates combining (1), (2) and (3). For example, U.S. states are all exposed to U.S. common shock such as a monetary policy shock or commodity price shocks. If these impacts are not taken into account in a multi-layer structure of regional economies, the co-movement of regional business cycle can potentially misrepresent the dependency structure between regional economies. To avoid this possible misunderstanding of regional interdependency induced from the co-moving behavior of regional economies, a multi-level structure dynamic factor model needs to be developed.

A dynamic factor model with interdependency and a multi-level structure can be found in Bai and Wang (2012). A simulation study suggests that the multi-level structure dynamic factor model prevents possible exaggeration of the effects of neighboring economies, thus providing a more accurate estimate of the overall spatial structure of regional economies. In essence, the application of a multi-level dynamic factor model to the study of a multi-regional economy is appropriate in assessing the regional interdependency structure, and in identifying the source of the shock.

The Bai and Wang (2012) model uses Bayesian MCMC algorithm for estimation, and estimates the observation equation and the state equation simultaneously. Thus, the confidence intervals for the estimated coefficients including the latent dynamic factors are more accurate with a finite number of observations.<sup>2</sup>

The organization of the remainder of this paper is as follows. Section 2 introduces some previous studies dealing with spatial dependency in regional economies including one of the few

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<sup>2</sup> One drawback of this estimation strategy is that it requires significant computing power, because acquiring a sampling size of 500 (with some burn-in and thinning) using Intel Core i-3 processor with 4GB RAM takes 2~3 days.

studies to address multi-level issues. Section 3 describes how an applied Bai and Wang (2012) type of model can be used in a multi-regional system. Section 4 presents simulation results using this model, and compares the implication of the model with that of the single level structure dynamic factor model, while section 5 estimates the model using data for the six Great Lakes states. Section 6 offers some conclusions and issues for future research.

## **2. Literature Review**

### **2.1 Topics on Regional Business Cycle**

#### *Temporal Disaggregation and the Sources of the Shocks on the Regional Business Cycles*

At state level in US, most analyses of regional business cycles have focused on the employment fluctuations. The results of decompositions of change have revealed a significant sensitivity to the frequency of data. For example, using annual variation in state employment, 66 percent of the variation could be attributed to a national component and only 34 percent is due to state-specific components (Blanchard *et al.*, 1992). When quarterly employment data for U.S. macro regions was used, the variance of cyclical innovation in regional employment could be decomposed into roughly 39 percent ascribed to national shocks with 41 percent accounted for by region-specific shocks on average (Clark, 1998; Clark and Shin, 1998). When Clark and Shin (1998) adapted a dynamic factor model, the share of the common component and of region-specific component were 54.3 percent and 45.7 percent respectively.

The ratio of each shock to total variance depends on the frequency of the utilized data: the lower the temporal frequency, the higher the portion accounted for by the national components. What has remained unexplored is how these multi-level estimates affect the estimation of regional spillover effects, one of the prime sources of motivation for this paper. A further complicating factor, explored in part by Park and Hewings (2012), is the role of industry mix and the interdependence between sectors within and between regions. As a result, one may ascribe a higher percentage of a region's business cycle to idiosyncratic (local) behavior when in fact it may reflect the way differences in national shocks are mediated through a regional system via different responses of different industries. Even with the similar industry mixes, regions may

still respond differently as a result of their firms' positions in value chains of production (Hewings, 2008).

### *Interstate Trade Flows and Spillover Effects*

The integration of a multiregional econometric-input-output model with interstate trade flow data from the Commodity Flow Survey (BTS, 2010) forms the basis of a system that can be used for impact analysis and forecasting. Recently, analysis was conducted to explore the interregional impacts of the 2007~2009 recession; in essence, the model provided insights into the spatial spread (spillover) of job losses in one state on surrounding states. The model was calibrated for six regions (Wisconsin, Illinois, Indiana, Ohio, Michigan and The Rest of the US) and table 1 provides the estimates of the interstate trade flows.

<<insert table 1 here>>

When integrated into the model, it was possible to present an estimate of the spillover effects, shown in table 2. For Illinois, about 20% of the indirect (spillover) effects were concentrated in the remaining Midwest states.

<<insert table 2 here>>

Based on the estimates derived from the Midwest Regional Econometric Input-Output Model (MREIM), the spillover effects in total accounted for about 2.2 percent of the total effects in terms of employment (table 3).

<<insert table 3 here>>

The main difference between MREIM and the model in this paper is that since MREIM is based on the assumption of Leontief production function, the spillover effects are always positive. On the contrary, although the model in this paper does not address the issue whether the competitive or complementary relationship is originated from the production factor substitution or goods/services market competition, the estimation results show that there can be negative spillover effects. Thus, in the presence of this competitive relationship between regional economies, the multi-regional input-output model based on Leontief production function might exaggerate or bias the spillover effects.

### *Regional Business Cycle Coordination*

Park and Hewings (2012) explored the degree to which five of the six states (Minnesota was not included) that are integrated in a significant trading relationship (between 30-40% of each state's exports are derived from the other four states) share similar business cycles and the degree to which their cycles are similar to those at the national level. This paper tackled the following issues. First of all, lead and lag relationship among regions and the role of the industry mix effect to this phenomenon were explored. Secondly, concurrent and lagged effects of the industry mix on the regional economic fluctuations were measured explicitly based on the identification of the national shock using a principal components method.<sup>3</sup> Finally, a VAR model in which both national disturbance and regional transmission mechanisms was considered explicitly is set up and simulation is performed to examine the degree to which the model mimics the real economy. The empirical analysis focused on five Midwest states (Illinois, Indiana, Michigan, Ohio and Wisconsin)<sup>4</sup> and the frequency of the data used is monthly (the highest frequency available at the state level). In addition, state coincident indexes developed by the FRB of Philadelphia were utilized to identify the lead and lag relationship among the Midwest states and the CFNAI (Chicago Fed National Activity Index) was used as one estimate of the national common shock.

In order to test the coordination among the Midwest states, Granger-causality test was performed with monthly non-farm employment data from January 1975 to July 2003. The specification of a bivariate VAR model for the Granger-causality test is shown in equation (1).

$$\Delta y_{it} = \alpha + \sum_{l=1}^L \beta_l \Delta y_{i,t-l} + \sum_{l=1}^L \gamma_l \Delta y_{j,t-l} + \varepsilon_t \quad (1)$$

where  $\Delta y_{it}$  denotes state  $i$ 's log-differenced employment and  $\varepsilon_t$  denotes error term. Based on this set-up, joint hypothesis  $\gamma_1 = \dots = \gamma_L = 0$  was tested.

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<sup>3</sup> Most of the literature treats the national aggregate of a specific data as a national common shock (*i.e.* in the analysis of state employment fluctuation, national employment is used as a proxy for the common component). In this paper, common disturbances are identified from the principal component analysis.

<sup>4</sup> As Blanchard *et al.* (1992, p.29) pointed out, only 26 percent of annual state employment variation is common to Census regions and as such, the majority of state employment is idiosyncratic. In addition, when the shares of industry employment to total employment are similar among Census regions. In this regard, data analysis in state level is better choice to capture industry mix effects. These five states are, for the most part, each other's largest trading partner (see Hewings *et al.*, 1998b).

As expected, most of the cases rejected the null hypothesis that employment changes in one state did not affect other neighborhood states at the 5 percent and 10 percent significance levels. That means that the Midwest states display statistically significant business cycle transmissions among themselves. However, the Granger-causality test could not distinguish between national and regional shocks and transmission effects in the state employment fluctuations. This phenomenon can happen when the industry mix effect of national components on regional business cycles is large enough. For example, if Illinois has a large share of an industrial sector that the national shock affected but with some delay, then Illinois' total employment response would lag those of other states, resulting in the rejection of Granger-causality hypothesis.

These results raise some questions about the extent to which regional business cycle differ: which state leads or lags other states? In order to find empirical evidence, state coincident indexes developed by the Federal Reserve Bank of Philadelphia were utilized. A state's coincident index, representing a state's business cycle, was generated from a dynamic factor model<sup>5</sup>.

For the purpose of finding the lead and lag relationship among states with the coincident indexes, the Hodrick-Prescott filtering method<sup>6</sup> was applied to the indexes and cyclical parts in the indexes are compared among states. The cyclical part of each state index (C\_\_) is compared to the national counterpart (CXCI)<sup>7</sup>, and it has been shown that all Midwest states (CIN, CMI, COH, CWI) except Illinois coincide with the national economy while the Illinois business cycle (CIL) lags the national cycle (CXCI); cross-correlation coefficients between the state cycle and national cycle (from January 1979 to April 2003) revealed that the Illinois cycle lagged the national cycles by 3 to 4 months while other state cycles were mostly moving together with the national cycle. From this result, it can be said that the Illinois business cycle followed the national from 3 to 4 months later while the other Midwest states move concurrently or lag one month the national economy.

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<sup>5</sup> The overview of the dynamic factor model is provided in equations (6)~(8) in section 2.2.

<sup>6</sup> Hodrick-Prescott filter method decomposes the time series  $y(t)$  into trend and cyclical parts. The trend component ( $\tau(t)$ ) minimizes  $\sum_{t=1}^T (y(t) - \tau(t))^2 + \lambda * \sum_{t=1}^T \{[\tau(t+1) - \tau(t)] - [\tau(t) - \tau(t-1)]\}^2$ . Here the penalty weight  $\lambda = 14,400$  is used as generally recommended.

<sup>7</sup> National counterpart (called as "experimental coincident index") is available in NBER website.



The national common shock has been shown to be closely related to the levels of national and regional manufacturing production (Park *et. al.*, 2002). Thus, if the manufacturing sector reacts to the national common shock promptly and services sector responds with time lags, it is possible for a manufacturing-oriented state to lead the national cycle and for a services-oriented state to follow the national cycle. Secondly, the explanatory power of the national common shock through industry mix effects in the state employment fluctuations was estimated. In the literature, the variances of *annual* state employment were divided into 66 percent of nation component and 34 percent of a region-specific shock (Blanchard and Katz, 1992). Using *quarterly* state employment data, the variance of cyclical innovation in regional employments is decomposed into roughly 39 percent of national shock, 41 percent of region-specific shock on average (Clark, 1998; Clark & Shin, 1998). Using *monthly* state employment data, the national common shock through the industry mix channel explains 40 percent of the state employment fluctuation on average in the Midwest states. When the direct effect is estimated by substituting  $F_{st}$  for CFNAI, the indirect effects turn out to cause, on average, 2 percent of the variance. This result suggests a larger effect of the national shock on regional employment fluctuations compared to the result of the principal component analysis and the previous literature. Even though higher frequency data are used, which tends to lower the national effects from a statistical point of view, considering industry mix effects and identifying the national shock with many national data (that is, using CFNAI) explicitly may result in larger effects attributed to the national shock. From this result, it can be said that the industry mix effects of the national shock produce differences in the business cycle behaviors along with propagation of the national shock through autoregressive forces and transmission mechanism among states.

Some parallel work by Hayashida and Hewings (2009) in Japan found that many of the differences in regional business cycle behavior could be attributed to differences in responses by regions to different macroeconomic variables. For example, a three fold division of regions revealed a set influenced by (1) production and consumption, another by (2) production and employment, and the final group by (3) industrial production, the consumption change and the employment situation. These differences in influence had profound impacts on regional business cycle turning points. Similar differences were found by Owyang (2005) who noted variations in regimes (expansion/contraction) by region across cycles; in particular, recession growth rates were related to industry mix, whereas expansion growth rates were related to education and age

composition. In addition, he found that states differ significantly in the timing of switches between regimes, indicating large differences in the extent to which state business cycle phases are in concord with those of the aggregate economy. Partridge and Rickman (2005) found over time decreases in national volatility was accompanied by an increase in regional asymmetry. Are these findings a reflection of some unobserved variations or, as Del Negro (2002) has suggested the result of some regional variability being underestimated by measurement errors?

However, there is limited empirical evidence on whether and to what extent regional business cycles differ – the exception being some work by Artis and Okubo (2009, 2010) in the UK and Japan and Hayashida and Hewings (2009) also in Japan and Park and Hewings (2007) in the Midwest of US. The main focus of macroeconomists has been to use intra-national business cycles as a way to test propositions advanced in connection with traditional and real business cycle theory and the theory of optimal currency areas.

The following sample of five important contributions to the measurement of regional business cycles illustrates some of the challenges. Hess and Shin (1997) conducted a two-level analysis, countries within EU and states within US) They explored intra-national cycles as a way to illuminate implications when countries join common markets and create common currency areas (e.g. Euro). For international productivity (real business cycles, RBC), geography mattered; industries within individual countries moved more than individual industries across countries. For intranational productivity – geography less important – cycles more symmetric. International evidence suggests that the volatility of terms of trade greater than RBC theory would suggest but is lower within countries (although indirectly measured because data not available).

Barrios *et al.* (2003) explored the co-fluctuations between UK regions and the Euro Zone. They confirmed the finding that the British cycle was persistently out of phase with the main euro-zone countries. Together with Artis and Okubo (2009), they found only minor cyclical heterogeneity among UK regions, noting that differences in sectorial specialization generated some of the asymmetry in GDP fluctuations but these were not significant in explaining UK-EU business cycle correlations.

Artis and Okubo (2009) analyzed co-fluctuations among UK regions both intra UK cyclical correlations and correlations between UK regions and the main Euro area countries. As with

Barrios *et al.* (2003), they found that most of the heterogeneity among UK regions could be explained by sectoral differences. Many UK regions have quite high cyclical correlations with individual Euro area countries (although not the case for UK as a whole). They highlighted the fact that Scotland's affiliation was not more notable than others (such as London and SE) – thus providing little support for rationalizing Scotland's devolution on the basis of greater ties to the Euro zone than other UK regions.

Ghosh and Wolf (1997) focused on the geographical and sectoral shocks in the US business cycle and examined the degree to which the aggregate US business cycle was driven by geographic shocks (all sectors in one state) or sectoral shocks (same sector across all states). At level of an individual sector in a state, shocks to output growth were driven by the sector not the state (i.e. textiles in Texas driven by textiles in US not in state). Shocks to sector growth have lower correlation across sectors compared to shocks to state growth across states. It turns out that spatial aggregation important; geographical shocks more important at higher levels (e.g. state as a whole). US business cycle volatility affected equally by changes in volatility of state and sector business cycles and changes in the correlation across sectors and states

Owang *et al.* (2009) built on Ghosh and Wolf's (1997) work by creating a dynamic factor model that identifies common factors underlying fluctuations in state-level income and employment growth to determine the degree to which each state's economy is related to the national business cycle. They found a great deal of heterogeneity in the nature of the links between state and national economies. Of interest to the focus on the current paper, they found that the closeness of state economies to the national business cycle was not only related to differences in industry mix but also to non-industry variables such as agglomeration and neighbor effects. Confirming other findings, the common factors tend to explain a large proportion of the total variability in state-level business cycles, but there is still significant cross-state heterogeneity.

## 2.2 Models with Spatial Interactions

*Clark and Shin (1998) VAR model*

One of the notable studies regarding the study of regional economics is Clark and Shin (1998), using a restricted VAR form of the model:

$$X_{r,i,t} = \mu_{r,i} + \sum_{p=1}^P \alpha_{r,i,p} X_{t-p} + \sum_{p=1}^P \beta_{r,i,p} X_{r,t-p} + \sum_{p=1}^P \gamma_{r,i,p} X_{i,t-p} + e_{r,i,t} \quad (2)$$

$$e_{r,i,t} = \delta_{r,i}c_t + \theta_{r,i}u_{r,t} + \lambda_{r,i}n_{i,t} + v_{r,i,t} \quad (3)$$

where  $X_{r,i,t}$ : growth rate of industry  $i$  in region  $r$ ,

$X_t = \sum_{r=1}^R \sum_{i=1}^I w_{r,i}X_{r,i,t}$ ,  $\sum_{r=1}^R \sum_{i=1}^I w_{r,i} = 1$ : fixed-weight average of region-industry variable,

$X_{r,t} = \sum_{i=1}^I a_{r,i}X_{r,i,t}$ ,  $\sum_{i=1}^I a_{r,i} = 1$ : fixed-weight average of industry variable,

$X_{i,t} = \sum_{r=1}^R b_{r,i}X_{r,i,t}$ ,  $\sum_{r=1}^R b_{r,i} = 1$ : fixed-weight average of region variable, and weights correspond to output shares or employment shares.

In this model, each region-industry variable is a function of a lagged aggregated variable, a lagged industry-level aggregated variable, and a lagged region-level aggregated variable. The error term  $e_{r,i,t}$  is further decomposed into a region-industry common shock ( $c_t$ ), region specific shock ( $u_{r,t}$ ), industry specific shock ( $n_{i,t}$ ), and an idiosyncratic error ( $v_{r,i,t}$ ).

The standard estimation procedure for the above model is:

- ① Normalize the standard deviations of structural shocks,  $c_t$ ,  $u_{r,t}$ ,  $n_{i,t}$  and  $v_{r,i,t}$ , equal to 1.
- ② Estimate equation (2) using OLS.
- ③ Regress residuals from ② to estimate equation (3) using MLE.

While the estimation procedure is simple, there is a drawback in this model, as noted in Clark and Shin (1998). The resulting estimates are not maximum likelihood estimates because, formally, equation (2) and (3) should be estimated simultaneously, but since the number of coefficients exceeds the number of observations, they are estimated by a two-step procedure. Thus, the calculated confidence intervals for the coefficients of equation (3) are not accurate.

### *Spatial Panel and Spatial VAR model*

Alternatively, with a set of spatial panel data, one of the most widely used models will be the spatial panel model.<sup>8</sup> A simple univariate version of a spatial autoregressive (SAR) model is presented in equation (4) using Anselin, *et al.* (2008):

$$Y_t = \mu + \rho WY_t + \alpha Y_{t-1} + \varepsilon_t \quad (4)$$

<sup>8</sup> See Magalhães *et al.* (2001) for an example using a non-linear relative dynamics formulation and Marquez *et al.* (2013) for a case study using a spatial vector autoregressive approach

where  $r = 1, \dots, R$  denotes region,

$t = 1, \dots, T$  denotes time, and

$Y_t = (y_t^1, \dots, y_t^R)'$  is the vector of endogenous regional observations, and

$W$  is a spatial weight matrix.

In this equation, the error term  $\varepsilon_t$  can have spatial dependency structure such as  $\varepsilon_t = \lambda W \varepsilon_t + u_t$  where  $u_t \sim iid N(0, \sigma^2)$  (*spatially and temporally uncorrelated*). Also, if there are additional spatio-temporal lag terms ( $WY_{t-1}$ ) on the right hand side, and  $y_t^r$  is a  $p \times 1$  vector of endogenous variables, then equation (4) becomes a multivariate version of Beenstock and Felsenstein (2007)'s spatial VAR model.

#### *Multi-level Structure Spatial Model*

The concept of multi-level structure in the context of interdependency is also described in Corrado and Fingleton (2011). Although, it is not directly mentioned in their work, the application of their multi-level structure on the regional economic analysis can be described as in equation (5):

$$Y_{ijt} = \mu + \rho W Y_{ijt} + \beta X_{ijt} + \gamma Z_{jt} + \delta_t + \varepsilon_{ijt} \quad (5)$$

where  $Y_{ijt}$  is the observation of the individual regional unit  $i$  in regional group  $j$  at time  $t$ ,

$Z_{jt}$  is the regional group  $j$  specific effect at time  $t$ , and

$\delta_t$  is the region common effect at time  $t$ .

Equation (5) states that the individual regional observation  $Y_{ijt}$  is determined by the endogenous evolution of its neighboring regions ( $\rho W Y_{ijt}$ ), group specific effects ( $Z_{jt}$ ), and the shock common to all observed regions ( $\delta_t$ ).

Putting aside the estimation procedure of the above model, the spatial lag coefficient  $\rho$  can be interpreted as the degree of dependency between regions. That is, if  $\rho$  is significant, then one can conclude that there exists spatial dependency. Also, regarding the sign of  $\rho$ , if it is positive, then there exists positive spillover effects.

While this spatial dynamic model is very useful in that it can capture the spatial dependency or fix the omitted variable bias problem, one drawback is that the identification of spatial dependency heavily depends on the specification of the spatial weight matrix,  $W$ . Since each element of the spatial weight matrix is predetermined by the analyst's belief, the estimation can bias the true relationships between regions if the belief is wrong. In the present paper, the spatial dependency structure is represented by a non-restricted coefficient matrix in order to prevent the possible bias originated from setting the spatial dependency structure as  $\rho W$ <sup>9</sup>. Loosening this restriction provides more freedom in the estimation of the spatial dependency. However, if the dimension ( $p$ ) of endogenous variable  $y_t^r$  is large, as it is in most cases, the spatial lag coefficient has  $R \times p(p - 1)$  elements, and this sometimes causes under-identification of the equation.<sup>10</sup> Fortunately, the problem regarding the large dimension of endogenous variable set can be resolved by a Stock and Watson (1989) type of dynamic factor model.

*Bernanke, et al. (2005)'s Dynamic Factor Model with Interdependency (FAVAR)*

Factor models are useful in many aspects. For example, in a data rich environment, a factor model, such as principal components method, can reduce the dimension of the variables of interest, enabling evaluation of the overall movement of economy. If a dynamic structure is added to the factor model, as suggested by Stock and Watson (1989), the factor model becomes more generalized in that it incorporates the evolution of the factor over time. Consider a  $p$ -dimensional time series variable ( $y_t$ ), each of which is stationary and standardized to have mean zero and unit variance. Assuming that there exists a single latent dynamic factor determining the movement of each variable, the specification of this single dynamic factor model can be described in equations (6)~(8):

$$y_t = \gamma(L)f_t + u_t \tag{6}$$

$$D(L)u_t = \varepsilon_t \tag{7}$$

$$\varphi(L)f_t = \eta_t \tag{8}$$

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<sup>9</sup> Alternatively, Bhattacharjee and Jensen-Butler (2013) suggests a spatial weight matrix whose off-diagonal elements can be either positive or negative. Thus, the spatial weight matrix suggested in their paper also can capture the competitive or complementary relationship between regional economies.

<sup>10</sup> There can also be an under-identification problem originated from the number of regional units, i.e., if the number of regional units is large, the number of elements in coefficient matrix is also large, exhausting the degree of freedom of our data set. However, the model used in this paper does not address this issue, thus, for this paper's empirical analysis, only those six Great Lake states were chosen.

where  $t = 1, \dots, T$  denotes time,

$y_t$  is  $p \times 1$  vector of macroeconomic variables that are hypothesized to move contemporaneously with overall economic conditions,

$f_t$  is common unobservable scalar variable (latent factor),

$\gamma(L), D(L)$  and  $\varphi(L)$  are lag polynomials, and

$(u_{1t}, \dots, u_{pt}, f_t)$  are mutually uncorrelated at all leads and lags, thus  $D(L)$  is bloc diagonal, and  $(\varepsilon_{1t}, \dots, \varepsilon_{pt}, \eta_t)$  are mutually and serially uncorrelated.

Thus, from equations (6)~(8), each variable is decomposed into a single common shock ( $f_t$ ) and a variable specific idiosyncratic shock ( $u_t$ ).

To incorporate the spatial dependency into a multi-regional framework, the above equation can be manipulated into a VAR type of state equation using the Bernanke, *et al.* (2005) factor-augmented VAR.<sup>11</sup> Assuming that there exists one dynamic factor for each region, and the dynamic factors have a VAR(1) specification, the dynamic factor model system (6)~(8) can be transformed into equations (9)~(11):

$$\begin{bmatrix} y_t^1 \\ \vdots \\ y_t^R \end{bmatrix} = \begin{bmatrix} \gamma^1(L) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \gamma^R(L) \end{bmatrix} \begin{bmatrix} f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} + \begin{bmatrix} u_t^1 \\ \vdots \\ u_t^R \end{bmatrix} \quad (9)$$

$$D^r(L)u_t^r = \varepsilon_t^r \quad \forall r \in \{1, \dots, R\} \quad (10)$$

$$\begin{bmatrix} f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \dots & \varphi_{1R} \\ \vdots & \ddots & \vdots \\ \varphi_{R1} & \dots & \varphi_{RR} \end{bmatrix} \begin{bmatrix} f_{t-1}^1 \\ \vdots \\ f_{t-1}^R \end{bmatrix} + \begin{bmatrix} \eta_t^1 \\ \vdots \\ \eta_t^R \end{bmatrix} \quad (11)$$

where  $y_t^r$  is  $p \times 1$  vector of endogenous region  $r$  observations,

$f_t^r$  is unobservable fundamental forces that affect the dynamics of  $y_t^r$ , and

$$E(\eta_t^r | f_t^1, \dots, f_t^R, f_{t-1}^1, \dots, f_{t-1}^R, \dots) = 0 \quad \forall t \in \{1, \dots, T\} \text{ and } \forall r \in \{1, \dots, R\}$$

The logic behind this model is that the observed variables of each region  $y_t^r$  can be decomposed into shocks comprised of two parts,  $f_t^r$  and  $u_t^r$ .  $u_t^r$  is  $p \times 1$  vector of idiosyncratic error

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<sup>11</sup> Although, Bernanke, *et al.* (2005) does not involve multi-regional framework in its model, the direct application of their work into multi-regional framework can easily be expressed into equation (79)~(91).

( $E(u_t^r | f_t^1, \dots, f_t^R) = 0 \quad \forall t \in \{1, \dots, T\}$  and  $\forall r \in \{1, \dots, R\}$ ) that is uncorrelated both within region ( $E(u_t^{ri} u_t^{rj} | f_t^1, \dots, f_t^R) = 0 \quad \forall i \neq j \in \{1, \dots, p\}, \forall t \in \{1, \dots, T\}$  and  $\forall r \in \{1, \dots, R\}$ ) and across regions ( $E(u_t^a u_t^b | f_t^1, \dots, f_t^R) = 0 \quad \forall t \in \{1, \dots, T\}$  and  $\forall a \neq b, r \in \{1, \dots, R\}$ ), but it can be temporally correlated as in equation (10). The other part of  $y_t^r$ , that is  $f_t^r$ , is a scalar value that governs the movements of the all observed variables in region  $r$ , but it is not only temporally correlated but also spatially correlated as in equation (11). Thus, if the off-diagonal terms of the coefficient matrix in equation (11) are all zero ( $\varphi_{ab} = 0 \quad \forall a \neq b$ ), equations (9)~(11) becomes a usual dynamic factor model described by equations (6)~(8) with additional superscripts  $r$  indicating the region.

### 3. Model

Bai and Wang (2012) further developed the dynamic factor model with a dependency structure into a multi-level dependency structure. This multi-level structure of a dynamic factor model has been widely used in the literature of business cycle identification such as in Kose, *et al.* (2003). The multi-level dynamic factor model assumes multiple layers of unobservable fundamental forces that govern the movements of observed variables. For example, a business cycle of a country can be decomposed into country specific shocks, continental specific shocks that affects every country within that continent, and world shock that affects every country in the world. Bai and Wang (2012) paper combines this multi-level structure dynamic factor model with a dynamic factor model with factor dependency. Assuming that there exists two layers of dynamic factors for each region, and the dynamic factors have a VAR(1) specification, the model can be described in equations (12)~(14):

$$\begin{bmatrix} y_t^1 \\ \vdots \\ y_t^R \end{bmatrix} = \begin{bmatrix} \delta^1(L) & \gamma^1(L) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \delta^R(L) & 0 & \dots & \gamma^R(L) \end{bmatrix} \begin{bmatrix} g_t \\ f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} + \begin{bmatrix} u_t^1 \\ \vdots \\ u_t^R \end{bmatrix} \quad (12)$$

$$D^r(L)u_t^r = \varepsilon_t^r \quad \forall r \in \{1, \dots, R\} \quad (13)$$

$$\begin{bmatrix} g_t \\ f_t^1 \\ \vdots \\ f_t^R \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \dots & \varphi_{1(R+1)} \\ \vdots & \ddots & \vdots \\ \varphi_{(R+1)1} & \dots & \varphi_{(R+1)(R+1)} \end{bmatrix} \begin{bmatrix} g_{t-1} \\ f_{t-1}^1 \\ \vdots \\ f_{t-1}^R \end{bmatrix} + \begin{bmatrix} \eta_t^g \\ \eta_t^1 \\ \vdots \\ \eta_t^R \end{bmatrix} \quad (14)$$

where  $y_t^r$  is  $p \times 1$  vector of endogenous regional observations,



$f_t^r$  is unobservable fundamental forces that affect the dynamics of  $y_t^r$ , and

$g_t$  is unobservable fundamental forces that affect the dynamics of  $(y_t^1, \dots, y_t^R)$

All specifications are the same as in the equation system (9)~(11) except that now there is a region common shock,  $g_t$ . Thus, this time,  $y_t^r$  is decomposed into three shocks, a region common shock ( $g_t$ ), a region specific shock ( $f_t^r$ ) and an idiosyncratic shock ( $u_t^r$ ). If the off-diagonal elements of the coefficient matrix in equation (14) are zero, then the above model becomes the Kose, *et al.* (2003) type of multi-level dynamic factor model. In this set up, since the innovations to the factors,  $(\eta_t^g, \eta_t^1, \dots, \eta_t^R)$  in equation (14) are independent of  $(u_t^1, \dots, u_t^R)$  of the measurement equation (12), and are i.i.d. normal with a diagonal covariance matrix, the factors are independent, conditional on the history. However, since the coefficient matrix in the state equation (14) has non-zero elements in its off-diagonal entries, factors are unconditionally correlated with each other. Thus, this model allows identification of the effect of the shock from one region to its neighbors.

To uniquely identify the model coefficients and the latent factors, Bai and Wang (2012) proposes some restrictions on the model parameters. In the specification above, it is required that  $(\eta_t^g, \eta_t^1, \dots, \eta_t^R)' = H_t \sim iid N(0, I_{R+1})$ , i.e., the factors, have unit variance, and some of the coefficients of the contemporaneous factor loading in equation (12) should be strictly positive.<sup>12</sup>

The estimation of this model is implemented by the Gibbs Sampling Algorithm for two reasons. First of all, in most of the cases, the above specification of the dynamic factor model fails to achieve convergence in most econometrics' packages when using MLE type of estimation procedures.<sup>13</sup> Also, since the Bayesian estimation does not rely on asymptotic theory, the inferences about the estimated parameters are exact.

The estimation can be done using WinBUGS<sup>14</sup> that simply requires the likelihood and the priors for the model parameters. Using the likelihood and the priors, the Gibbs Sampling algorithm draws samples from the posterior distribution of parameters including the latent factors. After sufficient burn-in period, the appropriate thinning period is set to eliminate the autoregressive

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<sup>12</sup> Bai and Wang (2012) discusses the required restrictions needed to identify the dynamic factor model in more general cases.

<sup>13</sup> The Dynamic Factor model treats the latent factors as parameters to be estimated, thus the dimension of parameters to be estimated is very large, which inevitably generates a lot of ridges in the model's likelihood function.

<sup>14</sup> Bayesian Inference Using Gibbs Sampling for Windows

relationship between each round of sampling, and the posterior distribution of each parameter is derived. Appendix 1 describes the likelihood and the priors used in the model for equations (12) through (14).

## **4. Simulation Study**

### **4.1 Conceptual Description of Dependency Structure: Dual Level vs. Single Level**

The competitive and complementary relationships between regions are comparable to those between corporations. Suppose, for example, company *A* and company *B* are in competition. A positive demand shock for the whole economy will likely result in better performances for both companies. However, a technology innovation in company *A* will likely result in a negative effect on company *B*. This relationship is illustrated in figure 1. In this setting, it is the source of the shock (demand or supply) and the competitive or complementary relationship between the two companies that determines the signs of the spillover effects.

*<<insert figure 1 here>>*

The regional economic counterpart of the above figure will be a region common shock for the demand shock, and region specific shocks for the supply shocks of companies. If the region common shock is not evaluated for the study of regional economies, then the resulting identification of the relationship between the two regions will be likely to exhibit more positive relationships since those two economies are moving in tandem with the region common shock.

For example, if the source of the positive shock to Indiana is from a region common shock, then it is likely that the neighboring states will react in the same positive manner against this shock. However, if the positive shock originates from an Indiana specific shock, it might negatively affect its competing neighbors.

The difference between the corporation framework and the regional economy setting is that the technology innovation of one company is not likely to be transmitted to the other company in the short run, while the individual state shock is likely to be transmitted to the whole region in a relatively short period of time through either the capital/labor market or the goods/service market. Thus, in a situation where the companies' performances (or regional economies' performances) are dependent on aggregate demand (or region common shock), a single level dynamic factor approach will likely to demonstrate positive spillover effect between companies (or between

regions). On the contrary, if the effect of the common shock is considered, it will be possible to more accurately identify whether two companies (or two regions) are in a competitive relationship or complementary relationship.

Conceptually, the estimated dynamic factors of regions can be described as shown in figure 2. Suppose observed regional economic variables are composed of a region common component and a region specific component, and the overall regional economic performance is heavily dependent on its region common component. Since the historical performances of observed economic indicators are mostly driven by the region common shock, it is likely that most of the dynamic factors extracted from the observed macroeconomic indicators will move coherently, possibly inducing positive dependency among states. However, the performances of the states net of the region common shock might reveal negative dependency among the states. Graphically, using some illustrative data, the three region case is shown in figure 2; the results reveal that the single level factor model exhibits positive correlation between each region, while the multi-level factor model exhibits both positive and negative correlation.

<<insert figure 2 here>>

For economic policy makers, this implies that, for example, investment in a local industry might harm a neighbor region with a similar industry, while benefiting other neighbor regions with complementary industries. If the latter are part of a supply chain, then it is possible to consider both complementary and competitive relationships existing between regions (see Hewings, 2008). Since the single level dynamic factor approach does not distinguish the source of the shock, any regional economic model that does not take into account the region common component might lead to misleading conclusion about the magnitude and sign of the spillover effects of an investment in a local economy.

#### **4.2 Simulation Results**

Before the actual estimation of the dynamic factor model using the real data, some simulated data were generated to compare the single level dynamic factor model and the multi-level dynamic factor model. The data were generated assuming that there are two competitive regions (A and B). Three macroeconomic variables ( $x, y$  and  $z$ ) are assigned for each region. Thus, there are one region common factor and two region specific factors, i.e., total of three unobserved true dynamic factors, that govern the observed behavior of each region's

macroeconomic variables. The length of the time period is 100. The true data generating process is described in equation (15) and (16). The true standard deviations of the Gaussian error terms ( $\varepsilon_t^g$ ,  $\varepsilon_t^A$  and  $\varepsilon_t^B$ ) are set to 1, and those of the observed macroeconomic variables are set to be (0.1, 0.2, 0.1, 0.2, 0.1, 0.2) in equation (16).

Unobserved Dynamic Factor

$$\begin{bmatrix} g_t \\ f_t^A \\ f_t^B \end{bmatrix} = \begin{bmatrix} 0.5 & 0.02 & 0.01 \\ 0.7 & 0.2 & -0.1 \\ 0.6 & -0.1 & 0.2 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ f_{t-1}^A \\ f_{t-1}^B \end{bmatrix} + \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^A \\ \varepsilon_t^B \end{bmatrix} \quad (15)$$

Observed Macroeconomic Variables

$$\begin{bmatrix} x_t^A \\ x_t^B \\ y_t^A \\ y_t^B \\ z_t^A \\ z_t^B \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.8 & 0 & 0.3 \\ 0.4 & 0.2 & 0 \\ 0.5 & 0 & 0.2 \\ 0.7 & 0.3 & 0 \\ -0.8 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} g_t \\ f_t^A \\ f_t^B \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{xA} \\ \varepsilon_t^{xB} \\ \varepsilon_t^{yA} \\ \varepsilon_t^{yB} \\ \varepsilon_t^{zA} \\ \varepsilon_t^{zB} \end{bmatrix} \quad (16)$$

Note that the (2,3) and (3,2) elements of the coefficient matrix of the unobserved dynamic factor equation is negative, to reflect the competitive nature of the two region, A and B. Also, the region common factor ( $g$ ) is not much affected by the region specific factors ( $f$ ), but much of the movement of the region specific factors can be explained by the region common factor.

Also, in the observation equation, each variable is affected by the region common factor and the region specific factor simultaneously, but the region specific factor from the other region only affects each region's variables through the interaction of the dynamic factors.

Using the samples generated from this true process, the point estimates are shown in equation (17)~(20):

Point Estimates of Single –level Dynamic Factor Model

Unobserved Dynamic Factor

$$\begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} = \begin{bmatrix} 0.217 & 0.576 \\ -0.031 & 0.074 \end{bmatrix} \begin{bmatrix} f_{t-1}^A \\ f_{t-1}^B \end{bmatrix} + \begin{bmatrix} \varepsilon_t^A \\ \varepsilon_t^B \end{bmatrix} \quad (17)$$

Observed Macroeconomic Variables

$$\begin{bmatrix} x_t^A \\ x_t^B \\ y_t^A \\ y_t^B \\ z_t^A \\ z_t^B \end{bmatrix} = \begin{bmatrix} 0.069 & 0 \\ 0 & 0.079 \\ 0.043 & 0 \\ 0 & 0.051 \\ 0.069 & 0 \\ 0 & 0.079 \end{bmatrix} \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{xA} \\ \varepsilon_t^{xB} \\ \varepsilon_t^{yA} \\ \varepsilon_t^{yB} \\ \varepsilon_t^{zA} \\ \varepsilon_t^{zB} \end{bmatrix} \quad (18)$$

Point Estimates of Multi –level Dynamic Factor Model

Unobserved Dynamic Factor

$$\begin{bmatrix} g_t \\ f_t^A \\ f_t^B \end{bmatrix} = \begin{bmatrix} -0.010 & 0.100 & 0.529 \\ 0.136 & -0.042 & 0.723 \\ -0.251 & 0.152 & 1.037 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ f_{t-1}^A \\ f_{t-1}^B \end{bmatrix} + \begin{bmatrix} \varepsilon_t^g \\ \varepsilon_t^A \\ \varepsilon_t^B \end{bmatrix} \quad (19)$$

Observed Macroeconomic Variables

$$\begin{bmatrix} x_t^A \\ x_t^B \\ y_t^A \\ y_t^B \\ z_t^A \\ z_t^B \end{bmatrix} = \begin{bmatrix} 0.039 & 0.057 & 0 \\ 0.023 & 0 & 0.075 \\ 0.020 & 0.038 & 0 \\ 0.014 & 0 & 0.048 \\ 0.028 & 0.066 & 0 \\ -0.018 & 0 & 0.080 \end{bmatrix} \begin{bmatrix} g_t \\ f_t^A \\ f_t^B \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{xA} \\ \varepsilon_t^{xB} \\ \varepsilon_t^{yA} \\ \varepsilon_t^{yB} \\ \varepsilon_t^{zA} \\ \varepsilon_t^{zB} \end{bmatrix} \quad (20)$$

The results are not satisfactory even with the true specification of the dynamic factor model. This outcome occurs because the observed macroeconomic variables are a complicated mixture of three error terms, i.e., a region common shock, a region specific shock and variable specific idiosyncratic shock, and the exact identification of the coefficient matrix is not easy to estimate.<sup>15</sup> However, for the identification of the regional interaction, the estimation results using the true specification are consistent with the true data generating process in terms of an impulse response analysis and variance decomposition. Conclusively, a multi-level dynamic factor model with dependency structure appropriately mimics the true structure of regional business cycles in terms of dynamic evolution of shocks, but it is not so capable of identifying the coefficients of the associated coefficients.

*Variance Decomposition and the Portion of the Shock from its Neighbor*

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<sup>15</sup> Even in the situation where the variances of the error terms are all zero, the point estimates of the coefficients are far from the true values of the model.

To eliminate the effect of orderings of the dynamic factors, a generalized variance decomposition was used to decompose a 12-step ahead forecast error variance, following Pesaran and Shin (1998). The variance decomposition results of the multi-level structure dynamic factor model, although the individual values deviate somewhat from the true value, are consistent with the true structure of regional economies (table 4). For example, region A's region specific dynamic factor is mostly accounted for by its own variance (58%) and the region common factor (42%), and the multi-level dynamic factor model is consistent with those figures (53% and 46% respectively). However, the single level dynamic factor model exaggerates the effect from its neighbor (49%), whereas the true effect of its neighbor is almost negligible (1%).

<<insert table 4 here>>

#### *Impulse Response Analysis (IRF) and the Sign of the Effects of the Shock from its Neighbor*

Using the point estimates of the dynamic factors extracted from the models, impulse responses are shown in figures 4 and 5. The IRFs from the estimation of the true specification of the model (figure 4) are similar to those of the true factors (figure 3). For example, the shock from region B has negative effect on region A's dynamic factor both in true IRFs and multi-level IRFs. On the contrary, the estimated IRFs from the single level structure misrepresent the true effect of the shock from region B on region A. Since the observed macroeconomic variables are moving together because of the presence of the region common shock, overlooking the effect of the region common shock will lead to the conclusion that the shock from region B will positively affect the economy of region A.

<<insert figure 3,4 and 5 here>>

In the next section, results from the empirical application to the six Great Lakes states region will be presented.

## 5. Estimation Results for Six Great Lake States



In this section, cross-correlated dual-layer structure dynamic factor model is applied to the monthly macroeconomic time series of six U.S. Great Lake states, Illinois, Indiana, Michigan, Minnesota, Ohio and Wisconsin. The estimation was conducted using Bayesian MCMC algorithm embedded in WinBUGS program, thus the outcomes are the posterior joint distribution of the model parameters and the latent factors.

### 5.1 Data Description

A balanced panel of monthly data from January 2003 to September 2012 is collected for each state of the Great Lakes region, Illinois, Indiana, Michigan, Minnesota, Ohio and Wisconsin. The variables for each state are the employment rate that is simply 1—unemployment rate, employment in the nonmanufacturing sector, average weekly hours of production employees in the manufacturing sector, and building permits. Thus, there are a total of 24 variables for the estimation (6 regions  $\times$  4 variables each).

Each first-differenced variable is transformed to have zero mean to eliminate the trend component. The detailed description of the data set is reported in table 5.

<<insert table 5 here>>

### 5.2 Estimation Results

The model is specified to have one region common factor and one region specific factor affecting the four variables of each state, as described in section 3. Although, it is reasonable to use Bayesian Information Criteria in determining the lag orders of the factor loadings in the measurement equation, the law of movement of the factors, and the law of movement of the idiosyncratic error of each variable, some predetermined lag orders are assigned because of the limited computing power.<sup>16</sup> For the observation equation, one temporal lag for each of the region common factor and the region specific factors is entered on the right hand side. For the

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<sup>16</sup> For this model, using 1.3Ghz Intel i3 processor with 4Gb RAM, estimation took about 3 days for 10,000,000 iterations.

state equation of the idiosyncratic error term of the observed variables, AR(1) is assumed. Finally, the movements of factors are assumed to have a reduced VAR(1) form. To assess how the consideration of region common factor affects the resulting estimation, a single level dynamic factor model is also estimated. The estimation results of parameters are presented in Appendix 2, and the estimation results of dynamic factors with their confidence intervals are provided in Appendix 3<sup>17</sup> for the multi-level dynamic factor, and in Appendix 4 for the single level dynamic factor.

#### *Forecast Error Variance Decomposition*

The two-step ahead forecast error variance decomposition of the multi-level dynamic factor model and that of single level dynamic factor model are presented in table 6. The multi-level dynamic factor model reveals that much of the region specific factor can be explained by the region common factor. For example, about 87% of the Illinois specific dynamic factor can be explained by a region common factor, while only 4% accounts for the effect from its neighbors (2.4% from Ohio and 0.4~0.5% from other states). Among those six states, Minnesota's dynamic factor is the one that is most independently evolving, while Indiana is the state that is most affected by its neighbors.

<<insert table 6 here>>

Comparing the variance decomposition results with those of the single level dynamic factor model, the region specific dynamic factors are shown to be much more affected by their neighboring regions if there is no consideration of the region common factor. For example, over 73% of the Illinois specific dynamic factor is explained by the effects from its neighbors. If one believes there is an hierarchical structure in the evolution of regional economy, then the conclusion is that the single level dynamic factor model exaggerates the effect of neighbors by almost 70%.

To summarize, the region common factor does not seem to respond much against state specific factors. On the contrary, at the state level, the region common factor seems to play a significant role. Additionally, it is worthwhile noting that in the variance decomposition of the forecasting

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<sup>17</sup> The estimated region common factor is, in fact, can be regarded as the common shock to the U.S. states. As we can see in Appendix 5, the extracted region common shock and the CFNAI (Chicago Fed National Business Activity Index) are almost identical except for the scales.



of each regional dynamic factor, the share of the region common factor increases as the forecasting horizon expands. Figure 6 presents the variance decomposition results with a time varying forecasting horizon of 1~6 steps ahead. This implies that for higher frequency regional economic observations, the model with spatial dependency will be more effective in assessing the evolution of economic variables, but for lower frequency observations such as annual GDP series, the consideration of spatial dependency might not add much meaningful explanatory power to the model.

<<insert figure 6 here>>

### *Impulse Response Functions*

Some significant interactions between region specific factors can be found from IRF analysis both in single level dynamic factor model and in multi-level dynamic factor model. However, the signs of the impact in the single level dynamic factor model are sometimes reversed in the multi-level structure model, a finding that was expected from the simulation results. The response functions of Illinois against a positive unit shock from its neighbors are illustrated in figure 7.<sup>18</sup>

<<insert figure 7 here>>

In a single level structure model, dynamic factors have primarily a positive impact on Illinois except for Minnesota. However, in the multi-level structure model, Ohio and Wisconsin have positive impacts on Illinois whereas Indiana, Michigan and Minnesota have negative impacts. Ignoring the region common shock, because of the co-movement of business cycles of those states, the single level dynamic factor model generates a positive response against the impulses of Indiana and Michigan. However, in multi-level dynamic factor model, the dynamic effects of Indiana and Michigan net of the region common factor are negative, implying that positive local shocks on those two regions will harm Illinois.

This presence of negative IRFs implies that there can be negative spillover effects between neighboring regions, as opposed to the conventional belief that positive growth of one region is likely to benefit its neighbors. For Illinois, the summary of the impact from and to its neighbors and the region common is depicted in figure 8; the thicker the arrow the larger the amount of the

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<sup>18</sup> The whole pictures of IRFs are in Appendix 6.

impact. The relationships between Illinois and other regions are mostly asymmetric because of the complex dependency structure between multiple regional units. As a whole, the impacts from Illinois on its neighbor regions are all positive, but very negligible in magnitude. The impacts from neighbor regions to Illinois are positive for Wisconsin and Ohio, and are negative for Indiana, Michigan and Minnesota; among them, Ohio has the largest impact on Illinois. However, compared to the region common factor, the region specific impacts are very small and negligible, which implies that macroeconomic behaviors aggregated at the state level can be mostly explained by the region common determinants such as a national level monetary/fiscal policy shock or an international commodity price shock.

<<insert figure 8 here>>

## 6. Conclusion

At the national level, a VAR model with GDP, capital stock and employment usually provides a reasonably comprehensive picture of business activity. On the contrary, at the regional level, GDP is only available annually, and capital stock series are not available. Thus, for the identification of the business cycles of regional economies, it is more appropriate to rely on a latent factor model since the identification of the business cycle inevitably involves the analysis of multiple types of macroeconomic behaviors. An application of the Bernanke, *et al.* (2005) factor-augmented VAR model to regional economies adds a spatial interdependency structure between regions to the dynamic factor model. The Bai and Wang (2012) model additionally provides a multi-level structure for the regional factor analysis. The multi-level dynamic factor analysis in this paper suggests that omitting the region common factor might bias and exaggerate the magnitude of the interdependency between regions.

This model has two implications for regional VAR analysis. One is that, in modeling regional dependencies, incorporating covariates to control for the region common behavior is likely to be critical in identifying the regional interdependency. In the estimation results in section 5, it can be seen that, although the region common shock is set to be endogenous against region specific shocks, the magnitude of the effects from the region specific shocks compared to the region common shock is relatively small. This implies that, conceptually, the region common shock can be regarded as exogenous shocks that are common to the individual regions: such shocks might be generated by monetary/fiscal policy, oil price changes, natural disasters, and so on.

Thus, for example, in the spatial panel model,  $Y_t = \rho WY_t + X_t\beta + \varepsilon_t$ ,  $Y_t = (y_t^A, \dots, y_t^R)$ ,  $t = \{1, \dots, T\}$ , or in more generally, a VAR model<sup>19</sup> with regional variables,  $Y_t = AY_t + X_t\beta + \varepsilon_t$ ,  $Y_t = (y_t^A, \dots, y_t^R)$ ,  $t = \{1, \dots, T\}$ , the control variable  $X_t$  should account for the region common behavior if it is suspected that there is an exogenous effect common to all regions of interest.

Another implication is that the restriction of the coefficient matrix in the spatial VAR model,  $\rho W$ , might misrepresent the interaction structure between regional economies. Since, in the unrestricted coefficient matrix for the regional interaction term, some coefficients have positive and others negative point estimates, the finding would suggest that the regional economies are mixed with competitive and complementary relationships. If the signs of the elements of this coefficient matrix are restricted, the competitive or the complementary relationships between regions might not be found appropriately.

One of the limitations of this paper's analysis is that the selection of the region of interest, i.e., Illinois, Indiana, Michigan, Minnesota, Ohio and Wisconsin, is based on the U.S. Census Bureau's definition of U.S. regions, and thus is somewhat arbitrary. This selection of region assumes that those six states are clustered and more connected to each other than the states outside the Great Lakes region. The interstate trade data provide a strong case that this is in fact true; however, this choice may result in biased estimators caused by other omitted but economically closely connected states. At this time, however, limited computing power makes it difficult to extend the analysis to all U.S. states.

Another limitation of this paper is that if it is suspected that intra-state heterogeneity is an important issue, then it would be important to extend the system to capture sub-state variations as well as the potential impacts of differences across states in industrial structure and their "location" in significant value chains of production. Again, computing power precludes, at this time consideration of multi-level (>2) influences.

In sum, in identifying the competitive and complementary relationship between regional economies, a dynamic factor model with multi-level interdependency structure is useful in solving several problems associated with the identification of business cycle and spatial interdependency in regional economics. One problem is that, unlike the national level

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<sup>19</sup> The coefficient matrix in spatial panel model  $\rho W$  can be thought as a restricted version of the coefficient matrix  $A$  in regional VAR model.

macroeconomic observations, regional level macroeconomic data are mostly limited to employment data, necessitating the use of a factor model. For another instance, in identifying the spatio-temporal dynamics of regional economies, coefficient restrictions involving the spatial weight matrix cannot identify the dynamic structure of regional economies when the regional relationships are mixed with competitive and complementary relationships; thus, the coefficient matrix of the regional interaction term should be allowed to have both negative and positive signs. Additionally, if we suspect the regional economies have multiple layer structure, and the higher level shocks are believed to be significant driving forces of the observed lower level economic behaviors, omitting the region common component can bias the effects of interdependency, thus providing a strong case for the adoption of a Bai and Wang (2012) type of multi-level structure dynamic factor model.

## References

- Aguilar, G. and West, M. (2000) "Bayesian Dynamic Factor Models and Portfolio Allocation," *Journal of Business and Economic Statistics*, 18, 338-357.
- Anselin, L., Gallo, J. L., and Jayet, H. (2008) Spatial panel econometrics. Ch. 19 in L. Mátyás and P. Sevestre, eds., *The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice*, Springer-Verlag, Berlin, 625-660.
- Bai, J. and Ng, S. (2002) "Determining the Number of Factors in Approximate Factor Models," *Econometrica*, 70:1, 191-221.
- Bai, J. and Ng, S. (2006) "Confidence Intervals for Diffusion Index Forecasts and Inference with Factor-Augmented Regressions," *Econometrica* 74, 1133-1150.
- Bai, J. and Wang, P. (2012) "Identification and Estimation of Dynamic Factor Models," Discussion Paper No.:1112-06, Department of Economics, Columbia University, New York, NY 10027
- Beenstock, M. and Felsenstein, D. (2007) "Spatial Vector Autoregressions," *Spatial Economic Analysis*, 2:2, 167-196.
- Belviso, F. and Milani, F. (2006) "Structural Factor-Augmented VARs (SFAVARs) and the Effects of Monetary Policy," *B.E. Journal of Macroeconomics: Topics in Macroeconomics*, 6 Issue 3, 1-46.
- Bernanke, B., Boivin, J. and Elias P. (2005) "Measuring the Effects of Monetary Policy: a Factor-Augmented Vector Autoregressive (FAVAR) Approach," *The Quarterly Journal of Economics* 120(1), 387-422.
- Bhattacharjee, A. and Holly, S. (2013) "Understanding Interactions in Social Networks and Committees," *Spatial Economic Analysis : the Journal of the Regional Studies Association*. - Cambridge : Routledge, Vol. 8.2013, 1, 23-53.
- Bhattacharjee, A. and Jensen-Butler, C. "Estimation of the Spatial Weight Matrix under Structural Constraints," *Regional Science and Urban Economics* 43 (2013) 617-634.
- Blanchard, O.J., Katz, L.F., Hall, R.E. and Eichengreen, B. (1992) "Regional Evolutions," *Brookings Papers on Economic Activity*, 92(1), 1-75.
- Bureau of Transportation Statistics (2010) 2007 Commodity Flow Survey: State Tables, [http://www.rita.dot.gov/bts/sites/rita.dot.gov/bts/files/publications/commodity\\_flow\\_survey/2007/state\\_s/index.html](http://www.rita.dot.gov/bts/sites/rita.dot.gov/bts/files/publications/commodity_flow_survey/2007/state_s/index.html)
- Carter, C. K. and Kohn, R. (1994) "On Gibbs Sampling for State Space Models," *Biometrika*, 81, 541-533.
- Chudik, A., Pesaran, M.H. and Tosetti, E. (2011) "Weak and Strong Cross-section Dependence and Estimation of Large Panels," *Econometrics Journal* (2011), volume 14, C45-C90.
- Clark, T. E. (1998) "Employment Fluctuations in U.S. regions and Industries: The Roles of National, Region Specific, and Industry-Specific Shocks," *Journal of Labor Economics*, 16, 202-229.
- Clark, T. E. and Shin, K. (1998) "The Sources of Fluctuations within and Across Countries," *Research Working Paper* 98-04, Federal Reserve Bank of Kansas City.
- Crone, T. M. (2002). "Consistent Economic Indexes for the 50 States," *Working Papers* 02-7 , Federal Reserve Bank of Philadelphia
- Corrado, L. & Fingleton, B. (2011) "Multilevel Modelling with Spatial Effects," *Working Papers* 1105, University of Strathclyde Business School, Department of Economics

- Di Giacinto, V. (2010) "On Vector Autoregressive Modeling in Space and Time," *Journal of Geographical Systems*, 12, 125-154.
- Gamerman, D. and Mignon, H. S. (1993) "Dynamic Hierarchical Models," *Journal of Royal Statistical Society Series B*, 55, 629-642.
- Glick, R. and Rogo, K. (1995) "Global versus Country-Specific Productivity Shocks and the Current Account," *Journal of Monetary Economics*, 35, 159-192.
- Hayshed, Motonari and Hewings, G. J. D. (2009) "Regional Business Cycles in Japan," *International Regional Science Review* 32, 110-147.
- Hewings, G.J.D. , (2008) "On some conundra in regional science," *Annals of Regional Science* 42, 251-265.
- Kascha, C. and Mertens, K. (2008) "Business Cycle Analysis and VARMA Models," *Norges Bank Working Paper*, ANO 2008/5 (Oslo).
- Kose, A., Otrok, C. and Whiteman, C. (2003) "International Business Cycles: World Region and Country Specific Factors," *American Economic Review* 93, 1216-1239.
- Kose, A., Otrok, C. and Whiteman, C. (2008) "Understanding the Evolution of World Business Cycles," *International Economic Review*, 75, 110-130.
- Lopes, H., Salazar, E. and Gamerman, D. (2008) "Spatial Dynamic Factor Model," *Bayesian Analysis* 3, 759-792.
- Magalhães, A., Sonis, M., and Hewings, G.J.D. (2001) "Regional competition and complementarity reflected in Relative Regional Dynamics and Growth of GSP: a Comparative Analysis of the Northeast of Brazil and the Midwest States of the U.S." In Joaquim J.M. Guilhoto and Geoffrey J.D. Hewings (eds.) *Structure and Structural Change in the Brazilian Economy*, London, Ashgate.
- Marquez, M.A., Ramajo, J., and Hewings, G.J.D. (2013) "Assessing Regional Economic Performance: Regional Competition in Spain under a Spatial Vector Autoregressive Approach," in Riccardo Crescenzi and Marco Percoco (Eds.) *Geography, Institutions and Regional Economic Performance* Heidelberg, Springer.
- Owyang, M.T, Rapach, D.E., and Wall, H.J. (2009) "States and the Business Cycle" *Journal of Urban Economics*, 65, 181-194.
- Park, Y., Seo, J. and Hewings, G.J.D. (2002). "Development of a Regional Economic Activity Index For the Chicago Metropolitan Area," *Discussion Paper* 02-T-5, University of Illinois, Urbana <http://www.real.illinois.edu/d-paper/02/02-t-5.pdf>
- Park, Y. and Hewings, G.J.D. (2012) "Does Industry Mix Matter in Regional Business Cycles?" *Studies in Regional Science*, 42, 39-60.
- Pesaran, M.H. (2006) "Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure," *Econometrica* Volume 74, Issue 4, 967–1012, July 2006
- Pesaran, M.H. and Shin, Y. (1998) "Generalized Impulse Response Analysis in Linear Multivariate Models," *Journal of Economics Letters*, 58, 17-29.
- Stock, J. H. and Watson, M. W. (1989) "New Indexes of Coincident and Leading Economic Indicators," *NBER Macroeconomic Annual*, 351-394.
- Stock, J. H. and Watson, M. W. (2005) "Implications of Dynamic Factor Models for VAR Analysis," *NBER Working Paper* 11467.

- Stock, J. H. and Watson, M. W. (2006) "Forecasting with Many Predictors," Chapter 10 in *Handbook of Forecasting*, Volume 1, Elliot, G., Granger, C.W.J., and Timmermann, A. (ed.), North Holland.
- Stock, J. H. and Watson, M. W. (2009) "The Evolution of National and Regional Factors in U.S. Housing Construction," *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*, Oxford University Press, Oxford.

<Table 1>

Interstate Trade Flows in the Midwest, US, 2007

	(\$ million)			(%)		
	Domestic	Foreign	Total	Foreign	Domestic	Midwest
IL	399,913	48,869	448,809	10.89	89.11	32.40
IN	252,023	25,956	227,979	9.34	90.66	33.82
MI	226,875	44,555	271,430	16.41	83.59	32.29
OH	369,824	42,562	412,386	10.32	89.68	27.62
WI	172,125	18,825	190,950	9.86	90.14	33.19

<Table 2>

Spillovers from Employment Changes in One State on the Remaining Midwest States

Change  
In state

Impact in state

	IL	IN	MI	OH	WI	Rest of Midwest Total	RUS
IL	-	5.98%	4.70%	5.13%	3.85%	19.66%	80.34%
IN	9.36%	-	6.19%	12.00%	2.33%	29.88%	70.12%
MI	5.78%	5.73%	-	13.10%	5.06%	29.66%	70.34%
OH	4.54%	6.47%	8.24%	-	1.98%	21.24%	78.76%
WI	7.91%	3.64%	8.35%	5.00%	-	24.91%	75.09%



<Table 3>

REIM Impact Analysis on Unit Shock on Regional Output

In terms of Output

(Mil. Dollars)

State	Direct Impact	Total Impact (2013)						Total
		IL	ID	MI	OH	WI	ROUS	
IL	1.000	<b>1.717</b>	0.042	0.041	0.023	0.023	0.301	2.146
ID	1.000	0.118	<b>2.051</b>	0.093	0.145	0.023	0.914	3.343
MI	1.000	0.075	0.056	<b>1.790</b>	0.121	0.049	0.457	2.549
OH	1.000	0.073	0.078	0.114	<b>1.882</b>	0.020	1.055	3.222
WI	1.000	0.096	0.034	0.099	0.028	<b>1.891</b>	0.426	2.573

In Terms of Employment

(1,000 persons)

State	Direct Impact	Total Impact (2013)						Total
		IL	ID	MI	OH	WI	ROUS	
IL	0.008	<b>0.013</b>	0.000	0.000	0.000	0.000	0.007	0.021
ID	0.009	0.001	<b>0.017</b>	0.000	0.001	0.000	0.009	0.029
MI	0.008	0.001	0.000	<b>0.014</b>	0.001	0.000	0.008	0.024
OH	0.009	0.000	0.001	0.001	<b>0.017</b>	0.000	0.009	0.028
WI	0.009	0.001	0.000	0.000	0.000	<b>0.016</b>	0.009	0.027

<Table 4>

12-step ahead Forecast Error Variance Decomposition (%)

	True Model			True Specification (Multi-level)			
	Source of Variation						
Factors	$g$	$f^A$	$f^B$		$g$	$f^A$	$f^B$
$g$	99.94	0.01	0.05	↔	98.61	0.15	1.23
$f^A$	41.77	57.62	0.62		46.17	53.25	0.57
$f^B$	65.74	0.68	33.58		58.32	2.74	38.94
		↕					
						← False Specification (Single level)	
$f^A$		51.13	48.87				
$f^B$		0.14	99.86				

<Table 5>

Data Description

Variables ( $y_t^r$ )	Description	Transformation
$y_t^{r1}$	Employment Rate (= $\frac{\text{Number of Employed Labor Force}}{\text{Total Labor Force}}$ )	1 <sup>st</sup> differenced and demeaned Originally seasonally adjusted
$y_t^{r2}$	Employment in Nonmanufacturing Sector (=Total Nonfarm Employment- Employment in Manufacturing Sector)	Log 1 <sup>st</sup> differenced, demeaned Originally seasonally adjusted
$y_t^{r3}$	Average Weekly Hours of Production Employees in Manufacturing Sector	Log 1 <sup>st</sup> differenced, demeaned Seasonally adjusted using Census X-12
$y_t^{r4}$	Building Permits (all units)	Log 1 <sup>st</sup> differenced, demeaned Seasonally adjusted using Census X-12

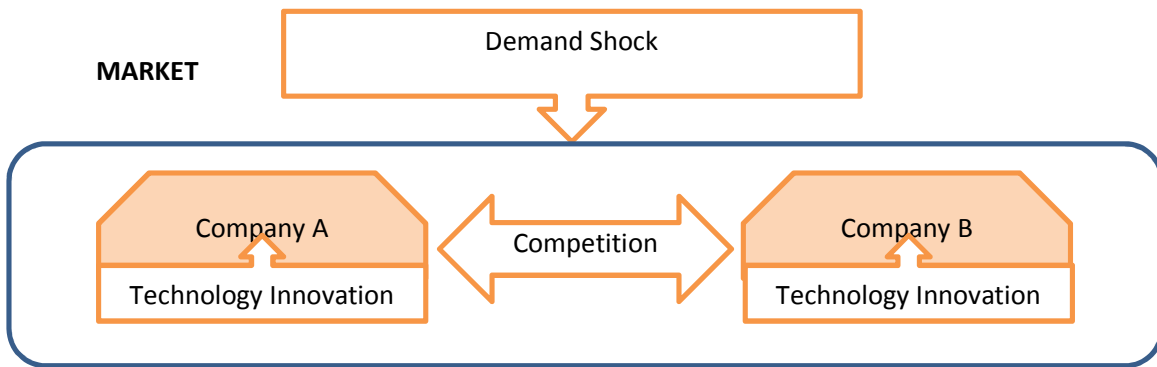
Source: Bureau of Labor Statistics and U.S. Census Bureau

&lt;Table 6&gt;

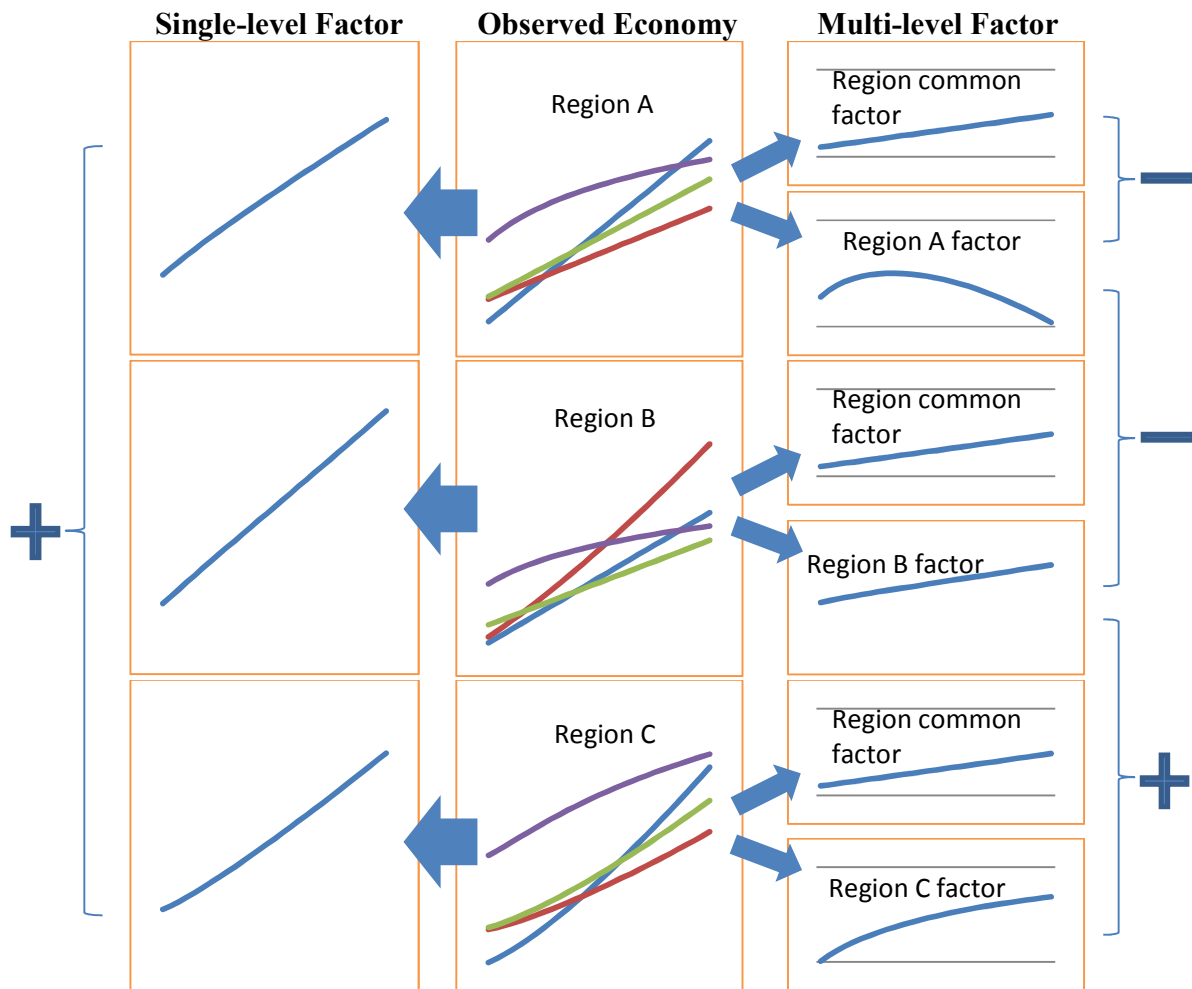
2-step-ahead Forecast Error Variance Decomposition (%)

Multi-level Dynamic Factor Model									
	IL	IN	MI	MN	OH	WI	Common	SUM	Neighbor
IL	9.4	0.4	0.4	0.5	2.4	0.4	86.5	100.0	4.1
IN	0.3	6.4	0.6	0.2	6.2	0.3	86.0	100.0	7.6
MI	0.2	0.1	10.5	1.2	2.5	2.3	83.1	100.0	6.4
MN	0.1	0.1	0.4	36.5	3.5	0.3	59.1	100.0	4.4
OH	0.1	0.2	0.4	0.2	29.8	1.0	68.3	100.0	1.9
WI	0.2	0.2	0.7	1.0	2.8	7.6	87.4	100.0	4.9
Common	0.0	0.1	0.6	0.8	0.0	0.1	98.3	100.0	
Single-level Dynamic Factor Model									
IL	26.8	25.7	0.6	5.7	31.6	9.5		100.0	73.2
IN	9.8	39.1	1.3	5.3	32.0	12.5		100.0	60.9
MI	40.3	12.6	7.1	4.7	23.3	12.0		100.0	92.9
MN	7.4	25.9	11.1	8.7	14.4	32.5		100.0	91.3
OH	2.1	11.7	0.3	0.8	82.8	2.4		100.0	17.2
WI	15.7	23.1	6.1	7.9	27.7	19.4		100.0	80.6

<Figure 1>

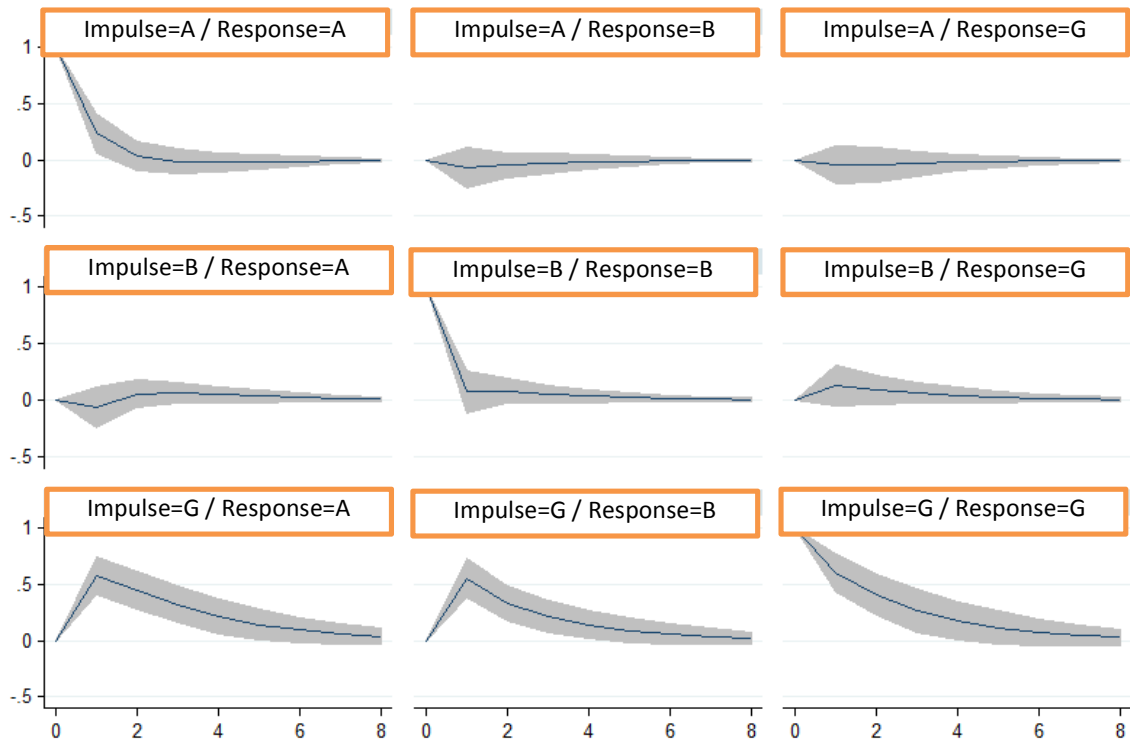


<Figure 2>



<Figure 3>

True IRF with 95% confidence interval



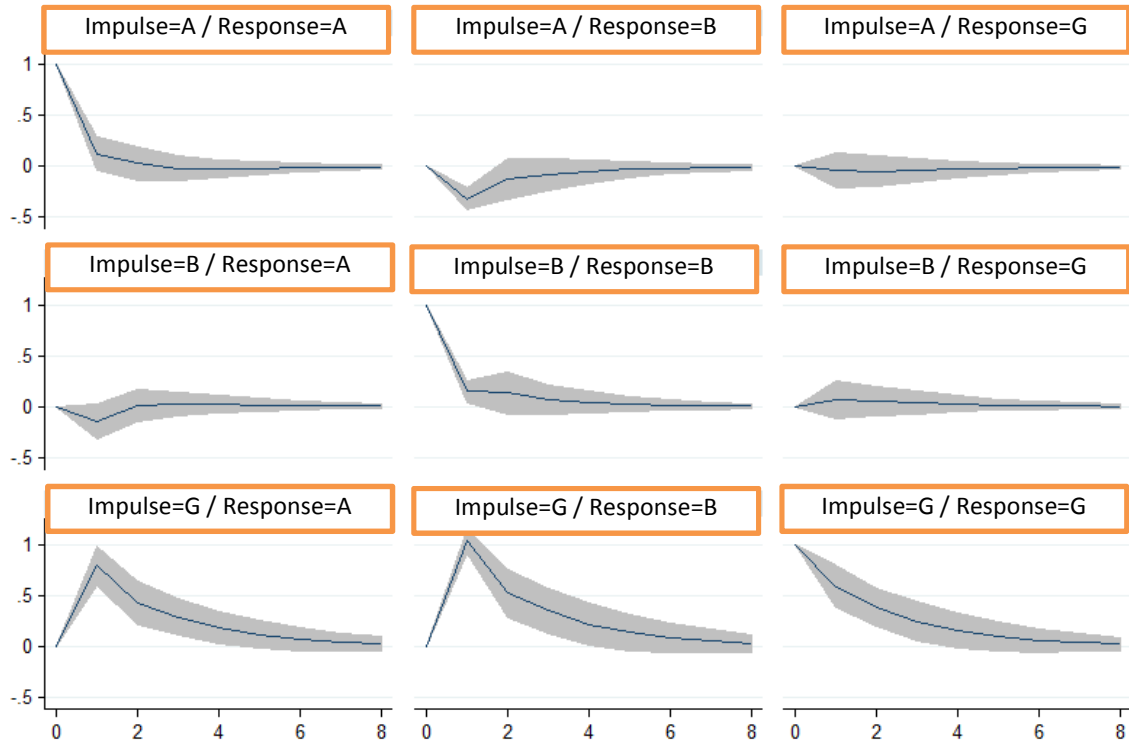
\* “A” denote region A specific factor, “B” denote region B specific factor, and “G” denote region common dynamic factor

\* Positive region common shock has significant and positive impact on individual regions and itself. On the contrary, positive region specific shock has negligible impact on region common shock.

\* Positive region specific shock has significant and positive impact on itself, and has small negative impact on its neighbors.

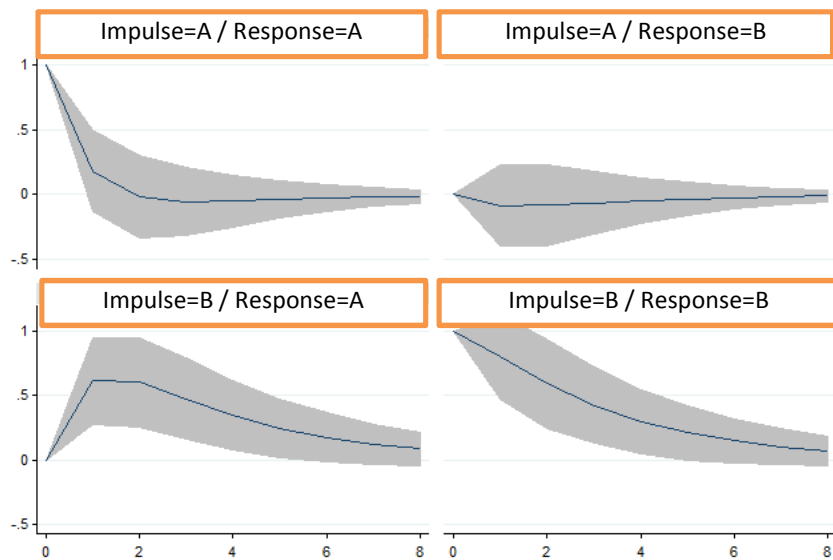
<Figure 4>

IRF from multi-level model with 95% confidence interval



<Figure 5>

IRF from single level model with 95% confidence interval

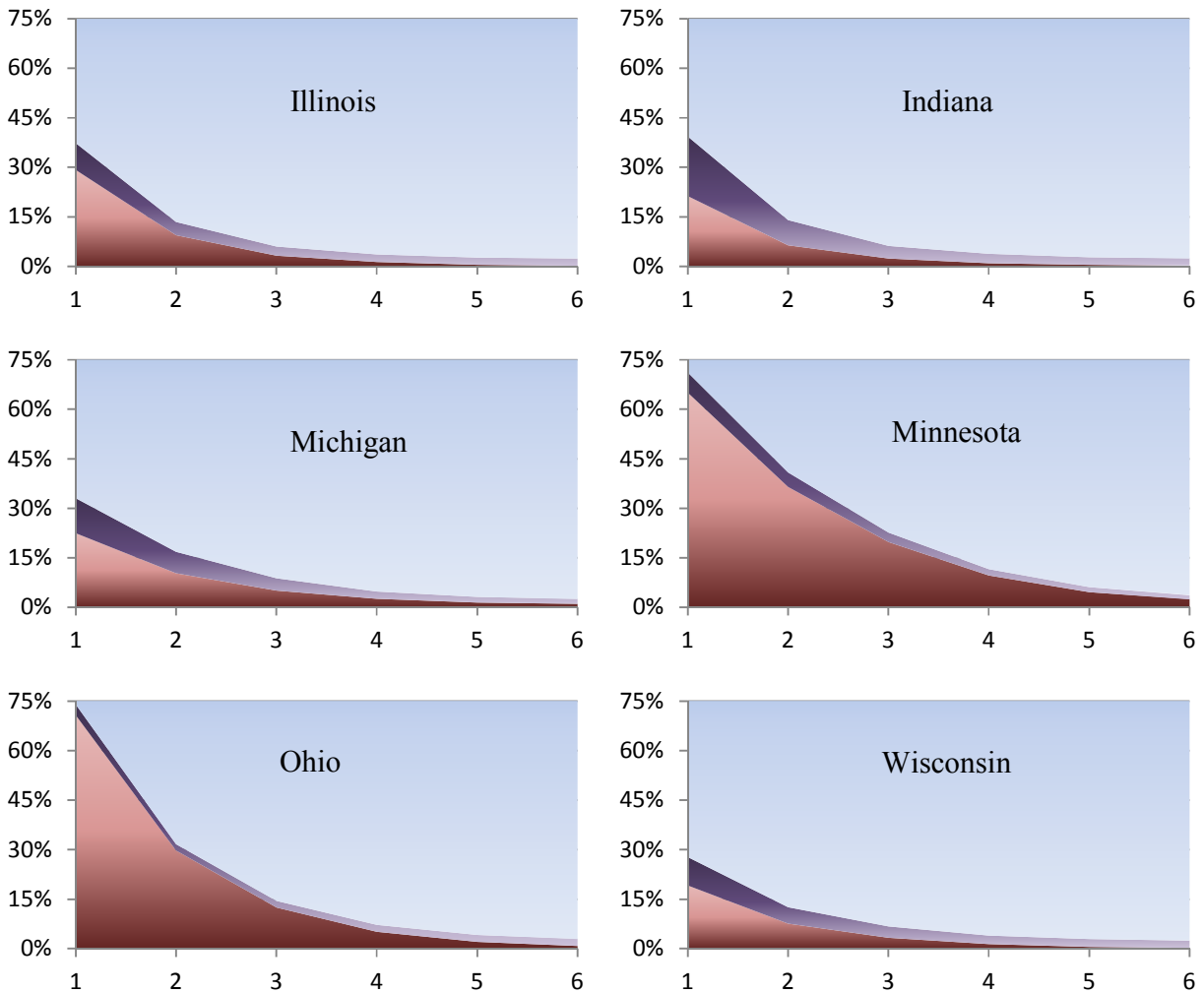


<Figure 6>

Forecast Error Variance Decomposition with Varying Forecasting Horizon\*

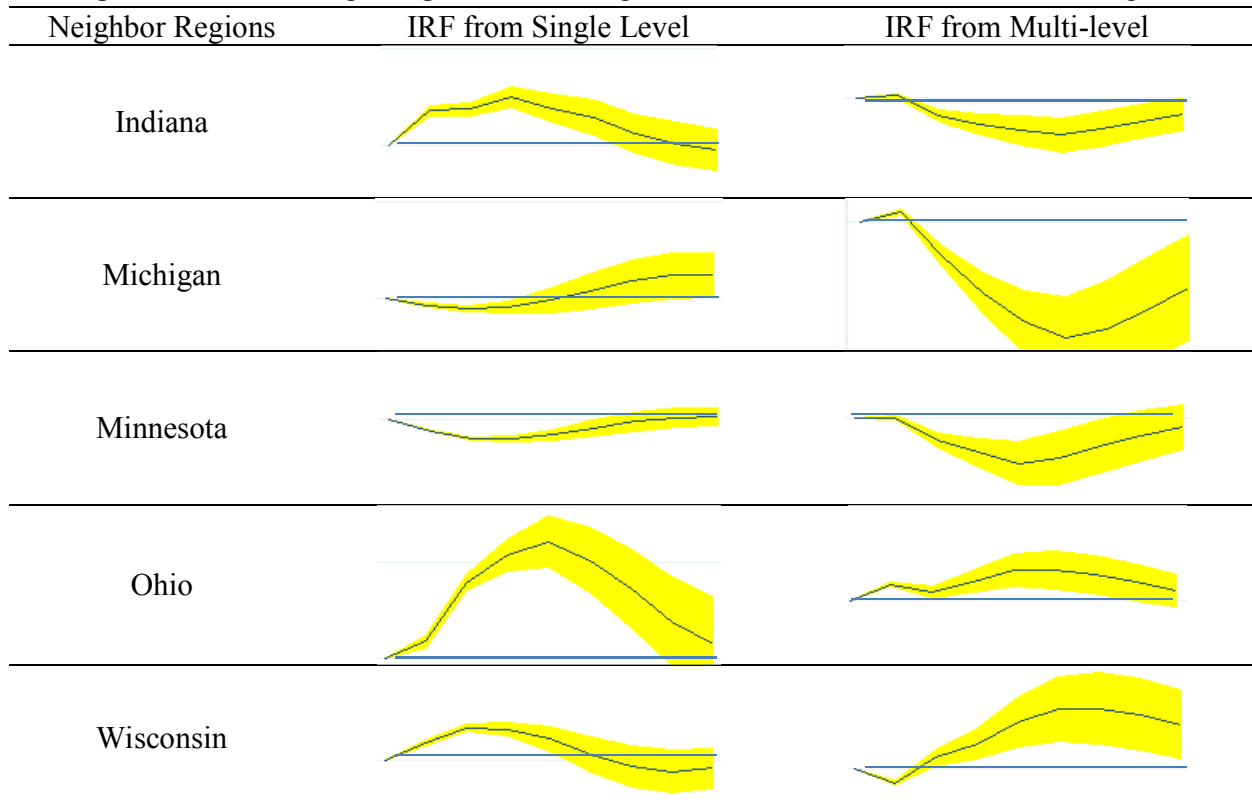
\* x-axis: forecasting horizon, y-axis: variance decomposition

The area describes, from the bottom, the variance from its own factor, the variance from its neighbor, and the variance from region common factor.



<Figure 7>

Response of Illinois Region Specific Factor against a Positive Unit shock from its Neighbors

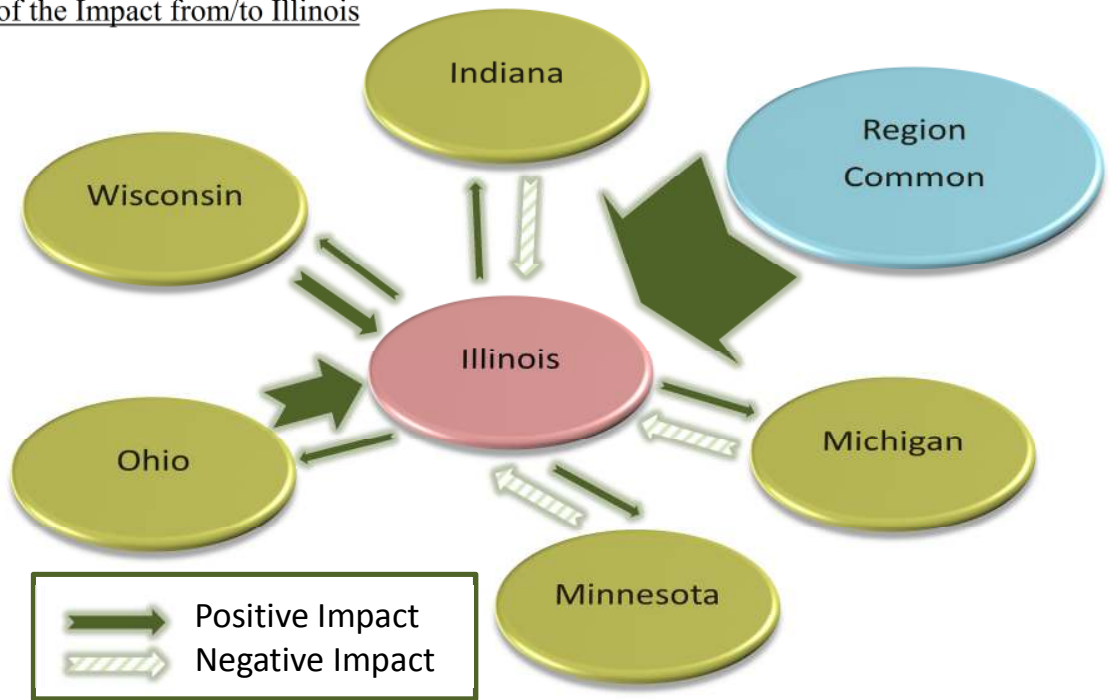


\* Horizontal line denotes 0



<Figure 8>

Summary of the Impact from/to Illinois



<Appendix 1>

### Posterior Distribution of Parameters

Suppose we have one lag factor term, and the error terms follow AR(1) process in observation equation (10). Then equation (10)~(12) can be expressed as (10)' ~ (12)'.

$$(10)' Y_t = \Lambda_0 f_t + \Lambda_1 f_{t-1} + u_t$$

$$(11)' u_t = \theta u_{t-1} + \varepsilon_t$$

$$(12)' f_t = \Phi f_{t-1} + H_t$$

$$\text{where } Y_t = (y_t^1, \dots, y_t^R)'$$

$$f_t = (g_t, f_t^1, \dots, f_t^R)'$$

$$u_t = (u_t^g, u_t^1, \dots, u_t^R)'$$

$$\varepsilon_t = (\varepsilon_t^g, \varepsilon_t^1, \dots, \varepsilon_t^R)'$$

Since  $\varepsilon_t$  are assumed to be mutually and serially uncorrelated, it is convenient to transform equation (10)' and (11)' into the following equation (10)'' in an actual implementation of WinBUGS.

$$(10)'' Y_t = \theta Y_{t-1} + \Lambda_0 f_t + (\Lambda_1 - \theta \Lambda_0) f_{t-1} - \theta \Lambda_1 f_{t-2} + \varepsilon_t$$

Along with equation (12)', the following describes the likelihood and the priors.

(Likelihood)

$$\text{For each observation, } y_t^r, f(y_t^r) = \text{Normal}(\mu_t^r, \Sigma^r), \text{ where } \Sigma^r = \begin{pmatrix} \sigma_1^{r^2} & 0 & \dots & 0 \\ 0 & \sigma_2^{r^2} & \dots & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & \dots & \sigma_p^{r^2} \end{pmatrix},$$

and for  $t = \{3, \dots, T\}$ ,  $\mu_t = \theta Y_{t-1} + \Lambda_0 f_t + (\Lambda_1 - \theta \Lambda_0) f_{t-1} - \theta \Lambda_1 f_{t-2}$  where  $\mu_t = (\mu_t^1, \dots, \mu_t^R)'$ .

Thus, the likelihood becomes:

$$L(\theta, \Lambda_0, \Lambda_1, \Sigma, f_1, \dots, f_T, u_1, u_2 | y_1, y_2) = \prod_{t=3}^T f(y_t | y_1, y_2)$$

(Priors)

For the unobservable factors, the equation (12)' suggests  $f_t \sim \text{Normal}(\mu_t^f, I_{R+1})$  where  $\mu_t^f = \Phi f_{t-1}$ . Note that the covariance matrix is restricted to have an i.i.d. unit variance to identify a unique set of latent factors.

For other parameters and the initial factor  $f_1$ , improper priors are assigned to reflect the ignorance of the prior distributions such as Normal distribution with precision 0.001 and Inverse Gamma distribution with parameter (0.001, 0.001).

<Appendix 2>

Dynamic Factor Model Estimation Results

Model for 6 regions ( $r \in \{IL, IN, MI, MN, OH, WI\}$ ) and 4 macroeconomic variables ( $i \in \{1, \dots, 4\}$ , (1,2,3,4)=(employment rate, non-manufacturing sector employment, manufacturing working hour, building permit)).

(1) Observation Equation:  $y_t^{ri} = \alpha_0^{ri} g_t + \alpha_1^{ri} g_{t-1} + \beta_0^{ri} f_t^r + \beta_1^{ri} f_{t-1}^r + \varepsilon_t^{ri}$

(Note that for single-level dynamic factor model,  $\alpha_0^{ri} = \alpha_1^{ri} = 0$ , and

For multi-level dynamic factor model,  $\alpha_1^{r1} = \beta_1^{r1} = 0$  and  $\alpha_0^{r1} > 0$  and  $\beta_0^{r1} > 0$ )

(2) State Equation for Idiosyncratic Error:  $\varepsilon_t^{ri} = \theta^{ri} \varepsilon_{t-1}^{ri} + u_t^{ri}$ ,  $u_t^{ri} \sim iid N(0, \sigma_{ri}^2)$

(3) State Equation for Dynamic Factor:  $F_t = \Lambda F_{t-1} + \eta_t$ ,  $\eta_t \sim N(0, I_R)$

where  $\Lambda = \begin{bmatrix} \lambda_{11} & \dots & \lambda_{1R} \\ \vdots & \ddots & \vdots \\ \lambda_{R1} & \dots & \lambda_{RR} \end{bmatrix}$  is  $R \times R$  matrix of coefficient,

$$F_t = \begin{cases} (g_t, f_t^{IL}, f_t^{IN}, f_t^{MI}, f_t^{MN}, f_t^{OH}, f_t^{WI})' & \text{for multi-level structure} \\ (f_t^{IL}, f_t^{IN}, f_t^{MI}, f_t^{MN}, f_t^{OH}, f_t^{WI})' & \text{for single-level structure} \end{cases} \text{ and } R = \begin{cases} 7 \\ 6 \end{cases}$$

	Multi-level Dynamic Factor Model				Single-level Dynamic Factor Model			
# of Iteration	1,700,000				2,300,000			
Burn-in	100,000				300,000			
Thining	4,000				4,000			
# of Sample	400				500			
-2logLik	10,087				9,978			
	mean	sd	2.5% QTR	97.5%QTR	mean	sd	2.5% QTR	97.5%QTR
$\beta_0(IL,1)$	0.0205	0.0148	0.0009	0.0581	0.0062	0.0063	0.0001	0.0246
$\beta_0(IN,1)$	0.0431	0.0381	0.0013	0.1358	0.4450	0.1006	0.2573	0.6368
$\beta_0(MI,1)$	0.0500	0.0341	0.0035	0.1276	0.2725	0.0490	0.1626	0.3680
$\beta_0(MN,1)$	0.1690	0.1104	0.0036	0.3979	0.0387	0.0136	0.0166	0.0673
$\beta_0(OH,1)$	0.0432	0.0239	0.0042	0.0945	0.0144	0.0155	0.0002	0.0571
$\beta_0(WI,1)$	0.0344	0.0233	0.0027	0.0928	0.0867	0.0251	0.0441	0.1390
$\alpha_0(IL,1)$	0.3405	0.0636	0.2279	0.4865				
$\alpha_0(IN,1)$	0.4232	0.0934	0.2613	0.6139				
$\alpha_0(MI,1)$	0.4530	0.0880	0.2960	0.6384				
$\alpha_0(MN,1)$	0.2669	0.0938	0.0734	0.4510				
$\alpha_0(OH,1)$	0.2582	0.0492	0.1681	0.3545				
$\alpha_0(WI,1)$	0.3980	0.0618	0.2893	0.5240				
$\beta_0(IL,2)$	0.0051	0.0079	-0.0127	0.0137	-0.0028	0.0016	-0.0065	-0.0004
$\beta_0(IN,2)$	0.0090	0.0039	0.0020	0.0178	0.0044	0.0016	0.0016	0.0077
$\beta_0(MI,2)$	-0.0043	0.0041	-0.0121	0.0038	0.0008	0.0006	-0.0003	0.0020
$\beta_0(MN,2)$	-0.0017	0.0079	-0.0107	0.0205	0.0004	0.0003	0.0000	0.0012
$\beta_0(OH,2)$	-0.0002	0.0037	-0.0101	0.0058	-0.0039	0.0027	-0.0090	0.0010
$\beta_0(WI,2)$	-0.0077	0.0058	-0.0169	0.0093	0.0004	0.0003	-0.0001	0.0012
$\alpha_0(IL,2)$	0.0087	0.0040	0.0013	0.0165				

$\alpha 0(\text{IN}, 2)$	0.0085	0.0056	-0.0014	0.0202				
$\alpha 0(\text{MI}, 2)$	0.0062	0.0035	-0.0004	0.0129				
$\alpha 0(\text{MN}, 2)$	0.0058	0.0033	-0.0005	0.0128				
$\alpha 0(\text{OH}, 2)$	0.0037	0.0049	-0.0061	0.0119				
$\alpha 0(\text{WI}, 2)$	0.0080	0.0047	-0.0017	0.0177				
$\beta 1(\text{IL}, 2)$	0.0024	0.0038	-0.0051	0.0095	-0.0009	0.0013	-0.0034	0.0020
$\beta 1(\text{IN}, 2)$	0.0015	0.0061	-0.0101	0.0129	-0.0008	0.0013	-0.0034	0.0015
$\beta 1(\text{MI}, 2)$	0.0025	0.0047	-0.0065	0.0121	0.0012	0.0006	0.0001	0.0024
$\beta 1(\text{MN}, 2)$	0.0011	0.0039	-0.0059	0.0106	-0.0001	0.0002	-0.0007	0.0003
$\beta 1(\text{OH}, 2)$	0.0109	0.0040	0.0014	0.0177	-0.0036	0.0029	-0.0097	0.0021
$\beta 1(\text{WI}, 2)$	-0.0001	0.0049	-0.0082	0.0126	0.0002	0.0003	-0.0004	0.0008
$\alpha 1(\text{IL}, 2)$	-0.0247	0.0331	-0.1133	0.0282				
$\alpha 1(\text{IN}, 2)$	-0.0271	0.0268	-0.0865	0.0036				
$\alpha 1(\text{MI}, 2)$	-0.0012	0.0132	-0.0241	0.0289				
$\alpha 1(\text{MN}, 2)$	-0.0060	0.0162	-0.0600	0.0050				
$\alpha 1(\text{OH}, 2)$	-0.0035	0.0140	-0.0280	0.0275				
$\alpha 1(\text{WI}, 2)$	0.0021	0.0155	-0.0293	0.0388				
$\beta 0(\text{IL}, 3)$	-0.0006	0.0067	-0.0121	0.0124	-0.0123	0.0052	-0.0239	-0.0034
$\beta 0(\text{IN}, 3)$	0.0089	0.0095	-0.0077	0.0299	0.0078	0.0062	-0.0043	0.0213
$\beta 0(\text{MI}, 3)$	-0.0050	0.0089	-0.0210	0.0158	0.0062	0.0039	-0.0015	0.0140
$\beta 0(\text{MN}, 3)$	0.0001	0.0108	-0.0212	0.0242	0.0024	0.0011	0.0008	0.0051
$\beta 0(\text{OH}, 3)$	0.0018	0.0178	-0.0318	0.0357	-0.0410	0.0281	-0.0921	0.0172
$\beta 0(\text{WI}, 3)$	-0.0020	0.0097	-0.0197	0.0204	-0.0003	0.0013	-0.0031	0.0022
$\alpha 0(\text{IL}, 3)$	0.0134	0.0074	-0.0006	0.0300				
$\alpha 0(\text{IN}, 3)$	0.0048	0.0140	-0.0233	0.0301				
$\alpha 0(\text{MI}, 3)$	0.0178	0.0135	-0.0084	0.0444				
$\alpha 0(\text{MN}, 3)$	0.0211	0.0090	0.0032	0.0398				
$\alpha 0(\text{OH}, 3)$	0.0180	0.0172	-0.0145	0.0506				
$\alpha 0(\text{WI}, 3)$	0.0018	0.0103	-0.0207	0.0226				
$\beta 1(\text{IL}, 3)$	0.0003	0.0045	-0.0094	0.0096	0.0087	0.0048	0.0006	0.0185
$\beta 1(\text{IN}, 3)$	-0.0071	0.0093	-0.0265	0.0137	-0.0031	0.0061	-0.0163	0.0084
$\beta 1(\text{MI}, 3)$	0.0015	0.0106	-0.0203	0.0223	-0.0042	0.0038	-0.0116	0.0034
$\beta 1(\text{MN}, 3)$	0.0074	0.0111	-0.0166	0.0296	-0.0020	0.0010	-0.0044	-0.0005
$\beta 1(\text{OH}, 3)$	-0.0159	0.0154	-0.0477	0.0130	0.0393	0.0279	-0.0186	0.0897
$\beta 1(\text{WI}, 3)$	0.0076	0.0083	-0.0079	0.0243	0.0011	0.0013	-0.0010	0.0040
$\alpha 1(\text{IL}, 3)$	-0.0113	0.0237	-0.0697	0.0358				
$\alpha 1(\text{IN}, 3)$	-0.0167	0.0367	-0.1140	0.0397				
$\alpha 1(\text{MI}, 3)$	-0.0107	0.0316	-0.0725	0.0627				
$\alpha 1(\text{MN}, 3)$	-0.0181	0.0135	-0.0438	0.0033				
$\alpha 1(\text{OH}, 3)$	-0.0091	0.0330	-0.0703	0.0684				
$\alpha 1(\text{WI}, 3)$	-0.0037	0.0215	-0.0551	0.0390				
$\beta 0(\text{IL}, 4)$	0.1184	0.3409	-0.6347	0.7333	-0.2887	0.1654	-0.6658	-0.0327
$\beta 0(\text{IN}, 4)$	0.1221	0.1116	-0.0905	0.3623	0.1443	0.0896	-0.0191	0.3305
$\beta 0(\text{MI}, 4)$	-0.0447	0.1207	-0.2799	0.2185	0.0667	0.0426	-0.0127	0.1610
$\beta 0(\text{MN}, 4)$	-0.9212	0.8147	-2.2420	0.6585	0.0048	0.0175	-0.0288	0.0412
$\beta 0(\text{OH}, 4)$	0.2722	0.1491	-0.0104	0.5846	-0.3701	0.1813	-0.7779	-0.0330
$\beta 0(\text{WI}, 4)$	-0.1492	0.1925	-0.4520	0.4006	0.0373	0.0209	0.0021	0.0865
$\alpha 0(\text{IL}, 4)$	0.5020	0.2885	-0.0086	1.1160				
$\alpha 0(\text{IN}, 4)$	0.2295	0.1751	-0.0649	0.6162				
$\alpha 0(\text{MI}, 4)$	0.2232	0.1563	-0.0376	0.5792				

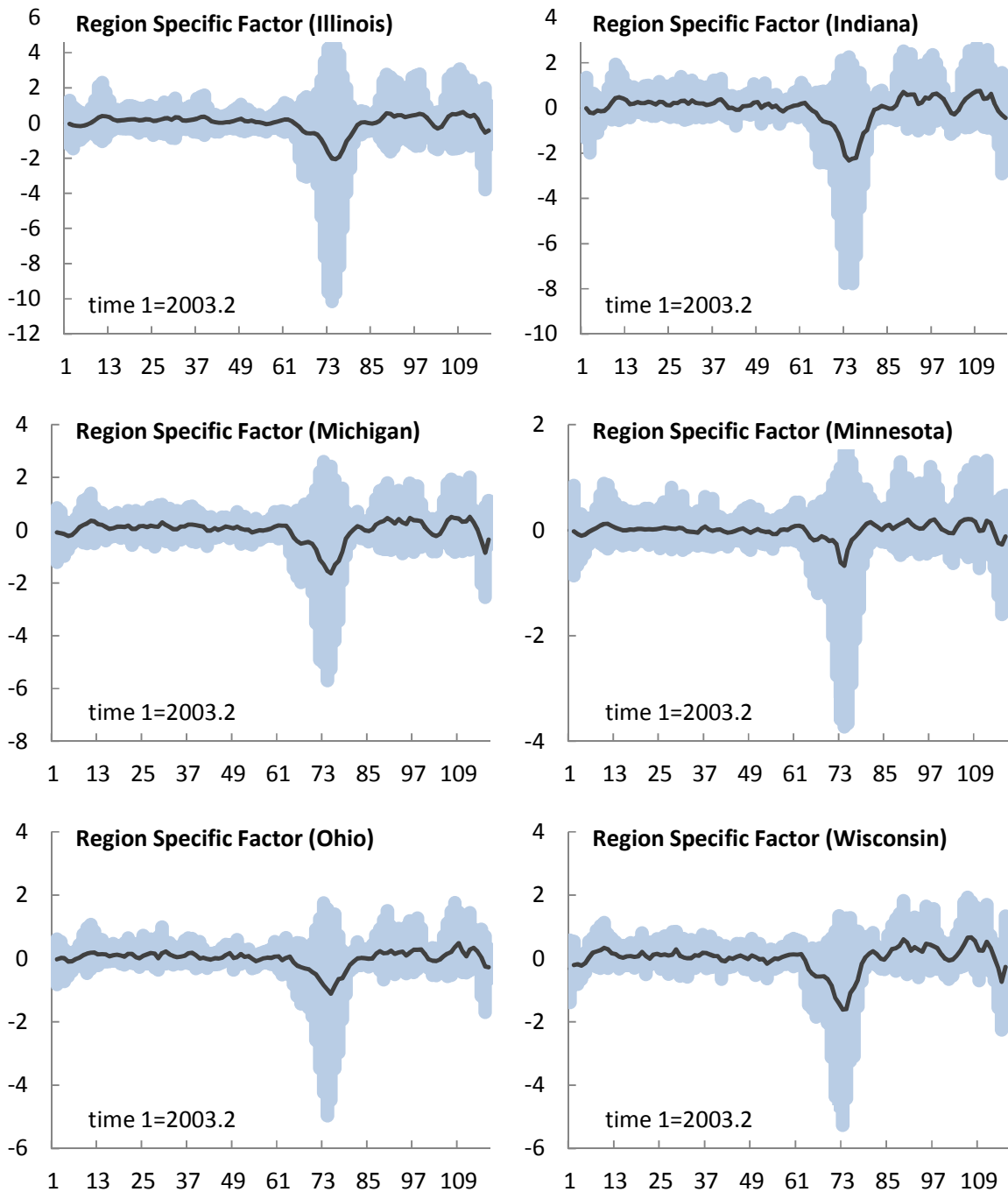
$\alpha 0(MN,4)$	0.3998	0.4496	-0.5055	1.2480				
$\alpha 0(OH,4)$	0.1609	0.1452	-0.1135	0.4797				
$\alpha 0(WI,4)$	0.4126	0.2094	0.0475	0.9144				
$\beta 1(IL,4)$	0.1242	0.1959	-0.2825	0.5468	0.2489	0.1581	0.0035	0.6021
$\beta 1(IN,4)$	0.0180	0.1313	-0.2413	0.3154	-0.1351	0.0904	-0.3190	0.0373
$\beta 1(MI,4)$	0.0437	0.1140	-0.1830	0.2784	-0.0512	0.0427	-0.1436	0.0303
$\beta 1(MN,4)$	0.9711	0.7888	-0.4890	2.3220	-0.0042	0.0174	-0.0414	0.0311
$\beta 1(OH,4)$	0.0062	0.1320	-0.2478	0.2510	0.3157	0.1791	-0.0156	0.7124
$\beta 1(WI,4)$	0.1741	0.1283	-0.0425	0.4345	-0.0341	0.0204	-0.0819	-0.0012
$\alpha 1(IL,4)$	-1.0640	1.3230	-4.4890	0.8777				
$\alpha 1(IN,4)$	-0.5583	0.5872	-2.1120	0.0964				
$\alpha 1(MI,4)$	-0.2256	0.3115	-0.8647	0.3757				
$\alpha 1(MN,4)$	-0.5312	0.5409	-1.6600	0.3175				
$\alpha 1(OH,4)$	-0.3381	0.3181	-1.1790	0.0544				
$\alpha 1(WI,4)$	-0.4569	0.3740	-1.2590	0.0577				
$\theta(IL,1)$	0.9020	0.0423	0.8159	0.9877	0.9358	0.0364	0.8708	1.0100
$\theta(IN,1)$	0.4402	0.1021	0.2358	0.6336	0.8133	0.0706	0.6638	0.9403
$\theta(MI,1)$	0.8786	0.0523	0.7651	0.9670	0.6736	0.2878	-0.2438	0.9525
$\theta(MN,1)$	0.5066	0.1034	0.2864	0.7122	0.6074	0.0833	0.4439	0.7616
$\theta(OH,1)$	0.9056	0.0426	0.8250	0.9877	0.9555	0.0256	0.9072	1.0040
$\theta(WI,1)$	0.8960	0.0494	0.7978	0.9891	0.4846	0.2492	-0.1740	0.8466
$\theta(IL,2)$	-0.0201	0.3351	-0.6685	0.8619	-0.0593	0.1014	-0.2564	0.1505
$\theta(IN,2)$	-0.1584	0.2216	-0.5777	0.3867	-0.2539	0.0935	-0.4277	-0.0656
$\theta(MI,2)$	-0.1615	0.2368	-0.6465	0.3014	-0.0640	0.0966	-0.2463	0.1251
$\theta(MN,2)$	-0.1480	0.1411	-0.3999	0.1296	-0.0312	0.1008	-0.2216	0.1640
$\theta(OH,2)$	-0.0496	0.2226	-0.5109	0.4549	-0.1831	0.1088	-0.3937	0.0165
$\theta(WI,2)$	-0.1078	0.2460	-0.6487	0.3966	-0.0691	0.0933	-0.2552	0.1323
$\theta(IL,3)$	-0.2438	0.0962	-0.4456	-0.0516	-0.2588	0.0985	-0.4414	-0.0645
$\theta(IN,3)$	-0.3197	0.1026	-0.4994	-0.0839	-0.3369	0.0948	-0.5287	-0.1505
$\theta(MI,3)$	-0.3873	0.0908	-0.5596	-0.2083	-0.3889	0.0916	-0.5567	-0.2213
$\theta(MN,3)$	-0.3607	0.0859	-0.5380	-0.1894	-0.3593	0.0896	-0.5359	-0.1785
$\theta(OH,3)$	-0.4028	0.0890	-0.5775	-0.2248	-0.3818	0.0925	-0.5686	-0.2156
$\theta(WI,3)$	-0.3082	0.0945	-0.5005	-0.1271	-0.3205	0.0898	-0.4998	-0.1520
$\theta(IL,4)$	-0.3707	0.0946	-0.5525	-0.1581	-0.4002	0.0915	-0.5811	-0.2283
$\theta(IN,4)$	-0.5426	0.0829	-0.7017	-0.3842	-0.5432	0.0845	-0.6986	-0.3728
$\theta(MI,4)$	-0.4211	0.0844	-0.5802	-0.2345	-0.4171	0.0936	-0.6071	-0.2378
$\theta(MN,4)$	-0.3253	0.2482	-0.6867	0.3555	-0.3599	0.0904	-0.5355	-0.1727
$\theta(OH,4)$	-0.3329	0.0971	-0.5071	-0.1385	-0.2941	0.0958	-0.4769	-0.0913
$\theta(WI,4)$	-0.4715	0.1056	-0.6460	-0.2712	-0.4502	0.0887	-0.6214	-0.2628
$\lambda[1,1]$	-0.3190	0.2568	-0.7870	0.3153	0.1557	0.3071	-0.4456	0.7041
$\lambda[1,2]$	0.1086	0.2660	-0.4618	0.7226	-0.7136	0.5247	-1.9640	0.1162
$\lambda[1,3]$	0.0618	0.3619	-0.5757	0.7606	0.0982	0.1680	-0.1916	0.4470
$\lambda[1,4]$	0.0453	0.2645	-0.5091	0.6441	0.2584	0.1641	0.0672	0.6893
$\lambda[1,5]$	0.5230	0.7558	-1.1130	1.9640	0.3545	0.3869	-0.3385	1.2520
$\lambda[1,6]$	-0.1149	0.4279	-0.9294	0.7047	-0.4406	0.2549	-1.0950	-0.0886
$\lambda[1,7]$	1.5400	2.9240	-3.7050	8.7750				
$\lambda[2,1]$	-0.2039	0.4501	-0.9994	0.6592	0.1957	0.2646	-0.2307	0.7392
$\lambda[2,2]$	-0.2256	0.3995	-0.9137	0.5774	0.2815	0.2915	-0.3612	0.8174
$\lambda[2,3]$	0.0493	0.5903	-1.1590	1.2190	-0.0780	0.1399	-0.3484	0.2027
$\lambda[2,4]$	-0.1122	0.4255	-0.9939	0.7825	-0.2421	0.1148	-0.5554	-0.0905

$\lambda[2,5]$	0.8931	0.6111	-0.0965	2.3150	-0.8715	0.4481	-2.0650	-0.2871
$\lambda[2,6]$	0.1734	0.4088	-0.6518	0.9574	0.5676	0.2826	0.1778	1.3230
$\lambda[2,7]$	1.7290	2.8470	-2.8030	8.0840				
$\lambda[3,1]$	0.0456	0.4223	-0.6769	0.9469	1.3320	0.5487	0.4101	2.6330
$\lambda[3,2]$	-0.0920	0.4629	-1.1230	0.7987	0.7676	0.4256	0.1292	1.7750
$\lambda[3,3]$	0.1558	0.4908	-0.8715	1.1240	0.5561	0.1888	0.1631	0.9509
$\lambda[3,4]$	0.0983	0.4297	-0.7197	0.8958	-0.4177	0.1983	-0.8615	-0.1232
$\lambda[3,5]$	-0.4922	0.7040	-1.8980	0.8046	-0.7491	0.4069	-1.6670	-0.0758
$\lambda[3,6]$	-0.4685	0.5193	-1.4810	0.7029	0.8968	0.3979	0.3259	1.8920
$\lambda[3,7]$	1.7450	1.9270	-2.1020	6.0300				
$\lambda[4,1]$	-0.0547	0.2463	-0.6202	0.4141	-0.2465	0.9335	-2.2310	1.7370
$\lambda[4,2]$	-0.0277	0.2319	-0.5471	0.4233	1.9980	1.0620	0.4648	4.5390
$\lambda[4,3]$	-0.0263	0.2961	-0.5788	0.6757	-2.1020	1.1510	-4.8770	-0.4390
$\lambda[4,4]$	0.0691	0.2855	-0.6426	0.5797	-1.1260	0.3124	-1.7290	-0.5364
$\lambda[4,5]$	-0.2892	0.6524	-1.5560	1.1030	-0.0701	0.7831	-1.8210	1.3760
$\lambda[4,6]$	0.0861	0.2399	-0.4080	0.5804	4.5430	2.3850	1.0550	9.7970
$\lambda[4,7]$	0.6673	1.1850	-1.2940	3.9330				
$\lambda[5,1]$	0.0460	0.3734	-0.7758	0.7326	0.1697	0.1793	-0.1212	0.5243
$\lambda[5,2]$	-0.0560	0.2209	-0.6531	0.3489	-0.3292	0.3746	-0.9840	0.5586
$\lambda[5,3]$	-0.0408	0.2519	-0.5662	0.4593	-0.0472	0.1104	-0.2230	0.1218
$\lambda[5,4]$	0.0673	0.3085	-0.4922	0.6298	0.0123	0.0484	-0.0787	0.1144
$\lambda[5,5]$	-0.0478	0.3510	-0.6293	0.6785	0.2133	0.2812	-0.3292	0.7300
$\lambda[5,6]$	0.1861	0.3935	-0.6830	1.0160	0.0190	0.0893	-0.1590	0.2217
$\lambda[5,7]$	0.6086	1.3150	-1.8250	3.6690				
$\lambda[6,1]$	0.0875	0.4556	-0.8776	0.8927	1.4660	0.8741	-0.3030	3.3770
$\lambda[6,2]$	-0.1695	0.3575	-1.0260	0.3472	2.4740	1.0920	0.9944	5.3760
$\lambda[6,3]$	-0.1782	0.4134	-1.0030	0.6117	-1.3410	0.7271	-3.0270	-0.2163
$\lambda[6,4]$	0.1310	0.5269	-1.0290	1.0960	-1.3680	0.8408	-3.9050	-0.4246
$\lambda[6,5]$	-0.6066	0.6182	-1.9070	0.5836	-1.3320	0.8110	-3.3570	-0.0511
$\lambda[6,6]$	-0.0404	0.5052	-0.8688	1.1980	3.0770	0.3699	2.4090	3.8610
$\lambda[6,7]$	1.9440	1.6360	-0.8010	5.5080				
$\lambda[7,1]$	-0.0478	0.1908	-0.3463	0.3271				
$\lambda[7,2]$	-0.0702	0.1611	-0.3665	0.2427				
$\lambda[7,3]$	-0.1283	0.1822	-0.4693	0.2474				
$\lambda[7,4]$	-0.1533	0.1519	-0.4379	0.1517				
$\lambda[7,5]$	-0.0295	0.1959	-0.4010	0.3580				
$\lambda[7,6]$	0.0265	0.1578	-0.2911	0.3355				
$\lambda[7,7]$	1.8540	0.2063	1.4650	2.2680				
$\sigma(IL,1)$	0.1553	0.9901	0.2102	0.1145	0.3962	3.0340	0.5230	0.3091
$\sigma(IN,1)$	0.5189	3.6724	0.6873	0.4060	0.1992	0.3498	0.3672	0.1190
$\sigma(MI,1)$	0.1307	0.7262	0.1916	0.0946	0.0000	0.0000	0.0367	0.0000
$\sigma(MN,1)$	0.0364	0.0024	0.5984	0.2316	0.4699	3.6166	0.6127	0.3697
$\sigma(OH,1)$	0.0621	0.4036	0.0833	0.0460	0.1995	1.5863	0.2639	0.1596
$\sigma(WI,1)$	0.0339	0.0902	0.0663	0.0184	0.0000	0.0000	0.0004	0.0000
$\sigma(IL,2)$	0.0000	0.0000	0.0002	0.0000	0.0003	0.0017	0.0004	0.0002
$\sigma(IN,2)$	0.0000	0.0000	0.0004	0.0000	0.0005	0.0036	0.0007	0.0004
$\sigma(MI,2)$	0.0000	0.0000	0.0002	0.0000	0.0003	0.0020	0.0004	0.0002
$\sigma(MN,2)$	0.0001	0.0000	0.0008	0.0000	0.0007	0.0051	0.0009	0.0005
$\sigma(OH,2)$	0.0000	0.0000	0.0002	0.0000	0.0002	0.0010	0.0003	0.0001
$\sigma(WI,2)$	0.0001	0.0000	0.0004	0.0000	0.0005	0.0040	0.0007	0.0004

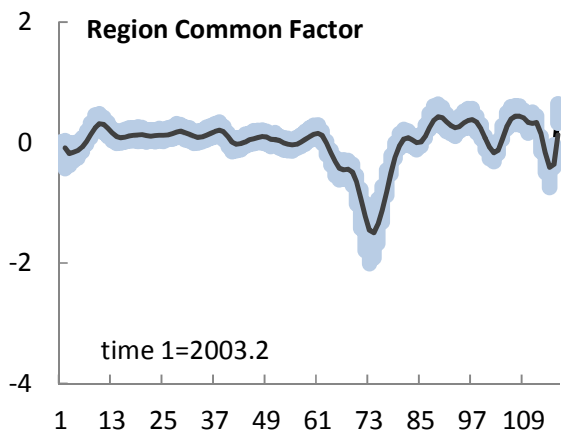
$\sigma(IL,3)$	0.0044	0.0314	0.0059	0.0034	0.0040	0.0293	0.0054	0.0031
$\sigma(IN,3)$	0.0104	0.0711	0.0140	0.0079	0.0112	0.0834	0.0150	0.0088
$\sigma(MI,3)$	0.0152	0.1117	0.0197	0.0120	0.0154	0.1116	0.0208	0.0118
$\sigma(MN,3)$	0.0105	0.0758	0.0139	0.0081	0.0107	0.0819	0.0143	0.0084
$\sigma(OH,3)$	0.0349	0.2492	0.0476	0.0268	0.0320	0.1272	0.0460	0.0209
$\sigma(WI,3)$	0.0106	0.0756	0.0140	0.0081	0.0109	0.0826	0.0146	0.0087
$\sigma(IL,4)$	4.6838	30.9502	6.2972	3.6643	4.7710	27.1444	6.3171	3.7580
$\sigma(IN,4)$	2.6709	19.1718	3.5651	2.0734	2.7064	20.5592	3.5804	2.1505
$\sigma(MI,4)$	2.1654	16.0308	2.8217	1.7039	2.2391	17.1969	2.9189	1.7590
$\sigma(MN,4)$	0.0001	0.0000	7.3099	0.0000	6.1881	44.3066	8.1766	4.7371
$\sigma(OH,4)$	1.6790	11.3895	2.2371	1.2577	1.5547	7.8247	2.2085	1.0330
$\sigma(WI,4)$	0.1736	0.0094	3.1397	1.6455	2.4691	18.3993	3.3557	1.9478

<Appendix 3>

Multi-level Dynamic Factor Model Estimation Results with 95% CI

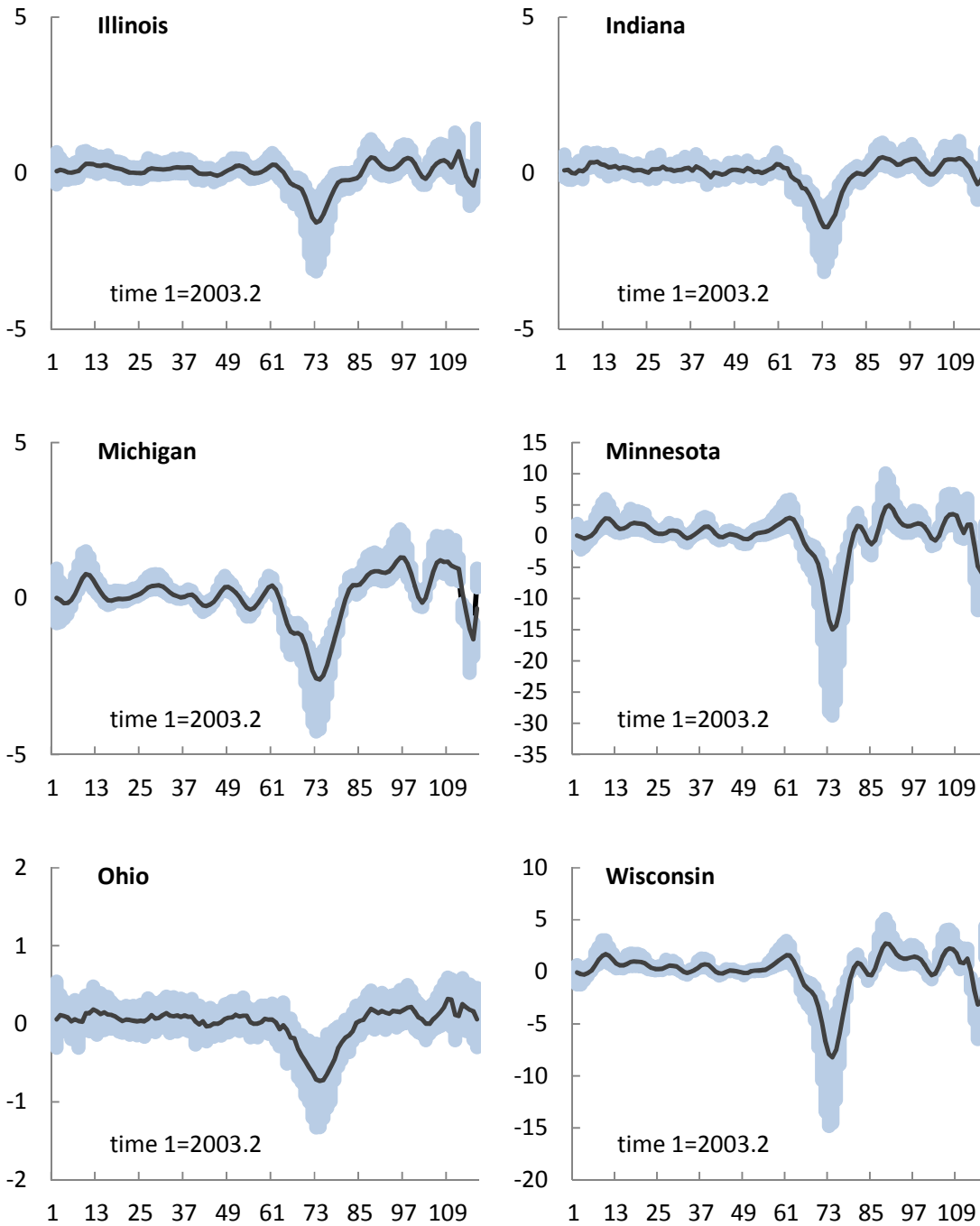






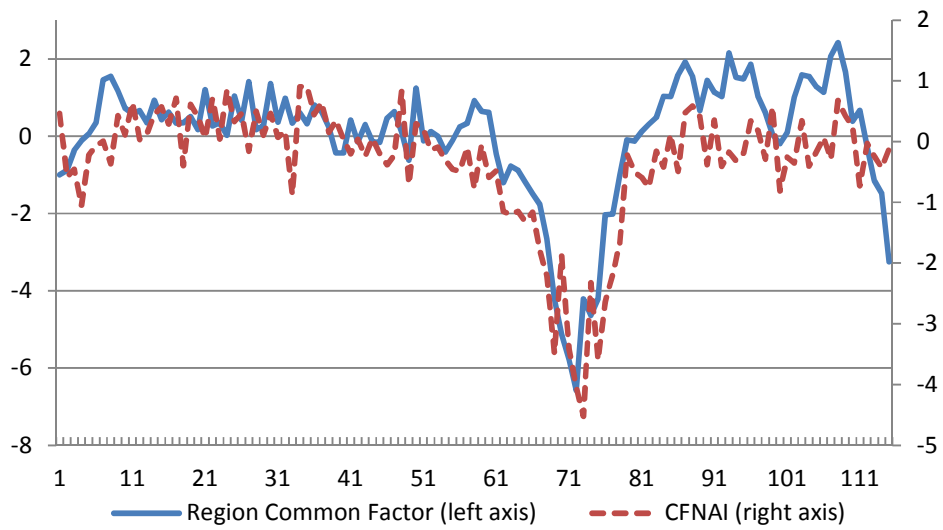
<Appendix 4>

Single-level Dynamic Factor Model Estimation Results with 95% CI



<Appendix 5>

The point estimates of Region Common Factor from Multi-level Dynamic Factor Model - Comparison with CFNAI

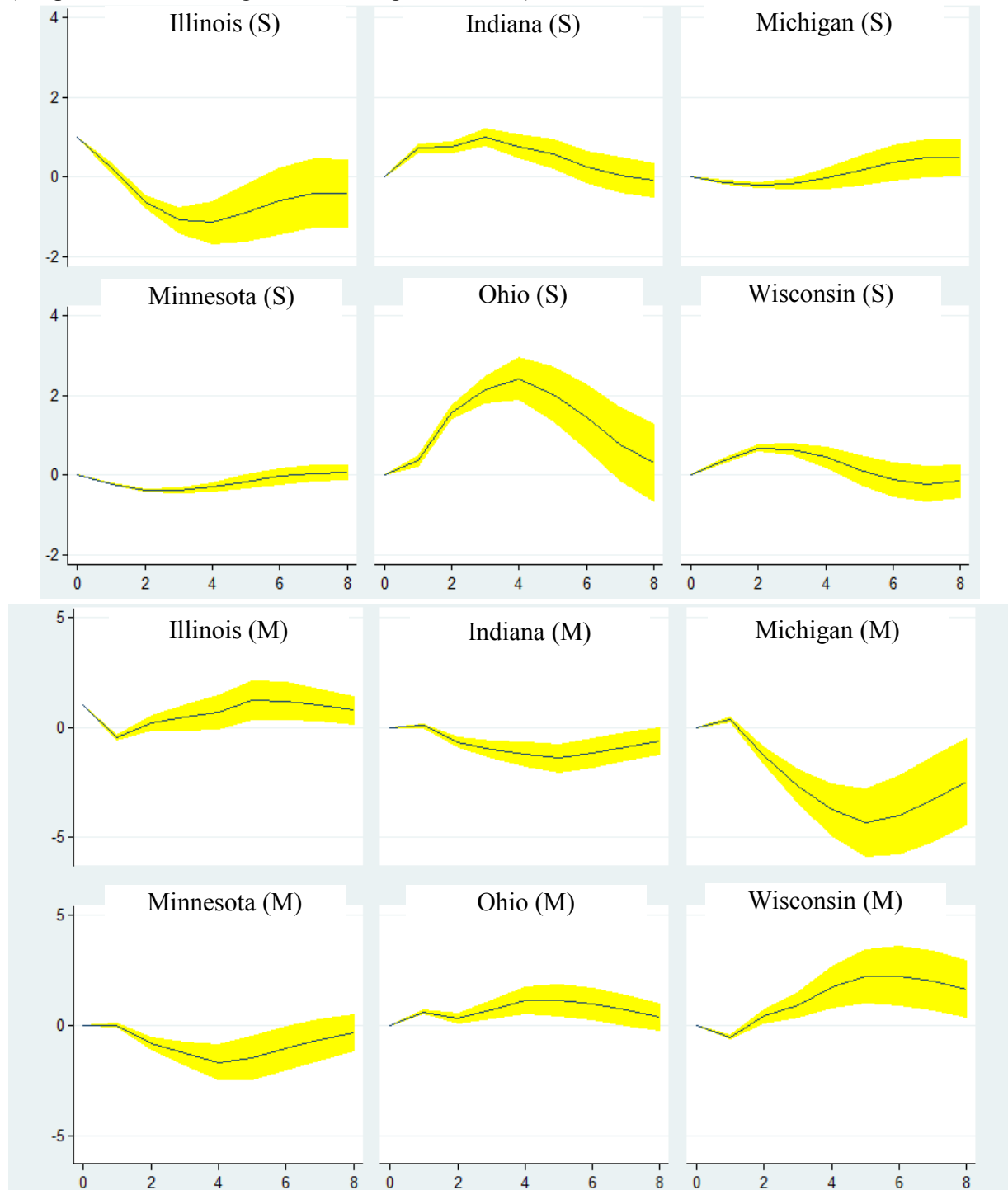


<Appendix 6>

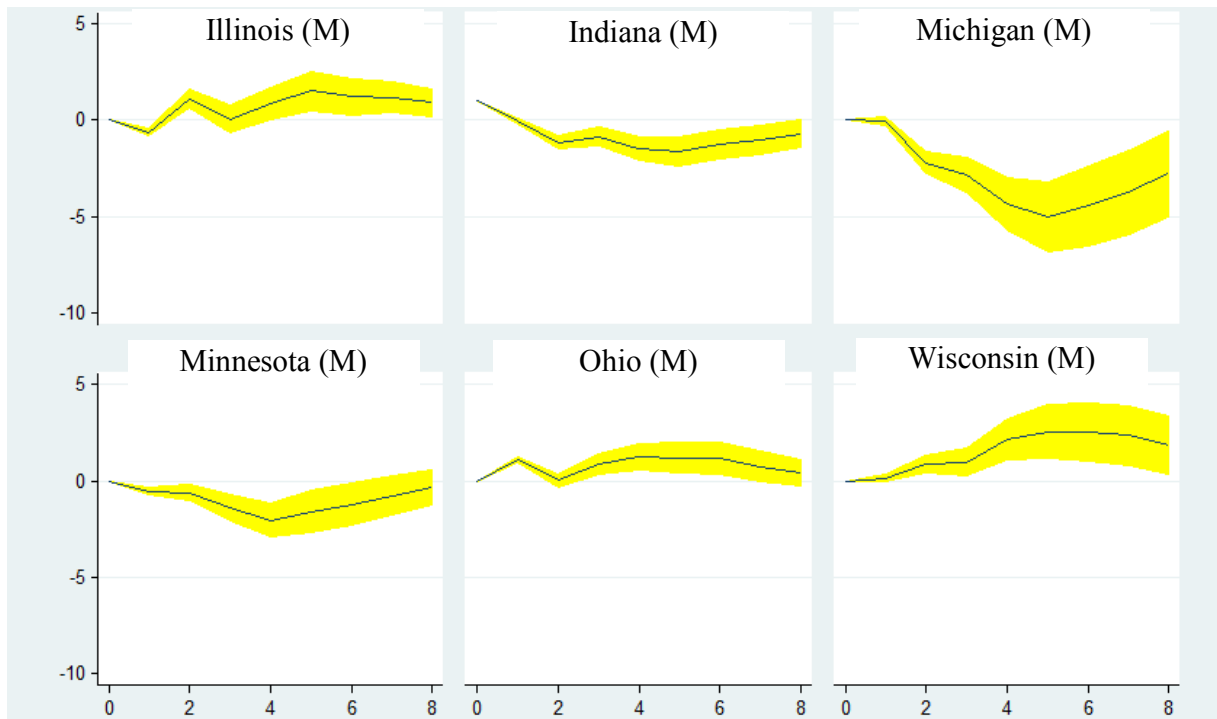
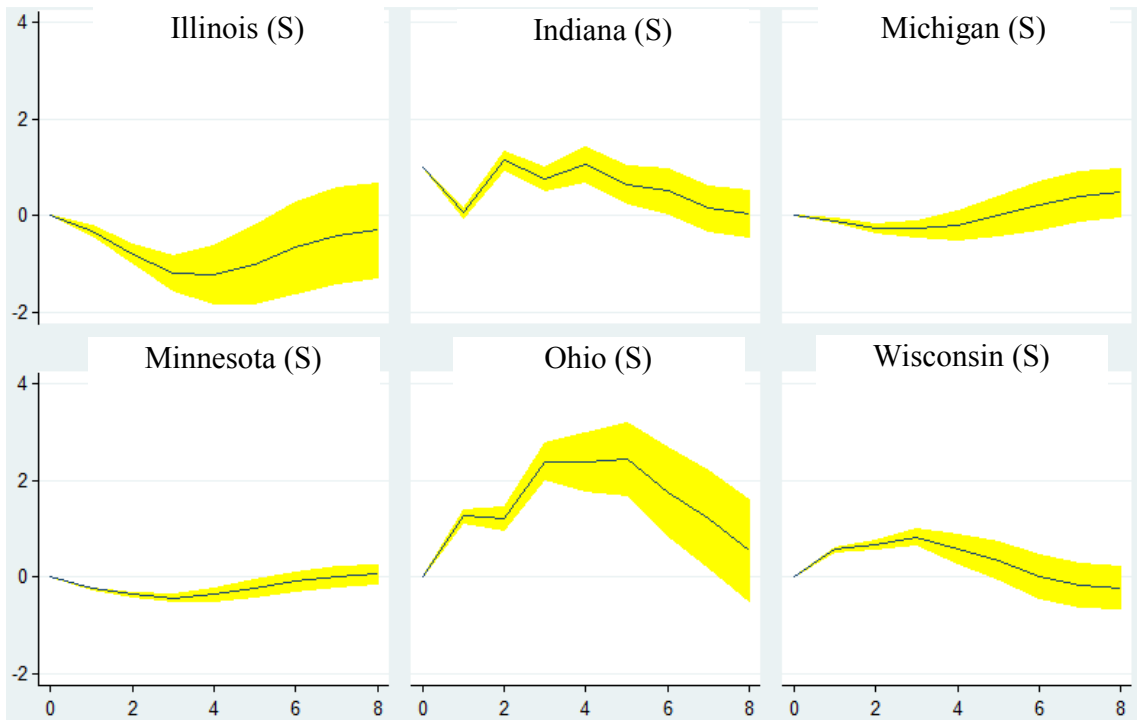
Impulse Response Functions for each Great Lake Region

(“S” denotes IRFs from single-level structure, “M” denotes IRFs from multi-level structure)

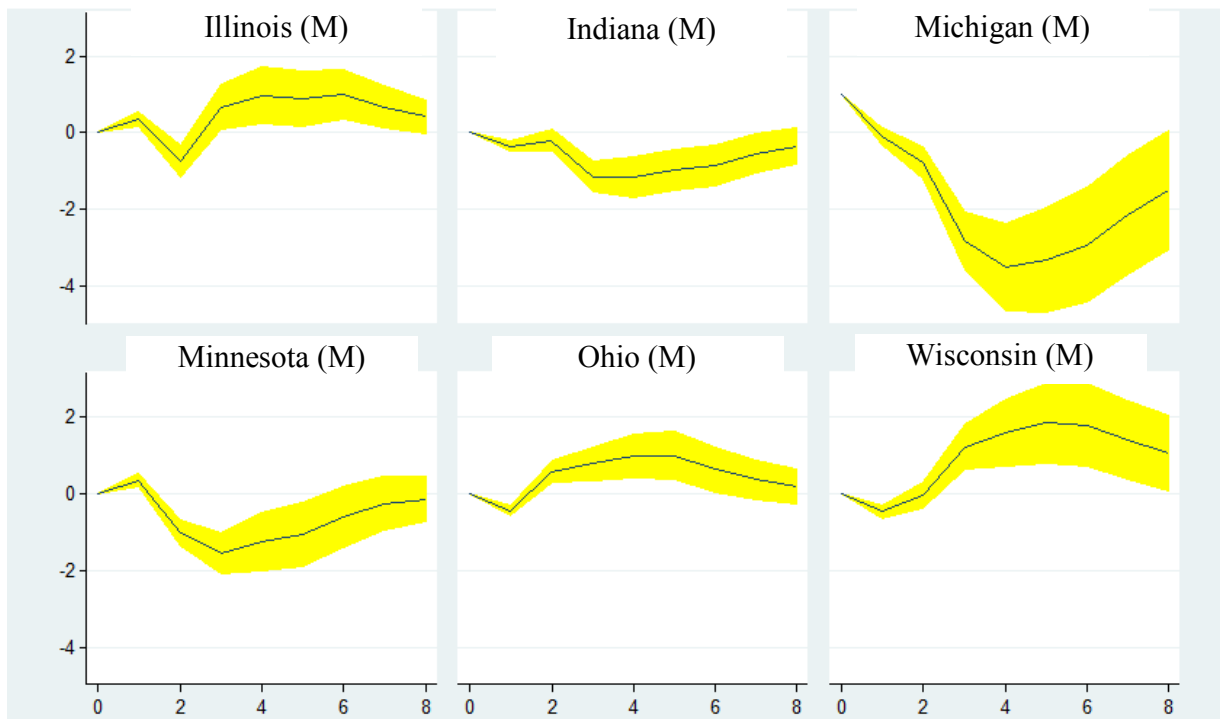
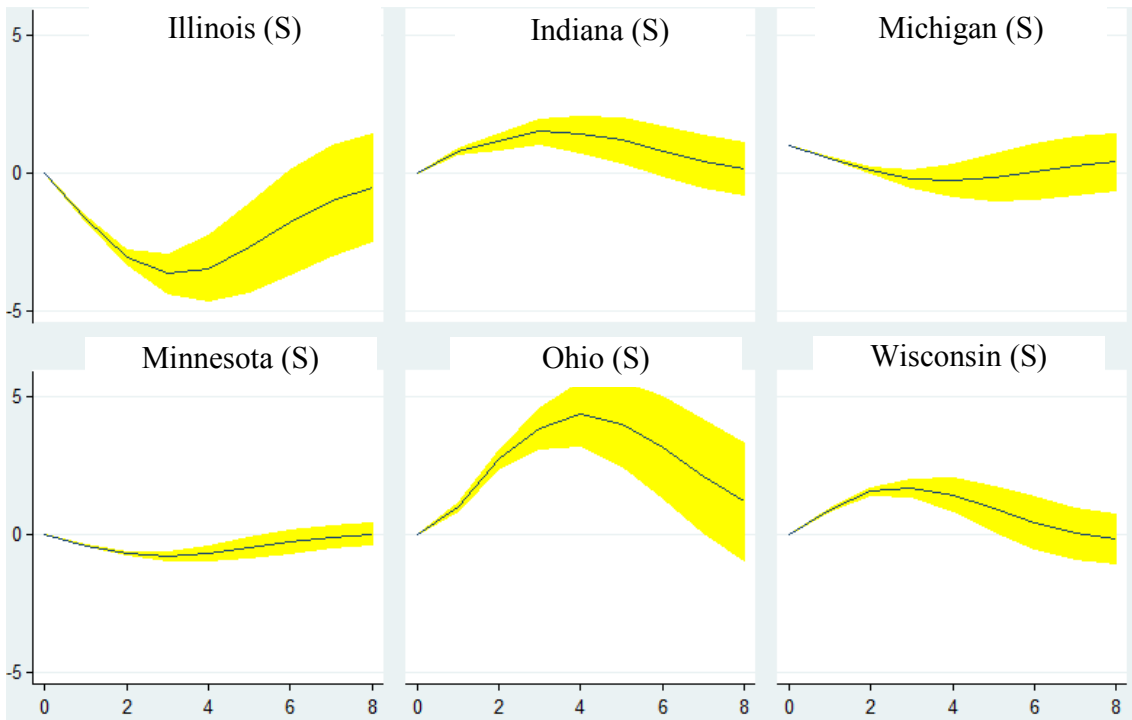
(Response of Illinois against each Regional Shock)



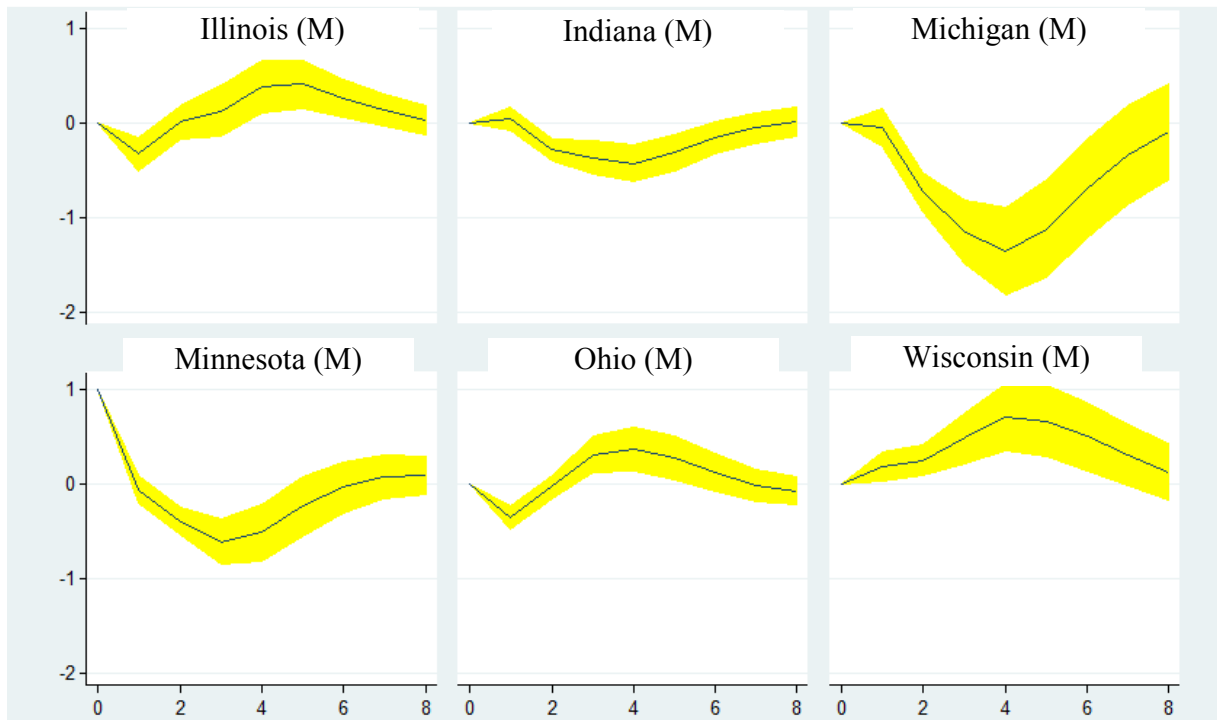
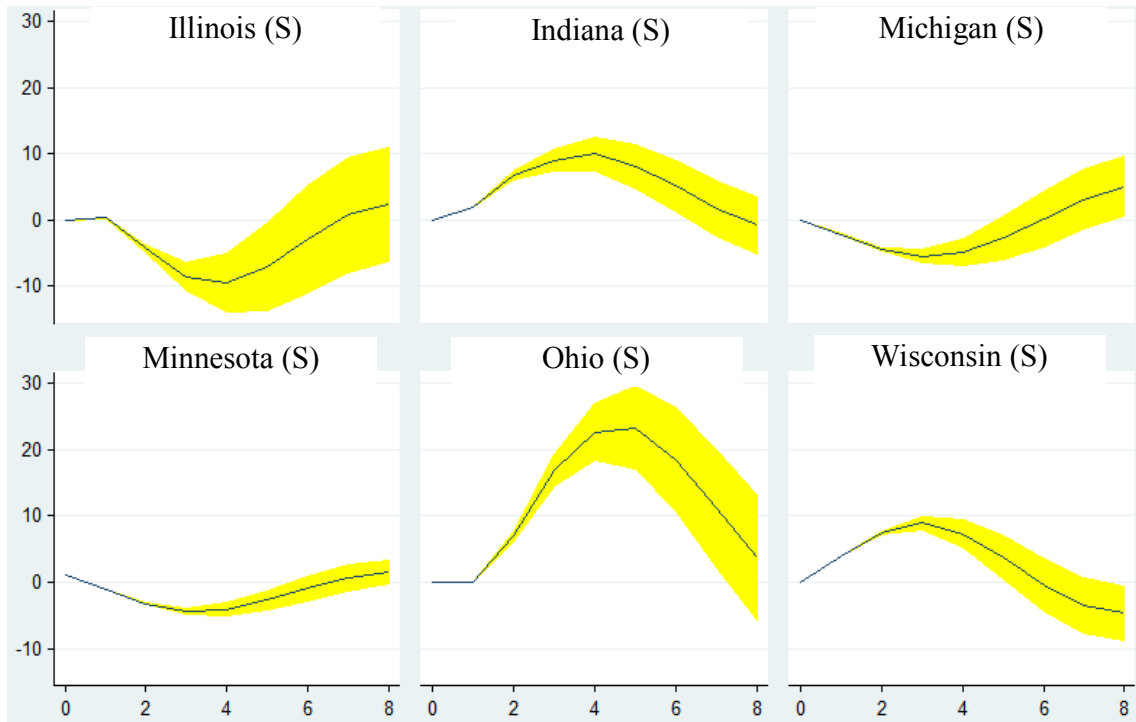
(Response of Indiana against each Regional Shock)



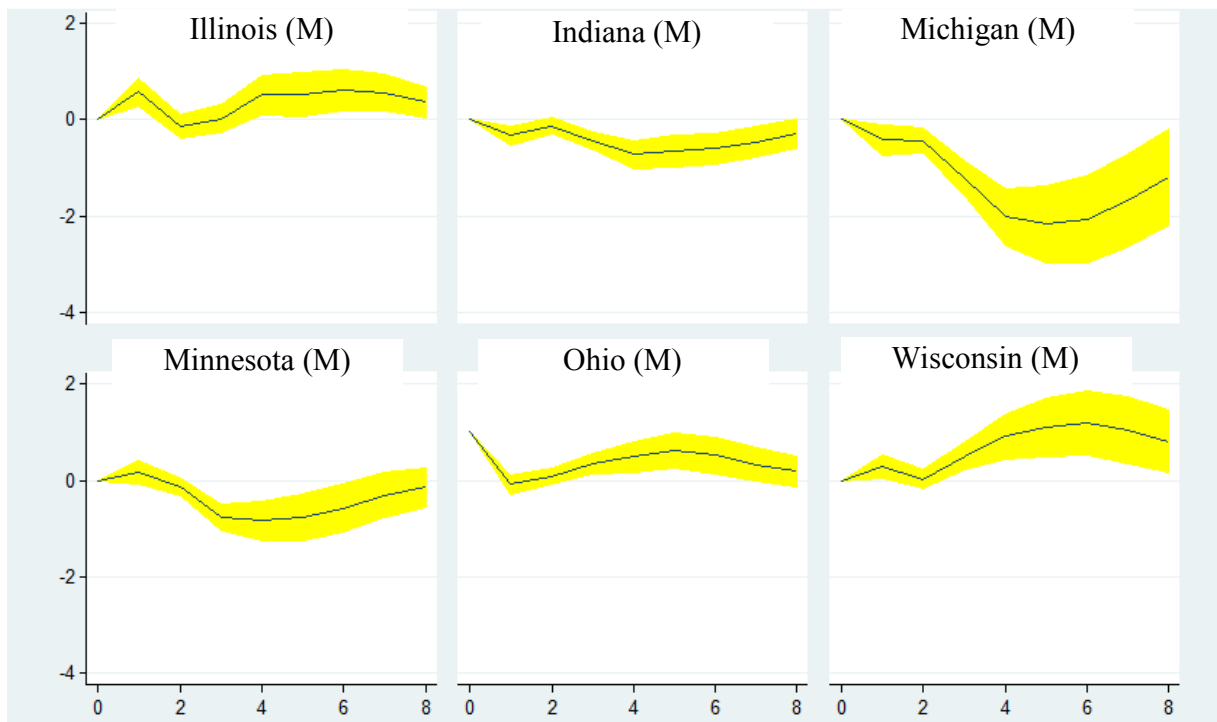
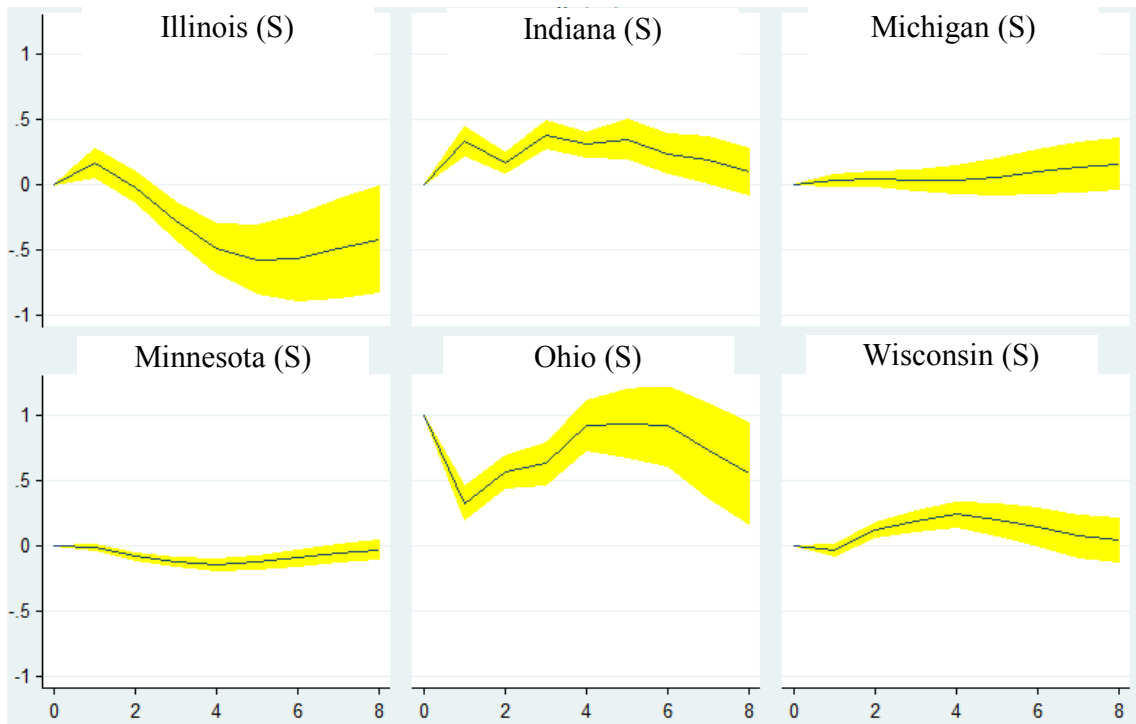
(Response of Michigan against each Regional Shock)



(Response of Minnesota against each Regional Shock)

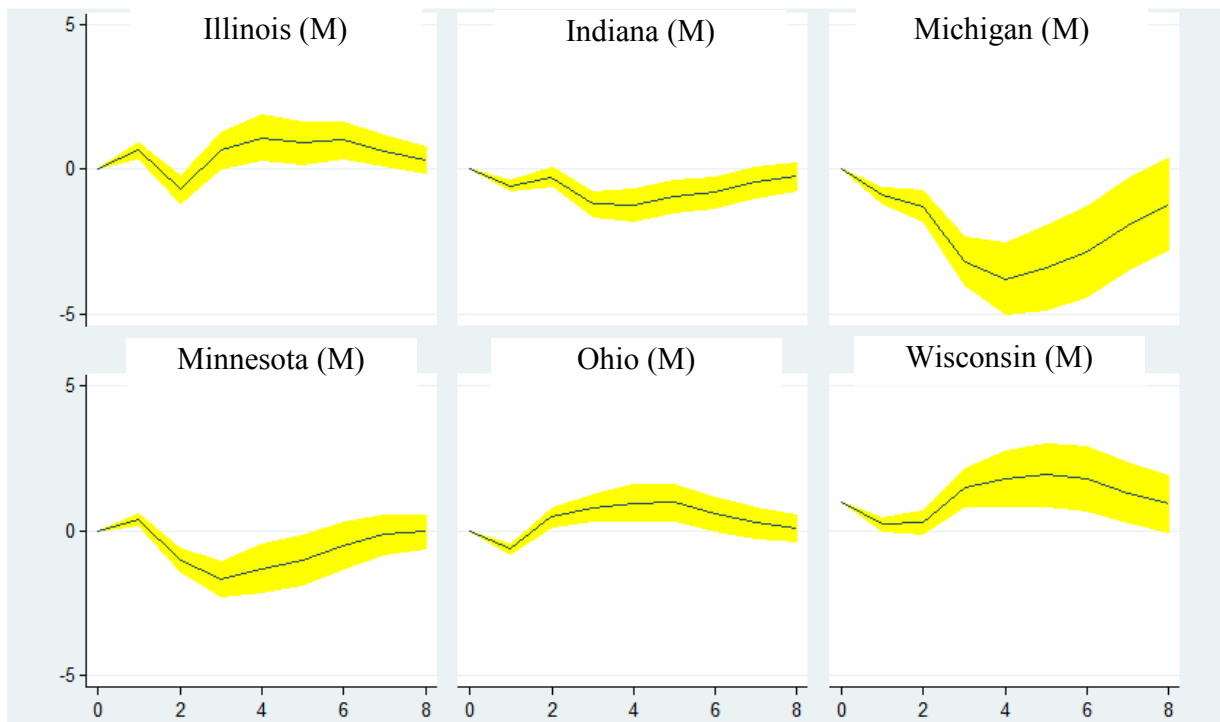
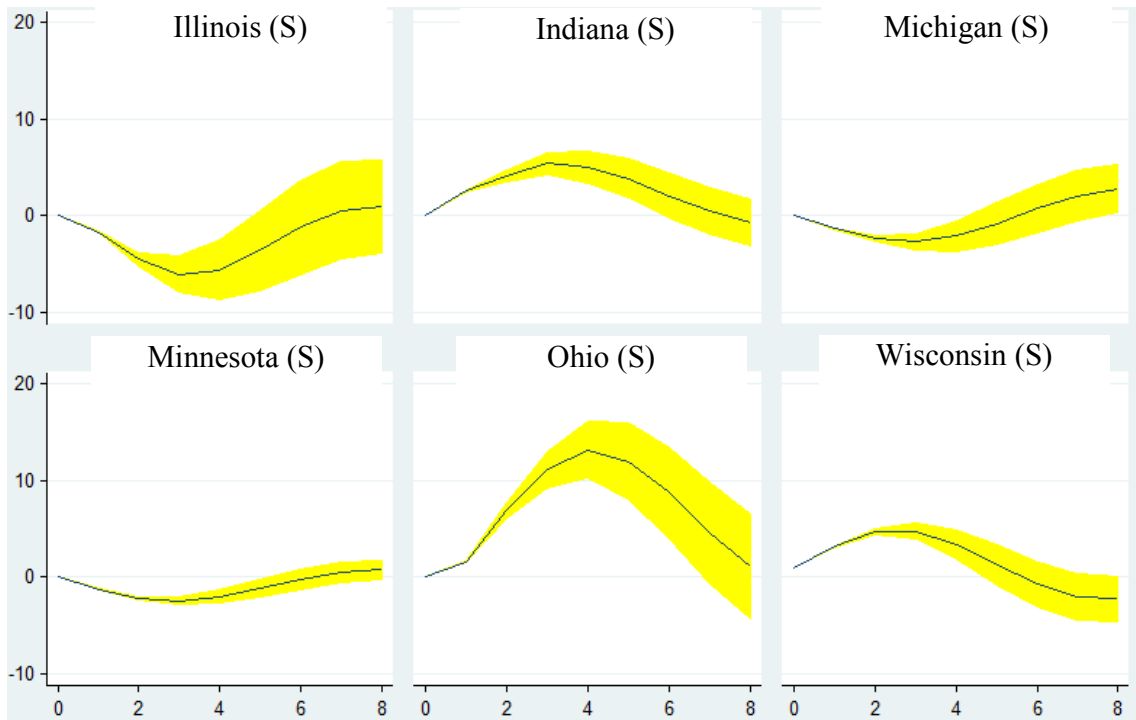


(Response of Ohio against each Regional Shock)

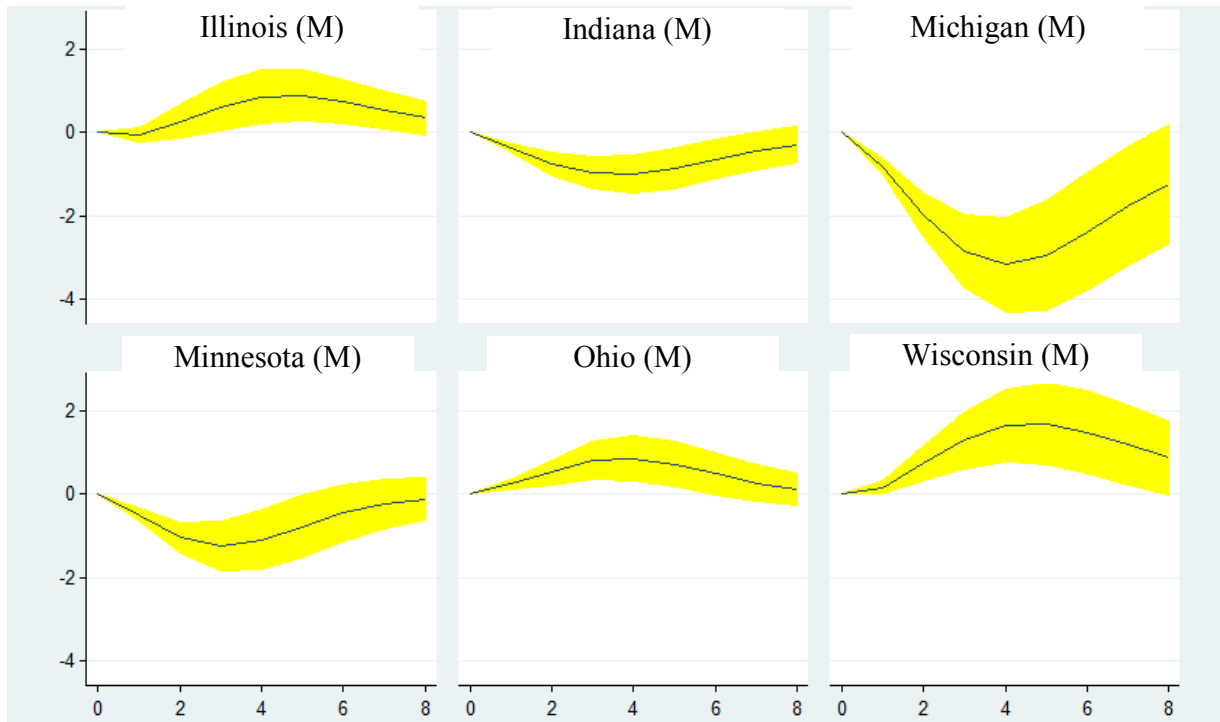




(Response of Wisconsin against each Regional Shock)



(Response of Region Common Factor against each Regional Shock)



(Response of each Regional Factor against Region Common Shock)

