REINTERPRETING THE FRISCH PARAMETER IN THE FIELD OF PERSONAL TAXATION: A LINK BETWEEN TAXABLE CAPACITY AND SOCIAL MARGINAL UTILITY IN OPTIMAL TAXATION

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Reinterpreting the Frisch parameter in the field of personal taxation: a link between taxable capacity and social marginal utility in Optimal Taxation

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Abstract: A non-existent meaning in the field of personal taxation is given to the Frisch parameter. Using an Indirect Addilog System, we formulate a relationship between Frisch parameter and the discretionary income based on the different degree of urgency or priority of needs routed in the values of households' consumption. As result, the concept of tax capacity can be interpreted in terms of utility, redefining and conciliating the approaches of taxable capacity and social welfare.

Keywords: Frisch parameter, taxable capacity, social marginal utility, Optimal taxation, Carter Report, Meade Report

JEL Classification: D63; H21; H24

1. Introduction

Measuring tax capacity has always been a challenge in personal taxation. The proposals arising from the Carter (1975) and Meade (1978) reports center around this concept that has its limitations in terms of definition and empirical implementation. In words of the Meade Report (1978, p.14): '*On examination, "taxable capacity" always turns out to be very difficult to define and to be a matter on which opinions will differ rather widely.*' Based on the aforementioned problems, over the last decades Optimal Taxation Theory (hereafter, OTT) (Mirrlees, 1971) and the so-called Dynamic OTT (Acemoglu *et al*., 2010, Golosov *et al*. 2011) have become the predominant strands in the literature of taxation. However, this theory is also lacking a fully objective nature since it is based on the use of utilitarian social welfare functions. According to Kay (2008, p.660): '*if one is to maximize a social welfare function based on an aggregation of individual circumstances, it is necessary to envisage some agreement on what the individual*

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arguments of that social welfare function (call them utilities) would be. (...) I believe it is difficult to argue that it is possible to define utilities but not to define taxable capacities'.

The present paper, by resorting to the well-known Frisch parameter (hereafter, FP) and incorporating it into the field of personal taxation, provides an objective meaning to the concept of taxable capacity that reconciles both approaches. We also establish a theoretical link between OTT and the proposals of the Carter and Meade Reports, solving at the same time Kay's (2008) criticism to both approaches.

Using IAS (Indirect Addilog System), we demonstrate that the FP provides a valuable contribution to the field of personal taxation, which to a large extent overcomes its pure cardinality connotation of marginal utility of income. Contrary to the long standing interpretation of the FP in Linear Expenditure Systems (LES), in IAS, the strict concept of a minimum consumption of subsistence (minimum fixed amount equal for everyone) can be substituted for a more flexible concept of necessary consumption that can be better routed in the values and consumption priorities of households according to their living standards and income levels. Further, it can also be estimated empirically, drawing from expenditure shares in households' budget surveys.

2. IAS and priority needs

Let us consider an IAS model in which consumption or budget shares are given by the following expression (see Somermeyer and Langhout, 1972): [3]

$$w_i = \frac{c_i (Y / p_i)^{\alpha_i}}{\sum_{k=1}^{n} c_k (Y / p_k)^{\alpha_k}} \qquad (1)$$

where w_i represents the budget shares on commodity i, p_k is the price on commodity i, Y is the total expenditure.

[3] The IAS model was independently proposed by Leser (1941) and Somermeyer and Wit (1972), in order to improve the explanatory capabilities of the main tools developed so far (Cobb-Douglas and LES functions).

The coefficients, c_k, are called "preference coefficients" and the α_k are "reaction parameters." In order to have a well behaved indirect utility function (De Boer *et al*. 2010) they must fulfill the conditions: $c_i \geq 0$ and $\alpha_i \geq -1$ for all *i*=1, …*n*, and with $\sum_{i=1}^{n} c_i = 1$.

The reaction parameters, α_k, capture the relative urgency of consumption needs; the lower the value of α_k (with the limit of -1), the more urgent is the consumption of *k*. Therefore, α_k modulates how "real income" or "purchasing power" $\left(Y / p_i \right)$ is allocated to the most urgent or priority needs in contrast to discretionary items.

The indirect utility function (*V*) associated with budget shares' (expression (1) (Houthakker, 1960, and Heij *et al*., 2004) is given by the following expression:[4]

$$V(p,Y) = \sum_{i=1}^{n} c_i \left[\frac{(Y / p_i)^{\alpha_i} - 1}{\alpha_i} \right] \qquad (2)$$

Income elasticities can be easily obtained from expenditure shares ($w_i = x_i p_i / Y$) and are given by the following expression:[5]

$$E(x_i, Y) = \frac{\partial x_i}{\partial Y} \frac{Y}{x_i} = 1 + \alpha_i - \bar{\alpha} \qquad (3)$$

$$\text{where } \bar{\alpha} = \sum_j w_j \alpha_j \qquad (4)$$

These expressions provide important insights. Necessary goods are those most urgently needed by low-income households.[6] Consequently, a commodity *i* is discretionary, $j \epsilon I_D$, or necessary,

[4] Commodities' demands (and consequently expenditure shares) are derived from the indirect utility function by applying Roy's identity: $x_i = \frac{\partial V(p,m) / \partial p_i}{\partial V(p,m) / \partial m}$.

[5] Since the expenditure elasticities are not equal to one, preferences are non-homothetic (De Boer, 2010)

[6] These goods are obtained first, and then households devoted their purchases to the discretionary spending. Therefore, their average share of expenditure, wi, are greater than their marginal share, $\omega i = \frac{\partial (pi.xi)}{\partial y}$, ie, income elasticity is less than unity.

$i \epsilon I_N$, if the income elasticity is greater or less than unity, respectively. Expression (3) classifies commodity i as necessary when $i \epsilon I_N \leftrightarrow \alpha_i < \bar{\alpha}$ and as discretionary when $j \epsilon I_D \leftrightarrow \alpha_i > \bar{\alpha}$.

3. The Frisch parameter as a measure of taxable capacity

The fundamental idea underlying the concept of taxable capacity as discretionary income is the one referring to human needs priorities. In IAS, we are able to define necessary consumption [$NC(Y)$] and discretionary income [$DY(Y)$] according to taxpayers' consumption priorities across different income levels and standards of living. The IAS approach has more explanatory power than the traditional LES one and also provides a richer cultural meaning because it is better situated in taxpayers' consumption priorities and living standards.

$$NC(Y) = \sum_{i \in I_N} x_i p_i \quad \text{where } \alpha_i < \bar{\alpha} \tag{5}$$

$$DY(Y) = \sum_{j \in I_D} x_j p_j \quad \text{where } \alpha_j < \bar{\alpha} \tag{6}$$

The concept of discretionary income is simply the counterpart of non-priority income, ($DY=Y-NC$). The marginal utility of income, λ, the Lagrange multiplier of consumers' optimizations can be obtained easily from the expression (2). According to the well-known envelop theorem, it is the instant variation rate of the optimal value with regard to the income constraint, Y:

$$\lambda = \frac{\partial V(p,Y)}{\partial Y} = \sum_{i=1}^{n} c_i p_i^{-\alpha_i} Y^{\alpha_i - 1} \tag{7}$$

The marginal utility of income ranges between $(\infty, 0)$. When the households' income is low ($y \rightarrow 0$), all consumption is devoted to the goods of higher priority and therefore the satisfaction that involves the purchase of the first unit of the good, λ, is infinite. As income increases, the marginal utility falls to zero and consumption of this good is replaced by the good of the next highest priority. This idea is closely related to the thinking of Foellmi (2005) and his notion of and hierarchy of wants. This notion of hierarchy of wants can be traced back to the early works of Menger (1871).

Therefore, FP, the elasticity of the marginal utility of income (λ) can be obtained as follows:

$$\varphi = \frac{Y}{\lambda}.\frac{\partial \lambda}{\partial Y} = \sum_{k=1}^{n} w_k (a_k - 1) = \sum_{k=1}^{n} w_k a_k - \sum_{k=1}^{n} w_k = (\bar{\alpha} - 1) \qquad (8)$$

where $\bar{\alpha} = \sum_k w_k \alpha_k$. The weighted sum of reactions parameters for average share expenditures, $\bar{\alpha}$, may be decomposed into two parts, corresponding to the necessary consumption, $\bar{\alpha}_N$, and discretionary, $\bar{\alpha}_D$, as follows:

$$\bar{\alpha}_N = \sum_{i \in I_N N} w_i . \alpha_i \qquad (9)$$

$$\bar{\alpha}_D = \sum_{j \in I_D} w_j . \alpha_j \qquad (10)$$

Therefore, FP can be defined according to the following expression:

$$\varphi = \left[\bar{\alpha}_N + \bar{\alpha}_D - 1 \right] \qquad (11)$$

The FP measures the willingness of consumers to substitute between consumption of high priority (necessaries) and non-priority (discretionary) goods. Its absolute value ranges between $\bar{\alpha}_N - 1$ and $\bar{\alpha}_D - 1$, respectively, from the bottom to the top of the income distribution.

At the lowest levels of income, there is no discretionary income ($\bar{\alpha}_D = 0$) and the purchases are focused on necessary goods. The FP reaches its maximum value close to -2.[7] If income levels rise continuously, purchases tend to expand to the non-necessary goods and consequently, decreases $\bar{\alpha}_N$ and increases $\bar{\alpha}_D$. In the limit, asymptotically, the shares of spending may be considered as concentrated in a single discretionary good with $\alpha_i = 1$, where FP reaches the value 0.[8]

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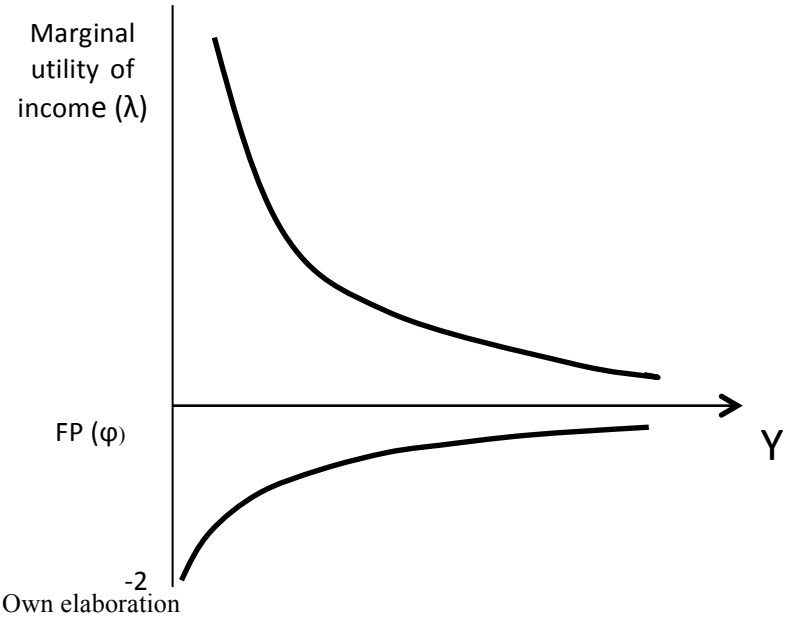
[7] At the lower level of income, purchases can be considered as concentrated in a single good with $\alpha_i = -1$. Therefore, $\varphi = \left[\bar{\alpha}_N + \bar{\alpha}_D - 1 \right] = \sum_j w_N \alpha_N + \sum_j w_D \alpha_D - 1 = 1*(-1) + 0 - 1 = -2$

[8] Bear in mind that the behavior of FP is based on a theoretical point of view and therefore we considered $\alpha_i = 1$ as the minimum value for a logical economic interpretation of these parameters.

Figure 1 describes the pattern of behavior of the FP. The FP decreases in absolute value as income rises and the discretionary coefficient of income goes up. The FP can be interpreted as an index of the degree of urgency of basic human needs along income levels, which can be empirically estimated. When the ability to pay taxes is related to the discretionary income, (equation 10), the FP is reinterpreted as an indicator of the decreasing ability to pay. In that way, new exciting research lines are open to formulate social assessment criteria based on consumers' values and expenditure priorities.



Source: Own elaboration

Figure 1. Marginal utility of income and Frisch parameter patterns

4. The Frisch parameter as a measure of social marginal utility in Optimal Taxation Theory

In the field of OTT, when using symmetric and additive social welfare functions à la Atkinson (1970), the parameter representing the social marginal utility of income for taxpayer h, β^h, becomes the individual marginal utility of income, λ, which can be estimated from consumers'

expenditure surveys by means of IAS (as we have seen in the previous section). Therefore, on the one hand, the FP can be associated with the assessment of social welfare in the OTT and, on the other hand, a link can be established between the definition of taxable capacities which lie at the heart of the Meade Report (1978) and the concept of social marginal utility of income in the OTT.

Departing from the very well-known result in Diamond and Mirrlees (1971, p. 270), at the optimum, changes in the social marginal utility (or welfare– society as a whole is the sum of individuals) are proportional to the total tax revenue after paying taxes. This variation has two components: a) evaluating the utility of such taxpayer, u^h, under a social welfare function, W, and b) the valuation of the loss of income of the taxpayer, h, according to the marginal utility of income level that corresponds to your living standard and income, λ^h:

$$\beta^h = \frac{\partial W}{\partial u^h}\lambda^h \qquad (11)$$

If, as in Diamond and Mirrlees, we consider social welfare functions, W, that (following Atkinson) are additively separable and individualistic, condition a) is equal to 1 (social welfare varies at the same rate as individual utility levels) and, therefore, the social marginal utility matches with the marginal utility of income of consumer h, λ^h.

$$\beta^h = \lambda^h \qquad (12)$$

The β^h parameter can be identified with the marginal utility of income (Lagrange multiplier λ studied in the previous section under IAS) of consumer h according to his living standard. The FP can be reinterpreted in the OTT as the elasticity of social marginal utility of income. In this way, a new line is open to link the social marginal value of income with the degree of relative urgency in the satisfaction of the basic human needs. An empirical assessment can be projected on the spending priorities of consumers (taxpayers) according to their different living standards.

5. Conclusions

This paper reinterprets the FP in the field of personal taxation drawing conclusions based on new developments in the theory of computable general equilibrium. First, the FP is reinterpreted as

an index of taxable capacity that is directly related with the share of discretionary income in households' budgets (the coefficient of discretionary income, CDY). Secondly, the FP can be associated with the assessment of the social marginal utility of income in the OTT, using symmetric and additive social welfare functions *à la* Atkinson (1970). Thirdly, the FP opens a connection between the definition of taxable capacities and the concept of social marginal utility of income. Fourthly, the FP can be estimated empirically in the field of personal taxation by using Household Budget Surveys and therefore taxable capacities and social marginal utility of income can be computed. This also solves, to a large extent, Kays' (2008) criticism of both concepts of taxable capacity and social welfare functions.

A very promising research line based on these theoretical results would be to estimate the FP using micro-data from Households Budget Surveys and then to explore its implications in the field of personal taxation. Moreover, this empirical approach would solve Kay's (2008) criticism as their results would be based on taxpayers' consumption patterns and expenditure priorities.

Therefore, on the one hand, the role of the government in designing tax methods would be simplified and, on the other, the tax burden could be defined according to the households' consumption priority needs. Once again, the coefficient of discretionary income becomes a key element to measure households' welfare and their taxable capacities (Carter Report, 1975). An important advantage of our results and their potential empirical implementation (using Households Budget Surveys) is that a precise meaning in terms of taxpayers' utility or welfare can be given to the concept of taxable capacity defined in the Carter Report (1975). Finally, another important outcome from this analysis is that we are able to offer a response to Kay's (2008) criticism that centered on the subjective nature of the concepts of taxable capacity (Carter Report, 1975) and to utility or welfare (OTT).

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