# INTER-REGIONAL ENDOGENOUS GROWTH UNDER THE IMPACTS OF DEMOGRAPHIC CHANGES

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ABSTRACT: This paper attempts to project the economic paths for the individual Midwest states (Illinois, Indiana, Michigan, Ohio and Wisconsin, as well as the Rest of the US) in the near future when the population ageing becomes more pronounced. To accomplish this task, a dynamic general equilibrium model is developed so that it could incorporate the inter-regional transactions and endogenous growth mechanisms within the framework of an overlapping generations (OLG) model. Key parameter values associated with the regional interconnections were assigned by using multi-regional Social Accounting Matrix (SAM) of the Midwest states. Two different steady-state results were presented with two different age-cohort population structures corresponding to year 2007 and 2030. These steady-state results imply that the rate of declining of per-capita output are projected to be heterogeneous across the regions due to the different developments of age-cohort population structures and consequently different levels of endogenously determined educational investment of workers. Also two steady-state simulation results revealed that the development of output price in a certain region reflects the dynamics of demographics of every region. Meanwhile, the dynamic simulation results reveal that the per-capita output of every region is projected to grow positively in the near future when the population ageing will be pronouncing. However, the growth rate of the per-capita output is projected to be heterogeneous across the regions: the regions with high-skilled workers hold the potential threat that population ageing could give more negative impacts on the economy due to the relatively sluggish growth of human capital stock. Also, the dynamic simulation results show that certain regions in Midwest will experience their terms-of-trade deteriorate in the near future, implying that careful attention should be given to their future trade conditions.

KEY WORDS: Human Capital; Overlapping Generations; Inter-Regional Transaction; Demographic Transition; Social Accounting Matrix (SAM); Terms of Trade

## I Introduction

This paper attempts to project the economic paths for the individual Midwest states (Illinois, Indiana, Michigan, Ohio and Wisconsin, as well as the Rest of the US) in the future when the population ageing becomes more pronounced. To accomplish this task, a dynamic general equilibrium model is developed so that it could incorporate the inter-regional transactions and endogenous growth mechanisms within the framework of an overlapping generations (OLG) model.

There has been expanding literature that has adopted OLG models to explore the issues of demographic change. In particular, the papers that used the OLG model and the endogenous growth mechanism showed that the negative impact of population ageing could be mitigated through the revelation of educational motive on the part of workers since educational investment in developing workers' human capital could improve the overall productivity in the corresponding economy and thus significantly attenuate the shortage of labor force. The literature includes the work of Sadahiro and Shimasawa (2002) and Ludwig *et al.* (2007). Although those two papers accepted different human capital technology under different scenarios of age-population projection, they found out that the individual's educational motive substantially adjusts the effect of population ageing. However, their papers did not pay attention to the interconnections between the regions, assuming implicitly that all transactions including exchanging intermediate inputs and consumption and investment goods are done in single economy. However, when multiple economies are interconnected with each other, then the different scale of demographic changes in one region should bring about the different flows of transactions between the regions.  It is important to develop a dynamic model that could recognize the interconnections between the regions.

Fougere *et al.* (2004) showed the effect of population ageing in Canada under some alternative scenarios.  In this context, they presented different demographic scenarios depending on the different number of immigrants and immigration destinations. The main contribution of this paper is to introduce an inter-regional OLG framework to capture the interactions of six regions of Canada.  For this, they assumed that the regional goods are imperfect substitutes each other; and each region's purchase of consumption and investment goods from the six regions are ruled by a constant elasticity substitution (CES) function.  They assumed no transactions of intermediate goods between the regions; and the productivity of each age-cohort was exogenously given.

A multi-regional social accounting matrix (SAM) records all the transactions between the regions in a certain fiscal year.  This valuable source could be useful to calibrating the parameters in the inter-regional model, especially related to the regional demand function for the goods produced in other regions.  There are not many papers that use a SAM in the process of calibrating a dynamic general equilibrium model.  Among them, Kehoe *et al.* (1995) and Kehoe (1996) could be regarded as a starting point for using SAM in general equilibrium models.  Kehoe *et al.* (1995) and Kehoe (1996) attempted to use social accounting matrices (SAMs) in muti-sectoral general equilibrium models.  Those papers used the transaction data of Spanish (national-level) SAM to calibrate the parameter in consumption, investment and production function.  Adopting the disaggregated model specification, where 12 production sectors, 9 consumption goods and 3 factors of production, those papers simulated the Spanish economy and presented the impacts of Spain's integration into the European Community.

This paper is organized as follows. In section II, the model description will be presented. In section III, the calibration procedure will be described, focusing on the regional production, consumption and investment demand function, by using an aggregated six-regional Midwest SAM for year 2007 that was compiled by the Regional Economics Applications Laboratory (REAL) at the University of Illinois at the Urbana-Champaign (UIUC). In section IV, computational results including steady-state results and dynamic results will be presented. In the final section, conclusions will be drawn and suggestions for further research will be briefly discussed.

## II Model description

The model represents the US economy through the specification of 6 regions- Illinois (IL=1), Indiana (IN=2), Michigan (MI=3), Ohio (OH=4), Wisconsin (WI=5) and the rest of US (ROUS=6). The economy is closed to the rest of the world; thus, there are no foreign imports or exports in the model. There are two types of economic agents in each region: a representative firm and households. Each year, there are 65 overlapped generations in the household sector. Also there is a federal government to operate a social security system in each region. The economy produces physical goods as well as human capital. Physical goods are tradable across regions; and the firm can purchase intermediate goods from each region. Also, consumers and investors purchase goods from all the regions for consumption and investment purposes respectively.

## 1 Production

Regional production functions specify intermediate input requirements of both factors, such as labor and capital, and other regional goods. Following Kehoe (1996), a requirement of fixed input per unit production is assumed for a composite input of regional goods as well as for value added. Factor input requirements are represented by Cobb-Douglas value-added functions. Defining $Y_{j,t}$ as the gross output in region $j$ at period $t$, the general form of production technology is as follows:

$$Y_{j,t} = \min(x_{1j,t}/z_{1j},......,x_{6j,t}/z_{6j},VA_{j,t}/z_{VA,j}) \tag{1}$$

where $z_{ij}$ is the amount of intermediate good produced in region $i$, required to produce one unit in region $j$, $z_{VA,j}$ is the fixed value-added requirement per unit production in the region $j$, $x_{ij}$ is the intermediate input of regional good produced in region $i$; and $VA_{j,t}$ is the value added. Value added technology is assumed to take a Cobb-Douglas form:

$$VA_{j,t} = A_j {K^d_{j,t}}^{\alpha_j} {L^d_{j,t}}^{1-\alpha_j} \tag{2}$$

while $A_j$ is a parameter of total factor productivity (TFP), $K_j^d$ is a demand for physical capital stock, $L_j^d$ is a demand for effective labor for the value added and $\alpha_j$ is a parameter of capital income share. We assume that labor is immobile. Since producers minimize cost, they never waste their inputs. Thus,

$Y_{j,t} = x_{1j,t}/z_{1j} = x_{2j,t}/z_{2j} = .... = A_j {K^d_{j,t}}^{\alpha_j} {L^d_{j,t}}^{1-\alpha_j} / z_{VA,j}$ holds.

The firm's optimization problem is to solve:

$$Max\ \pi_{j,t} = p_{j,t}Y_{j,t} - w_{j,t}L^d_{j,t} - rr_{j,t}K^d_{j,t} - \sum_{i=1}^{6} p_{i,t}x_{ij,t} \tag{3}$$

where $rr$ is a rental return of physical capital belonging to investors, $w$ is a wage rate for one effective labor unit. This problem leads to the following first order conditions:

$$rr_{j,t} = \frac{\alpha_j A_j}{z_{VA,j}} \left( \frac{K^d_{j,t}}{L^d_{j,t}} \right)^{\alpha_j - 1} \left( p_{j,t} - \sum_{i=1}^{6} p_{i,t} z_{i,j} \right) \tag{4}$$

$$w_{j,t} = \frac{(1-\alpha_j) A_j}{z_{VA,j}} \left( \frac{K^d_{j,t}}{L^d_{j,t}} \right)^{\alpha_j} \left( p_{j,t} - \sum_{i=1}^{6} p_{i,t} z_{i,j} \right) \tag{5}$$

where $p$ is output price. These conditions reveal that the marginal product of capital and labor should be depreciated by the cost of buying intermediate goods which complement the labor or capital input. That is, the rental return and wage rate is positively correlated with the terms of trade. If the output price in region $j$ becomes relatively higher than the other regions, then this relative increase should be reflected by factor prices. Also, conditions (4) and (5) imply that firms earn zero profit in every region at every period since the market is assumed to be perfectly competitive.

**2 Consumption**

In each region at every period, households are represented by 65 overlapping generations. Each individual is assumed to live 65 periods: each individual is born and enters the labor market at age 1 (real age 20), works until age 45; and lives until age 65 (real age 84).

A household's inter-temporal optimization problem consists of choosing a sequence of consumption and educational investment share over the life-time in order to maximize life-time utility subject to life-time wealth. The following formulation is a current period preference of a representative household of generation $g$ in region $j$ at period $t$:

$$u(c_{j,g,t}, e_{j,g,t}) = \frac{c_{j,g,t}^{\ 1-\gamma} + \theta\, e_{j,g,t}^{\ 1-\gamma}}{1-\gamma} \quad \gamma > 1,\ 0 < \theta < 1 \tag{6}$$

where $c$ denotes a consumption bundle, which is composed of the final goods produced in each region; and $e$ is a educational investment share of individual's time endowment (=1) with $\gamma$ determining the inter-temporal elasticity of substitution and $\theta$ being a parameter representing the degree of educational investment motive. Thus, life-time utility of a representative individual born at time $t$ in the region $j$ is as follows:

$$U_t = \sum_{g=1}^{45} \beta^{g-1} u(c_{j,g,t+g-1}, e_{j,g,t+g-1}) = \sum_{g=1}^{45} \beta^{g-1} \left( \frac{c_{j,g,t+g-1}^{1-\gamma} + \theta\, e_{j,g,t+g-1}^{1-\gamma}}{1-\gamma} \right) \tag{7}$$

where $\beta$ denotes the subjective discount factor.

The individual who was born in the region $j$ at time $t$ has a following life-time budget constraint:

$$\sum_{g=1}^{65} \left( \left( \prod_{k=t}^{t+g-2} \frac{1}{1+r_{j,k}} \right) p^{c}_{j,g,t+g-1} c_{j,g,t+g-1} \right) = \sum_{g=1}^{45} \left( \left( \prod_{k=t}^{t+g-2} \frac{1}{1+r_{j,k}} \right) (1-\tau^{p}_{j,t+g-1}) h_{j,g,t+g-1} w_{j,t+g-1} (1-e_{j,g,t+g-1}) \right)$$

$$+ \sum_{g=46}^{65} \left( \left( \prod_{k=t}^{t+g-2} \frac{1}{1+r_{j,k}} \right) pen_{j,g,t+g-1} \right) \tag{8}$$

where $r_{j,t}$ denotes the rate of return on capital stock in the region $j$ at time $t$, $p^{c}_{j,g,t}$ is consumption price index, $\tau^{p}_{j,t}$ is a pension tax on earnings in the region $j$ at time $t$ and $h_{j,g,t}$ denotes human capital stock of age-cohort $g$ in the region $j$ at the time $t$. This budget constraint means that present value of life-time consumption (i.e., left-hand side of the equation 8) should be exactly the same as the present value of life-time wealth composed of labor income and pension benefits (i.e., right-hand side of 8). It is assumed that there is no unexpected death until the age 65; thus there should not be unintended bequests. Note that the current period budget constraint is as follows:

$$
\begin{aligned}
&a_{j,g+1,t+1} = (1+r_{j,t})a_{j,g,t} + (1-\tau^{p}_{j,t})w_{j,t}h_{j,g,t}(1-e_{j,g,t}) - p^{c}_{j,g,t}c_{j,g,t} \quad (\text{if } g \le 45), \\
&a_{j,g+1,t+1} = (1+r_{j,t})a_{j,g,t} + pen_{j,g,t} - p^{c}_{j,g,t}c_{j,g,t} \quad (\text{if } g \ge 46).
\end{aligned}
\tag{9}
$$

Now, the inter-temporal Euler equations are computed from the household's optimization problem (7) subject to (8) as follows:

$$
c_{j,g+1,t+1} = \left[\frac{\beta_j(1+r_{j,t+1})p^{c}_{j,g,t}}{p^{c}_{j,g+1,t+1}}\right]^{1/\gamma} c_{j,g,t} \tag{10}
$$

$$
e_{j,g,t} = \left(\frac{\theta\, p^{c}_{j,g,t}}{(1-\tau^{p}_{j,t})w_t h_{j,g,t}}\right)^{1/\gamma} c_{j,g,t} \quad (\text{if } g \le 45) \tag{11}
$$

Then, aggregate consumption demand in the region $j$ at the period $t$ could be characterized as:

$$
C_{j,t} = \sum_{g} N_{j,g,t}c_{j,g,t} \tag{12}
$$

where $N_{j,g,t}$ is the number of population belonging to the age-cohort $g$ in the region $j$ at time $t$.

In the next optimization step, Armington's (1969) strategy is applied to allocate the household's consumption expenditure across each region's produced goods. Consumers are assumed to consider each region's goods as imperfect substitutes. Given this assumption, a CES type sub-utility function of households can be developed:

$$
c_{j,g,t} = \left[\sum_{i} \upsilon^{c\;\,1-\sigma^{c}_{j}}_{i,j}(d^{c}_{i,j,g,t})^{\sigma^{c}_{j}}\right]^{\frac{1}{\sigma^{c}_{j}}} \tag{13}
$$

where $d^{c}_{i,j,g,t}$ denotes the demand for the final good produced at the region $i$ by individuals of age-cohort $g$ living at the region $j$; and $\upsilon^{c}_{i,j}$ is a preference parameter determining a consumption distribution across regional goods; and $\sigma^{c}_{j}$ determines the elasticity of substitution across regional goods at the region $j$. Then, by the first order condition of household's optimization problem, the demand for region $i$ product by the region $j$ consumers is specified as follows:

$$d^{c}_{i,j,g,t} = \upsilon^{c}_{i,j} \left( \frac{p^{c}_{j,t}}{p_{i,t}} \right)^{\frac{1}{1-\sigma^{c}_{j}}} c_{j,g,t} \tag{14}$$

where $p_{i,t}$ is the output price of goods produced at the region $i$ at the time $t$. The consumption price index ( $p^{c}_{j,t}$ ) can be computed as a non-linear weighted average of each region's output price:

$$p^{c\,\frac{-\sigma^{c}_{j}}{1-\sigma^{c}_{j}}}_{j,t} = \sum_{i} \upsilon^{c}_{i,j} p_{i,t}^{\frac{-\sigma^{c}_{j}}{1-\sigma^{c}_{j}}} \tag{15}$$

**3 Investment**

After consumption and social security payments, the rest of an individual's disposable income is saved in the form of investment in physical capital for the next period. The aggregate supply of physical capital at the region $j$ can be defined as follows:

$$K^{s}_{j,t} = \frac{\sum_{g} N_{j,g,t} a_{j,g,t}}{p^{I}_{j,t}} \tag{16}$$

where $p^{I}$ denotes the unit price of the investment good and $K^{s}$ is a aggregate supply of physical capital.

The law of motion for the physical capital stock is as follows:

$$K^{s}_{j,t+1} = Inv_{j,t} + (1-\delta^{k})K^{s}_{j,t} \tag{17}$$

where $\delta^{k}$ denotes the depreciation rate of physical capital; and $Inv_{j,t}$ represents the aggregate investment bundle in the region $j$. Investment activity is supposed to be inter-regional, implying that the investment bundle in the region $j$ ($Inv_{j,t}$), which was purchased in region $j$ for the investment purpose, is composed of each region's produced good. The investment bundle ($Inv_{j,t}$) is formed as a CES function that combines the goods from the six different regions as follows:

$$Inv_{j,t} = \left[ \sum_{i} {\upsilon^{I}_{i,j}}^{1-\sigma^{I}_{j}} (d^{I}_{i,j,t})^{\sigma^{I}_{j}} \right]^{\frac{1}{\sigma^{I}_{j}}} \tag{18}$$

where $d^{I}_{i,j,t}$ is the quantity of goods produced in region $i$, that is demanded by the investor[1] of region $j$; $\upsilon^{I}_{i,j}$ is the preference parameter determining a regional distribution of investment goods and $\sigma^{I}_{j}$ determines the elasticity of substitution across the regional goods. Then, an investor chooses the optimal portfolio of regional goods according to the following equation:

$$d^{I}_{i,j,t} = \upsilon^{I}_{i,j} \left( \frac{p^{I}_{j,t}}{p_{i,t}} \right)^{\frac{1}{1-\sigma^{I}_{j}}} Inv_{j,t} \tag{19}$$

Now, $p^{I}_{j,t}$ can be computed as a non-linear weighted average of each region's output price:

[1] There is no investor in this model explicitly. However, for the purpose of interpretation of model specification, an investor could be understood as a group of individuals in each region; and an investor is supposed to decide the composition of portfolio of the aggregate investment in his/her region.

$$p_{j,t}^{I\,\frac{-\sigma_j^I}{1-\sigma_j^I}} = \sum_i \upsilon_{i,j}^I p_{i,t}^{\frac{-\sigma_j^I}{1-\sigma_j^I}} \tag{20}$$

The rate of return from investment in physical capital should be composed of rental return and a capital gain as follows:

$$1+R_{j,t} = \frac{rr_{j,t} + (1-\delta_j^k) p_{j,t}^I}{p_{j,t-1}^I} \tag{21}$$

where $R_{j,t}$ denotes the (net) rate of return from the investment in physical capital. There are no financial assets in this economy, so this rate of return will serve as a bench mark interest rate; thus $1+r_{j,t} = 1+R_{j,t}$ for all $j$.

## 4 Social security

The federal government operates a pay-as-you-go (PAYG) style pension system. Under the PAYG system, the government levies a social security tax ( $\tau^p$ ) on labor income and transfers the pension benefit to the retirees. There is neither public debt nor other forms of taxation from the governments. The pension benefit ( $pen$ ) is assumed to be a fraction of average life-time labor income and this fraction rate ( $\xi$ ) is identical across the regions. The pension benefit is fixed according to the following:

$$pen_{j,G\geq 46,t} = \xi \sum_{g=1}^{45} (1-e_{j,g,t-(G-g)}) w_{j,t-(G-g)} h_{j,g,t-(G-g)} \tag{22}$$

The social security tax is endogenously determined so that the federal government's pension system is assumed to be balanced every period as follows:

$$\tau_t^p \left( \sum_j \sum_{g=1}^{45} N_{j,g,t} \left( (1-e_{j,g,t}) w_{j,t} h_{j,g,t} \right) \right) = \sum_j \sum_{g=46}^{65} N_{j,g,t}\, pen_{j,g,t}\ . \tag{23}$$

## 5 Human capital

The human capital technology is governed by the following specification proposed by Sadahiro and Shimasawa (2002):

$$h_{j,g+1,t+1} = (1-\delta_h)h_{j,g,t} + B(mk_{j,t})^{\phi}(h_{j,g,t}e_{j,g,t})^{1-\phi} \tag{24}$$

where $k_t$ is the physical capital/labor ratio while $B$ is the parameter for the accumulation efficiency of human capital, $m$ is the portion of physical capital stock used for producing the human capital stock, $\delta_h$ is the parameter of depreciation rate of human capital stock and $\phi$ is the parameter of the elasticity of human capital formation function.

Human capital is transmitted between generations according the following rule:

$$h_{j,1,t} = \pi^{hc}\left(\left(\sum_{g=1}^{45} h_{j,g,t-1}N_{j,g,t-1}\right)/\sum_{g=1}^{45} N_{j,g,t-1}\right) \tag{25}$$

where $\pi^{hc}$ is the parameter of human capital transmission factor. This parameter can be interpreted as the degree of quality or efficiency to pass the available stock of knowledge from generation to generation. If a society can provide the individual a successful educational environment (either formally or in-formally) in childhood and youth so that the individual earns the cognitive ability and creativeness well in these period, this parameter value should be high since the human ability acquired early will make post-secondary learning easier.

The aggregate human capital stock of region $j$ at time $t$ is defined using (26); and the aggregate supply of labor can be computed using (27):

$$H_{j,t} = \sum_{g=1}^{45} h_{j,g,t}N_{j,g,t} \tag{26}$$

$$L_{j,t}{}^{s} = \sum_{g=1}^{45}(1-e_{j,g,t})h_{j,g,t}N_{j,g,t} \tag{27}$$

## 6 Competitive equilibrium

A competitive equilibrium of the economy is defined as a dynamic and spatial sequence of regional disaggregate variables $\{ c_{j,g,t}, e_{j,g,t}, a_{j,g,t} \}_{j,g,t}$; regional aggregate variables $\{ C_{j,t}, K^s_{j,t}, L^s_{j,t}, K^d_{j,t}, L^d_{j,t}, Inv_{j,t} \}_{j,t}$; regional demand variables $\{ d^c_{i,j,g,t} \}_{i,j,g,t}$ and $\{ d^i_{i,j,t} \}_{i,j,t}$; regional intermediate demand variable $\{ x_{ij,t} \}_{i,j,t}$; regional output price and factor prices $\{ p_{j,t}, rr_{j,t}, w_{j,t} \}_{j,t}$; the interest rate $\{ r_{j,t} \}_{j,t}$; and the regional pension contribution rate $\{ \tau^p_t \}_t$ where $i, j$ =IL, IN, MI, OH, WI and ROUS; $g$ is the age-cohort from 1 to 65; and $t$ denotes year which satisfy 1) through 4):

1) Given prices and interest rate, the allocations are feasible for every region at every period: $Y_{i,t} = \sum_j x_{ij,t} + \sum_j \sum_g N_{j,g,t} d^c_{i,j,g,t} + \sum_j d^I_{i,j,t}$, $L^s_{j,t} = L^d_{j,t}$ and $K^s_{j,t} = K^d_{j,t}$.

2) Output prices and factor prices $\{ p_{j,t}, rr_{j,t}, w_{j,t} \}_{j,t}$ satisfy (4) and (5) for every region at every period; and (21) holds for every period.

3) Given prices and the interest rate, disaggregate variables $\{ c_{j,g,t}, e_{j,g,t}, a_{j,g,t} \}_{j,g,t}$ satisfy (9), (10) and (11) for every generations for every region at every period.

4) Given prices and interest rate, the pension contribution rate $\{ \tau^p_t \}_t$ satisfies (23) for every region at every period.

# III Calibration

This section will focus on the estimation of the parameter values in the production, consumption and investment functions. There are very little data available necessary for statistically estimating the inter-regional elasticity of substitution in consumption and investment functions described in the previous section. For example, there are only four sets (that is, 1993, 1997, 2002 and 2007) of data from the Commodity Flow Survey (CFS). Also, there are no time-series of SAMs to provide the possibility for estimating the price elasticities of consumption and investment across the regions. However, there is some literature that has attempted to estimate the regional import elasticity of substitution in the US, focused on individual industries. For example, Bilgic *et al.* (2002) estimate the elasticity of import substitution for 20 industry groups between 48 states, based on the micro-level data of 1993 CFS. In this paper, for the regional elasticity of substitution, this work is used as a benchmark; thereafter, the preference parameters of consumers and investors in each region from the six-regional Midwest-SAM are developed. For the human capital technology, the parameter values are assumed to be identical across regions, drawing on those available for the US economy in Sadahiro and Shimasawa (2002).

**1 Production function**

Table 1 is provides the expenditure quantities across regions for all industries in each state derived from input-output table of the Midwest. Since this paper does not consider sectors of institutions as well as tax and transfer of government and international trade, we exclude the products of institution and indirect business taxes as well as exports and imports in the table when calibrating the parameters. Also it should be noted that the original values in the IO table are denominated by the consumer price index (CPI) of each corresponding region (see table 2 for the regional coverage of CPI). Thus, each number in the cell in the table 1 is a unit-free quantity.

The parameters determining the quantity of intermediate goods across regions (that is, $z_{ij}$) could be computed as shares of expenditures from the table 1. The capital and labor income shares are also computed from the same table. Table 3 reveals the parameter values, which will be assigned to the production function.

**2 Consumption and Investment functions**

The elasticity of substitution in consumption and investment across regional goods is set to be 1.103, derived from the result of Bilgic *et al.* (2002). This number is the estimated elasticity of import substitution of all commodities. This elasticity is assumed to be homogeneous across regions and generations. Using this parameter value, the preference parameter of each region in consumption and investment could be computed with the information of consumption and investment shares of each region (table 4). Note that the magnitude in this table is also a unit-free quantity.

From (14), the preference parameter of consumption ($\upsilon^c_{i,j}$) could be computed as follows:

$$\upsilon^c_{i,j} = \frac{d^c_{i,j}}{c_j} \times \left( \frac{p^c_j}{p_i} \right)^{-s_j{}^c} \tag{28}$$

where $s_j{}^c = 1/(1-\sigma^c_j)$ is nothing but an elasticity of substitution for consumption; and $p^{c\;\frac{-\sigma^c_j}{1-\sigma^c_j}}_{j,t} = \sum_i \upsilon^c_{i,j} p_{i,t}^{\frac{-\sigma^c_j}{1-\sigma^c_j}}$ . Also, from (18), the preference parameter of investment ($\upsilon^I_{i,j}$) can be estimated as:

$$\upsilon^I_{i,j} = \frac{d^I_{i,j}}{Inv_j} \times \left( \frac{p^I_j}{p_i} \right)^{-s_j{}^I} \tag{29}$$

where $s_j^{I} = 1/(1-\sigma_j^I)$ is a elasticity of substitution across regional investment goods; and $p_{j,t}^{I}{}^{\frac{-\sigma_j^I}{1-\sigma_j^I}} = \sum_i \upsilon_{i,j}^I p_{i,t}{}^{\frac{-\sigma_j^I}{1-\sigma_j^I}}$. Table 5 shows the calibration results.

## IV Computational results

### 1. Steady-state

In this section, before presenting the results of the dynamic simulation, the steady state simulation results will be briefly summarized. For this presentation, the age-cohort population structure is adopted from the Census Bureau's estimation for the year 2007. Figure 1 shows the age-cohort structure that was adopted into the model for the steady state simulation. Note that IL's total population is normalized to a unit ($\sum_{g=1}^{65} N_{j=IL,g,t=2007} = 1$). Table 6 reveals that OH has the highest dependency ratio; and IL has the lowest. Figure 1 reveals that IL has significantly more people belonging to the below-retirement age than OH; but IL has almost same number of people belonging to the retirement age as OH. This could be interpreted as the result of retirement migration out of IL[2]. For the steady state analysis, this age-cohort population structure is assumed to be maintained in the long-term; also, it is assumed that there will be no change in prices including output, consumption and investment prices as well as factor prices such as the rental return and the wage rate. These assumptions described above would not be

[2] According to the 2008 American Community Survey (ACS) data, Illinois ranked the 2nd state after New York in the volume of losing the elderly residents (age 65+) through the out-migration during the previous one year.

maintained in the dynamic simulation, the results of which will be presented in the next section.

The input-output table (table 7) could be constructed by using the simulation results of the transactions among the six regions. The IO table, which is one of the outcomes of steady state general equilibrium simulation, is very similar to the actual IO table presented before: (i) the industries of each state purchases the commodities from the industries of the same state in large part; (ii) the consumers and investors also buy the majority of their consumption and investment goods from their own states; (iii) the volumes of production are in the order of ROUS>IL>OH>MI>IN>WI; and (iv) the usage of each region's output is largely consistent with actual statistics. For example, according to the simulation result, 48.0% of IL's output is sold as intermediate input; 37.3% as consumption goods; and 14.7% as investment goods while actual IO table shows that 50.3% of output produced in IL are purchased for input, 40.2% for consumption and 9.5% for investment (see table 8).

There exists a noteworthy gap in per-capita output across the regions according to the simulation results (table 9). Simulation and actual statistics point out that the state with the lowest per-capita output among the five Midwest states is Michigan; and the state with the highest per-capita output is Illinois. It should be noted that one of the reasons for the discrepancy between simulation result and actual data could be attributed to ignoring the differences of the technology level across the regions in the simulation model. Also the assumption that there is no external trade may bias the estimation. Further, it is also clear that the economy of the US was not in steady state in 2007.

Therefore, it is only with some degree of probability that the steady-state simulation result and the actual statistics should not be entirely consistent with each other.

The gaps of investment in physical capital and human capital play a key role in achieving different level of per-capita output in the simulation model. Table 10 shows that ROUS and IL invest 17.1% and 16.2% of their output while IN, WI, MI and OH allocate only 12.2%, 13.1%, 13.6% and 14.2% of their output in physical investment. This difference in investment tendency is related to the rate of rental return (see table 12 for factor prices): household agents would more inclined to consume the goods rather than save and invest them when the rental return becomes relatively low (or is expected to become low in the dynamic model.)

Also, the educational attainment could be a major factor in determining the difference of economic performance (here, per-capita output) since the educational investment is directly linked to the improvement of the human capital stock or productivity in our model. It is very certain in this model that the regions with higher per-capita output tend to combine inputs such as physical capital and labor force with a higher level of productivity. Table 11 shows the average time share spent in educational investment across the regions: IN, MI, WI and OH spend apparently less time in education than ROUS and IL. Accordingly, there should be subsequent gaps in human capital stock across the regions: figure 2 shows the discrepancies of the age-productivity profile (or human capital stock).

There is a notable gap between two groups: high skilled region and less skilled regions. The high skilled region are ROUS and IL; and less skilled region consist of IN, MI, OH and WI. For example, the average worker at the retirement age in the high skilled region (that is ROUS and IL) is 36.8% more productive than the worker at the same age in the less skilled region (that is IN, MI, OH and WI). This simulation result is consistent with the statistics of labor productivity between the regions: the labor statistics shows that IL and ROUS is the leading region in terms of labor productivity (the last row in table 11). Again, these gaps in productivity are attributed mainly to the differences of time spent on educational investment (table 11) and also the level of physical capital stock in the six-regional economies according to the model specifications (see 24).

Finally, Table 12 shows the regional prices such as output, consumption and investment price as well as production factors. The gaps of goods prices between the regions are larger than the actual CPI presented in the table 2. However, the order of prices matches well with the actual CPI level except MI: The simulation results under-estimates the consumption price in MI, compared to the table 2. Also, the simulation results imply that renting physical capital and hiring one unit of labor cost the most in the ROUS; on the contrary, the least region is IN.

Another steady state result can be generated with the different age-cohort structure in order to obtain the insight of impact of population ageing on the economy. The Census Bureau expects that the population ageing process will accelerate over time (table 13). According to the projection of the Census Bureau, the number of people between 15 and 64 will decline in the Midwest from 2007 to 2030. On the contrary, the number of people above 65 will grow at a significant rate. In particular, in the ROUS, the number of people of age 65+ will almost double from 2007 to 2030.

Without any change of model specification, the steady state simulation was implemented with the projected age-cohort structure for the year 2030. It is very important to note that this steady state result does not reflect the dynamic changes of the human capital level of households in each region. In other words, the steady-state result in this section reflects the changes of human capital level only between the generations, but does not consider the changes of human capital stock along the time dimension. However, the dynamic simulation, the result of which will be described in the next section, will reflect the endogenous growth of human capital stock along the time dimension as well as between the generations. Furthermore, the changes of human capital- related variables would play a critical role in simulating the dynamics of the six regions' economies.

Table 14 shows the comparison of per-capita output under the two different age-cohort structures. The results are quite intuitive: due to the population ageing, per-capita output under the age-cohort structure in 2030 is less than per-capita output under the age-cohort structure in 2007 in every region. It should be noticed that the per-capita output in OH under the demographic scenario of 2030 does not decline so much from the level under the scenario of 2007. The number of people belonging to the working age (15-64) in OH declines faster than the other region from 2007 through 2030; subsequently the total population size (15+) grows at only 1.4% (table 13). On the contrary, it grows at 24.6% in the ROUS and 10.6% in the WI. The relative faster growth of the external demand mitigates the negative impact of population ageing to some extent. This positive effect from the external economy is reflected by the relative price changes: the demand growth from the growing population in the other regions and the limited supply of the good produced in OH (owing to drop of labor force) causes the improvement of the terms of trade for OH, assuming that the goods produced in each region are imperfect substitutes each other. As shown in figure 3, the growth of the relative output price of OH from 2007 through 2030 is the highest among 5 Midwest states, reflecting the improving terms of trade for OH.

## 2 Results of dynamic simulation

### 2.1 Dynamics of age-cohort population structure

The main origin of dynamics of the regional economies in this paper should be population ageing: in particular, the dynamic simulation is focused on the demographic change from 2007 through 2030. The panels in figure 4 show the changes of age-cohort structure of each region. Again note that the total population size of IL in 2007 in this simulation is normalized to be a unit ($\sum_{g=1}^{65} N_{j=IL,g,t=2007} = 1$). It is apparent that the population ageing process is becoming more important in every one of the six regions from 2007 through 2030. This paper assumes that the age-cohort structure and population size of each region after 2030 is same as those for 2030[3].

### 2.2 Outcomes

The results presented in this section should be different from what we have seen in the previous section since the agents are assumed to react to the expectation of future price development caused by the demographic changes in the dynamic simulation model. In the steady-state simulation, the price variables are assumed to be constant permanently.

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[3] This could be strict assumption in the dynamic simulation since the individuals respond sensitively to the movement of economic variables in the future. Thus it is quite desirable to get the projection of age-cohort population strucuture as long as possible for the dynamic simulation like the model in this paper.

Table 15 shows the per-capita output in 2007 and 2030 for each region (see the appendix for the detailed figures). Unlike the results shown in the previous section, the dynamic simulation demonstrates that the per-capita output will grow positively even though there will be a fast growing population ageing phenomenon. As shown in Kim and Hewings (2010), this is because individual's endogenous choice in educational investment mitigates the negative effects of population ageing to some extent thru improving the overall productivity in the corresponding economy during the transition. Furthermore, the simulation results projects that IN and MI will grow 22.9% and 19.6% respectively while IL and ROUS grow at 2.2% and 3.6%. Figure 5 shows the size of per-capita output, compared to 2007: it shows that IN and MI are growing relatively fast. However, notice that the growth of 22.8% (IN) for 23 years (2007 thru 2030) is still very low: it amounts to 0.9% per year. Also, note that ROUS and IL will produce most per worker; and MI produces least per worker still in 2030 (see the appendix).

In the economy that the model describes, the physical capital and human capital complement each other. In terms of human capital, higher human capital stock makes the combination of labor and physical capital more effective; consequently, the combination promotes the economic growth. This economic growth induces the physical capital stock to be built up more since the economy produces more per unit of input than before. However, as the physical capital stock per labor grows, the marginal return to investment in human capital stock decreases (see the human capital technology equation 24); and workers react less unfavorably to increasing their investment time more in their human capital. This reluctance in increasing educational investment decelerates the economic growth; and consequently lowers the growth of physical capital stock. Overall, in the region of higher human capital stock and physical capital stock, the growth of human capital stock and physical capital stock is relatively low. In our simulation, IL and ROUS have the highest level of physical capital per worker and human capital among the six regions in 2007 (see the 2nd and 3rd section in the table 16). On the contrary, IN and MI have the lowest average physical capital and human capital stock, implying the marginal return to education in IN and MI is higher than the other area. So the workers in IN and MI will increase their educational investment more rapidly than those in ROUS and IL. This conjecture is consistent with the simulation results: the first section of table 16 shows that educational investment in IN and MI grows at 14.2% and 12.8% respectively while ROUS and IL grow at 1.4% and 2.4% respectively from 2007 thru 2030. Thus, the growth of human capital stock and physical capital stock per worker in IN and MI is higher than the other regions; and those in ROUS and IL is less than the other regions. However, note that ROUS and IL would still lead the other regions in terms of amount of time spent in educational investment, average human capital stock and average physical capital stock per worker in 2030.

Population ageing will generally cause the wage rate to increase since a large number of retired persons will cause the labor force to decline compared to the other production factor (physical capital). Consistent with this notion, the wage rates in every one of the six regions increases from 2007 to 2030 according to the simulation results (see upper panel in figure 6). However, the growth rate of the wage rate is different across the regions. For example, OH and WI are projected to experience higher increase of wage rates than any other region. As shown in the first order condition (5), there exist two prime forces to influencing the wage rate in each region: one is the relative scarcity of labor and the other is the remaining aggregate region's output price together with the parameters representing the preference for each region's output as an intermediate good. The former is linked positively; and the latter is linked negatively with the movement of the wage rate. First, IN, MI and WI are the top 3 regions where the labor (joined with human capital stock) will become scarce more rapidly than the other regions (see the 3rd section in the table 16). So the wage rate in WI is projected to show a higher growth rate (+14.8%). However, IN and MI will not show a high increasing tendency in the wage rate even though the labor force will become scarce relatively rapidly. This is because their terms of trade are projected to deteriorate from 2007 through 2030. Figure 6 shows that output price of IN, ROUS and MI will decrease 3.4%, 1.2% and 1.0% during the period while the output price of the goods produced in OH will grow at 8.7%. The wage rate in OH will experience the upward pressure from the improving terms of trade (which means a decline in the relative output price in the other regions). The terms of trade of ROUS will deteriorate since its demand for the goods produced in the Midwest states will grow more rapidly than the Midwest states' demand for the goods produced in ROUS, taking the population growth projection in the table 13. However, IN and MI will receive a smaller benefit from this declining terms of trade in ROUS since ROUS' demand for the goods produced in IN and MI is relatively low, compared to the other Midwest states. Table 17 compares the preference parameter of ROUS fixed in the previous calibration procedure (note that the elasticity of substitution across the regions was set to be identical across the regions in the previous section). The parameters representing the preference of ROUS for the goods produced in IN and MI as intermediate input, consumption and investment goods are low, compared to the other regions. Also the goods produced in IN and MI will be relatively abundant in 2030, compared to 2007, thanks to the rapid increase of per-capita output in these regions as analyzed before.

## V Conclusion

In the last section, the simulation results were presented to explore two issues: one is how significantly the educational motive mitigates the negative impact of faster growing number of elderly people (65+); and the other is how each state affects each other during the population ageing era. According to the last section, the steady state results reveal that each region will grow negatively in sense of per-capita output from 2007 through 2030 even though there will be some differences of degree of deterioration across the regions.

However, dynamic simulation results imply that the economy in every region will show the positive growth during the period thanks to a prominent underlying force of economic growth: human capital. Also, it was shown that there exist the eventual interactions of demographics and terms of trade of each region through the mechanisms of demand and supply of the goods; further, these interactions substantially affect the development of factor prices. Incorporating the information from the multi-regional input-output within the dynamic OLG framework worked well in simulating the mechanism of demand and supply forces at the intermediate good, consumption and investment good markets.

This paper projects that the per-capita output of IN and MI will grow relatively rapidly from 2007 thru 2030, taking the expected demographic developments and endogenous growth mechanism into considerations. However, it should be noted that the per-capita output simulated in the last section should be understood as a concept of potential output: the highest sustainable level of output that the corresponding economy can produce by efficiently combining its every inputs such as labor, capital and technology. Thus, the simulation result regarding the growth of per-capita output just reveals that the negative impacts of population ageing in IN and MI could be mitigated more than the other regions due to human capital formation mechanism. To realize this potential, it is critical that there should be no formal or informal hindrance for the workers to implement their optimal decisions on educational investment. In practice, the regional government should encourage the workers to invest their time in improving their human capital stock by fiscal policy so that the economy could mitigate the negative impact from the population ageing (see Kim and Hewings (2010) for the detailed analysis); and follow the track presented in the last section. Meanwhile, it was shown that IN and MI would be ones where the output price will be declining during the aging period. The deterioration of terms of trade will result in a decline in the wage rate, implying that the worker's welfare would be undermined in these regions. Thus, while encouraging the worker to invest more time in improving his/her human capital stock as described above, the firms should try to build up the industrial relationship with the institutions outside the Midwest so that their goods could be demanded more than before as intermediate inputs, as well as consumption and investment goods by the regions outside the Midwest states.

For the highly developed region like ROUS and IL in terms of per-capita physical capital stock and human capital stock level, there exist underlying threats stemming from the population ageing since the human capital growth could be sluggish in the near future in these regions. Thus, the political effort should be concentrated on shifting the human capital stock formation function itself. One of the policies for consideration would be upgrading the educational environment, either through institutional or non-institutional settings, of the group with the lower human capital stock such as international immigrants and African-Americans. Upgrading the learning ability during the early period of life of the people in the lower-skilled group could shift upward the post-school human capital stock formation of the corresponding community. Kim and Hewings (2010) showed that this kind of upgrading of human capital stock formation technology corresponding to the African-American society in Illinois benefits the whole economic agents in Illinois substantially in the long run under a population ageing scenario.

This paper assumes that the household agent is immobile between the states. However, as this paper reveals in the previous section, the heterogeneous demographic change across the regions causes the dynamic movements of factor prices and different growth of physical and human capital stock across the regions. These effects of regional demographic changes may provide a portion of regional residents with the incentives to migrate between the regions to seek higher wage rates and/or higher returns to education and so forth. Empirically, Illinois is considered to be the number two state after New York in terms of retirement outmigration according the data analysis of 2008 American Community Survey (ACS); and as Yu (2009) analyzed, the fund outflows stemming from the retirement outmigration pattern negatively affect the regional economy. Thus it is desirable to extend the model presented in this paper to incorporate the migration of active labor and retirees. For this extension, the uninsurable idiosyncratic shock related to the skill factor (say) should be included in the model; and the revealed history of shocks makes each individual to be heterogeneous in terms of skill level. This heterogeneity of skill level and subsequently difference of expected life-time income will cause individuals to reveal a different pattern in choosing whether to stay or not during each period in which the model is run. For example, one might expect that highly skilled and unskilled workers would exhibit a different pattern of optimizing their choice on whether to out-migrate or not. As Basile and Lim (2006) have demonstrated, there needs to be additional consideration of the decision to migrate and the actual time when the migration takes place. Different factors may influence each part of this two-stage process.

Finally, as mentioned before presenting the simulation results, this paper assumed that the age-cohort population structure will be maintained after 2030. It is highly possible that this assumption may generate a distortion of projection to some extent especially in the later period near 2030 since the economic agents in the simulation model optimize their choices based on the expectation of future economic variables; and future economic variables are substantially influenced by demographics. For generating more plausible outcomes, the longer term dynamics of age-cohort population structure in each region should be generated. In other words, more rigorous way in obtaining the age-cohort population structure is desired instead of depending entirely on the relatively short-term population data provided by the Census Bureau.

## References


Armington, P. 1969. "A Theory of Demand for Products Distinguished by Place of Production." *International Monetary Fund Staff Papers* 16: 159-178.

Basile, R. and Lim, J. 2006. "Wages Differentials and Interregional Migration in the U. S.: An Empirical Test of the "Option Value of Waiting" Theory." ERSA conference papers, European Regional Science Association.

Bilgic, A., King, S., Lusby, A. and Schreiner, D. 2002. "Estimates of U.S. Regional Commodity Trade Elasticities of Substitution." *Journal of Regional Analysis and Policy* 32: 79-98.

Borsch-Supan, A., Ludwig, A. and Winter, J. 2005. "Aging, Pension Reform, and Capital Flows: A Multi-Country Simulation Model." *Computing in Economics and Finance* 123: 1-49.

Bouzahzah, M., Croix, D. and Docquier, F. 2002. "Policy reforms and growth in computable OLG economies." *Journal of Economic Dynamics & Control* 26: 2093-2113.

Fougere, M., Harvey, S., Merette, M. and Poitras, F. 2004. "Ageing Population and Immigration in Canada: An Analysis with a Regional CGE Overlapping Generations Model." *Canadian Journal of Regional Science* 27:209-236.

Hall, R. and Jones, C. 1999. "Why Do Some Countries Produce So Much More Output Per Worker Than Others?" *The Quarterly Journal of Economics* 114: 83-116.

Kehoe, T., Polo, C., and Sancho, F. 1995. "An evaluation of the performance of an applied general equilibrium model of the Spanish economy." *Economic Theory* 6: 115-141.

Kihoe, T. 1996. “Social Accounting Matrices and Applied General Equilibrium Models.” Working paper 563, Federal Reserve Bank of Minneapolis.

Kim, T. and Hewings, J. 2010. “Endogenous Growth of the Ageing Economy with Intra-Generational Heterogeneity over Race and Migration Status.” Discussion paper 10-T-2, Regional Economics Applications Laboratory, University of Illinois at Urbana-Champaign.

Ludwig, A., Schelkle, T. and Vogel, E. 2007. “Demographic Change, Human Capital and Endogenous Growth.” Working paper, MEA, University of Manheim.

Miller, R. and Blair, P. 1985. Input-Output Analysis. New Jersey: Prentice-Hall, Inc.

Plassmann, F. and Tideman, T. 1999. “A Dynamic Regional Applied General Equilibrium Model with Five Factors of Production.” Working Paper W9907, Binghamton University.

Plassmann, F. 2005. “The Advantage of avoiding the Armington assumption in multi-region models.” *Regional Science and Urban Economics* 35: 777-794.

Sadahiro, A. and Shimasawa, M. 2002. “The computable overlapping generations model with an endogenous growth mechanism.” *Economic Modelling* 20:1-24.

Shoven, J. and Whalley, J. 1992. Applying General Equilibrium. New York: Cambridge University Press.

Yu, Chenxi. 2009. “Illinois Migration Report 1992-2006.” Retrieved from http://www.real.illinois.edu/.

Table 1: Expenditure quantity of each region's industry in 2007

| | | IL | IN | MI | OH | WI | ROUS | Total |
|---|---|---|---|---|---|---|---|---|
| Intermediate purchases across regions | I L | 1,622 | 73 | 52 | 45 | 51 | 638 | 2,482 |
| | I N | 55 | 841 | 42 | 69 | 14 | 236 | 1,257 |
| | M I | 42 | 37 | 1,078 | 99 | 49 | 315 | 1,621 |
| | O H | 35 | 75 | 110 | 1,310 | 16 | 529 | 2,075 |
| | W I | 38 | 11 | 46 | 12 | 779 | 228 | 1,115 |
| | ROUS | 698 | 337 | 458 | 715 | 331 | 40,245 | 42,784 |
| | Intermediate total | 2,490 | 1,375 | 1,786 | 2,249 | 1,240 | 42,192 | 51,333 |
| Value Added | Employee Compensation | 1,742 | 728 | 1,157 | 1,381 | 699 | 32,459 | 38,166 |
| | Proprietary Income | 220 | 81 | 142 | 138 | 64 | 4,509 | 5,154 |
| | Other Property Income | 864 | 353 | 547 | 630 | 330 | 16,430 | 19,154 |
| | Indirect Business Taxes (A) | 224 | 87 | 143 | 162 | 80 | 4,262 | 4,959 |
| | VA total | 3,049 | 1,249 | 1,989 | 2,312 | 1,173 | 57,660 | 67,433 |
| | VA total – (A) | 2,826 | 1,162 | 1,846 | 2,150 | 1,093 | 53,398 | 62,474 |
| Institutions (B) | | 12 | 9 | 10 | 13 | 6 | 196 | 246 |
| Foreign Import (C) | | 117 | 88 | 111 | 141 | 58 | 5,502 | 6,017 |
| Total | | 5,669 | 2,721 | 3,896 | 4,715 | 2,478 | 105,550 | 125,029 |
| Total – (A) – (B) – (C) | | 5,316 | 2,537 | 3,632 | 4,399 | 2,333 | 95,590 | 113,807 |

Table 2: Consumer price index of each region in 2007

| Region | Coverage | CPI |
|---|---|---|
| IL | Chicago-Gary-Kenosha, IL-IN-WI | 204.8 |
| IN | Cincinnati-Hamilton, OH-KY-IN | 193.9 |
| MI | Detroit-Ann Arbor-Flint, MI | 200.1 |
| OH | Cleveland-Akron, OH | 196.0 |
| WI | Milwaukee-Racine, WI | 194.1 |
| ROUS | US city average | 205.7 |

Source: Bureau of Labor Statistics (www.bls.gov)

Table 3: Parameter values in production function

| | IL (j=1) | IN (j=2) | MI (j=3) | OH (j=4) | WI (j=5) | ROUS (j=6) |
|---|---|---|---|---|---|---|
| IL ( $z_{1j}$ ) | 0.3052 | 0.0288 | 0.0142 | 0.0102 | 0.0220 | 0.0067 |
| IN ( $z_{2j}$ ) | 0.0104 | 0.3314 | 0.0116 | 0.0156 | 0.0062 | 0.0025 |
| MI ( $z_{3j}$ ) | 0.0079 | 0.0147 | 0.2968 | 0.0226 | 0.0210 | 0.0033 |
| OH ( $z_{4j}$ ) | 0.0066 | 0.0298 | 0.0302 | 0.2977 | 0.0067 | 0.0055 |
| WI ( $z_{5j}$ ) | 0.0071 | 0.0043 | 0.0128 | 0.0027 | 0.3340 | 0.0024 |
| ROUS ( $z_{6j}$ ) | 0.1312 | 0.1330 | 0.1261 | 0.1625 | 0.1417 | 0.4210 |
| Value Added ( $z_{VA,j}$ ) | 0.5316 | 0.4580 | 0.5083 | 0.4887 | 0.4684 | 0.5586 |
| Labor income share (1- $\alpha_j$ ) | 0.6166 | 0.6262 | 0.6269 | 0.6425 | 0.6394 | 0.6079 |
| Capital income share ( $\alpha_j$ ) | 0.3834 | 0.3738 | 0.3731 | 0.3575 | 0.3606 | 0.3921 |

Table 4: Expenditure quantity of each region's consumption and investment in 2007

| | IL | IN | MI | OH | WI | ROUS | Total |
|---|---|---|---|---|---|---|---|
| **Consumption** | | | | | | | |
| IL | 1,528 | 35 | 31 | 25 | 23 | 342 | 1,984 |
| IN | 28 | 636 | 17 | 29 | 6 | 101 | 817 |
| MI | 25 | 15 | 1,029 | 50 | 22 | 176 | 1,317 |
| OH | 22 | 31 | 55 | 1,164 | 8 | 291 | 1,571 |
| WI | 26 | 5 | 28 | 7 | 622 | 123 | 812 |
| ROUS | 367 | 129 | 224 | 341 | 137 | 36,430 | 37,628 |
| Total | 1,997 | 851 | 1,384 | 1,616 | 817 | 37,463 | 44,129 |
| **Investment** | | | | | | | |
| IL | 435 | 2 | 2 | 2 | 2 | 28 | 471 |
| IN | 7 | 241 | 9 | 11 | 2 | 24 | 293 |
| MI | 10 | 6 | 338 | 24 | 8 | 59 | 445 |
| OH | 5 | 7 | 22 | 350 | 2 | 59 | 446 |
| WI | 6 | 1 | 4 | 5 | 220 | 11 | 247 |
| ROUS | 52 | 23 | 69 | 74 | 23 | 9,748 | 9,989 |
| Total | 515 | 279 | 445 | 466 | 257 | 9,930 | 11,892 |

Table 5: Regional preference parameter values in consumption and investment function

| | IL (j=1) | IN (j=2) | MI (j=3) | OH (j=4) | WI (j=5) | ROUS (j=6) |
|---|---|---|---|---|---|---|
| **Consumption** | | | | | | |
| IL ( $v^c_{1,j}$ ) | 0.7666 | 0.0434 | 0.0226 | 0.0160 | 0.0293 | 0.0091 |
| IN ( $v^c_{2,j}$ ) | 0.0134 | 0.7367 | 0.0121 | 0.0176 | 0.0066 | 0.0025 |
| MI ( $v^c_{3,j}$ ) | 0.0121 | 0.0184 | 0.7405 | 0.0311 | 0.0274 | 0.0046 |
| OH ( $v^c_{4,j}$ ) | 0.0107 | 0.0359 | 0.0390 | 0.7113 | 0.0095 | 0.0074 |
| WI ( $v^c_{5,j}$ ) | 0.0122 | 0.0059 | 0.0198 | 0.0043 | 0.7509 | 0.0031 |
| ROUS ( $v^c_{6,j}$ ) | 0.1851 | 0.1597 | 0.1661 | 0.2197 | 0.1763 | 0.9733 |
| **Investment** | | | | | | |
| IL ( $v^I_{1,j}$ ) | 0.8458 | 0.0072 | 0.0051 | 0.0050 | 0.0064 | 0.0029 |
| IN ( $v^I_{2,j}$ ) | 0.0121 | 0.8562 | 0.0194 | 0.0236 | 0.0064 | 0.0023 |
| MI ( $v^I_{3,j}$ ) | 0.0186 | 0.0235 | 0.7593 | 0.0515 | 0.0319 | 0.0057 |
| OH ( $v^I_{4,j}$ ) | 0.0101 | 0.0234 | 0.0480 | 0.7444 | 0.0089 | 0.0057 |
| WI ( $v^I_{5,j}$ ) | 0.0111 | 0.0028 | 0.0087 | 0.0102 | 0.8498 | 0.0010 |
| ROUS ( $v^I_{6,j}$ ) | 0.1023 | 0.0869 | 0.1596 | 0.1654 | 0.0965 | 0.9824 |

Table 6: Dependency ratio of each region in 2007

| IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|
| 18.04% | 18.54% | 18.33% | 20.11% | 19.39% | 18.70% |

Table 7: Steady state results with age-cohort structure in 2007- Regional input-output table

| | Intermediary Input | | | | | | | Consumption | | | | | | | Investment | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | IL | IN | MI | OH | WI | RS | Sub-total | IL | IN | MI | OH | WI | RS | Sub-total | IL | IN | MI | OH | WI | RS | Sub-total | |
| IL | 2.440 | 0.094 | 0.069 | 0.061 | 0.065 | 1.106 | 3.835 | 2.274 | 0.030 | 0.027 | 0.029 | 0.022 | 0.600 | 2.982 | 1.082 | 0.002 | 0.003 | 0.004 | 0.002 | 0.084 | 1.176 | 7.993 |
| IN | 0.083 | 1.079 | 0.057 | 0.094 | 0.018 | 0.413 | 1.743 | 0.052 | 0.676 | 0.019 | 0.042 | 0.007 | 0.217 | 1.013 | 0.020 | 0.352 | 0.014 | 0.024 | 0.003 | 0.087 | 0.500 | 3.256 |
| MI | 0.063 | 0.048 | 1.447 | 0.136 | 0.062 | 0.545 | 2.300 | 0.047 | 0.017 | 1.153 | 0.074 | 0.028 | 0.400 | 1.719 | 0.031 | 0.010 | 0.532 | 0.052 | 0.014 | 0.217 | 0.856 | 4.875 |
| OH | 0.053 | 0.097 | 0.147 | 1.787 | 0.020 | 0.908 | 3.011 | 0.036 | 0.028 | 0.052 | 1.443 | 0.008 | 0.547 | 2.113 | 0.015 | 0.008 | 0.029 | 0.639 | 0.003 | 0.184 | 0.878 | 6.002 |
| WI | 0.057 | 0.014 | 0.062 | 0.016 | 0.986 | 0.396 | 1.531 | 0.043 | 0.005 | 0.028 | 0.009 | 0.687 | 0.245 | 1.017 | 0.017 | 0.001 | 0.006 | 0.009 | 0.336 | 0.035 | 0.403 | 2.951 |
| RS | 1.049 | 0.433 | 0.615 | 0.975 | 0.418 | 69.489 | 72.978 | 0.536 | 0.109 | 0.191 | 0.388 | 0.132 | 62.654 | 64.009 | 0.128 | 0.027 | 0.083 | 0.124 | 0.031 | 27.627 | 28.019 | 165.01 |
| VA | 4.249 | 1.491 | 2.478 | 2.933 | 1.382 | 92.200 | | | | | | | | | | | | | | | | |
| Output | 7.99 | 3.26 | 4.87 | 6.00 | 2.95 | 165.06 | | | | | | | | | | | | | | | | |

Note: RS in the table indicates ROUS

Table 8: Percentage of usage of each region's output (%)

| | | Intermediary Input | Consumption | Investment |
|---|---|---|---|---|
| IL | Simulation | 47.97 | 37.31 | 14.72 |
| | Actual data | 50.27 | 40.19 | 9.54 |
| IN | Simulation | 53.54 | 31.11 | 15.35 |
| | Actual data | 53.11 | 34.52 | 12.38 |
| MI | Simulation | 47.18 | 35.26 | 17.56 |
| | Actual data | 47.92 | 38.93 | 13.15 |
| OH | Simulation | 50.17 | 35.21 | 14.62 |
| | Actual data | 50.71 | 38.39 | 10.90 |
| WI | Simulation | 51.88 | 34.47 | 13.65 |
| | Actual data | 51.29 | 37.35 | 11.36 |
| ROUS | Simulation | 44.23 | 38.79 | 16.98 |
| | Actual data | 47.33 | 41.62 | 11.05 |

Table 9: Per-capita output

| | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|
| Simulation | 0.9704 | 0.8036 | 0.7286 | 0.7990 | 0.7996 | 1.0000 |
| Actual data | 1.0729 | 0.8885 | 0.8442 | 0.8835 | 0.9197 | 1.0000 |

Note: 1. Numbers of ROUS is normalized to a unit.
2. Actual data is calculated by GSP (Gross State Product) excluding public sectors ÷ population estimation in 2007.

Source: BEA (www.bea.gov) for GSP; and Census Bureau for population estimation.

Table 10: Steady-state results- Investment-output ratio

| | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|
| Physical Investment (A) | 1.2911 | 0.3982 | 0.6625 | 0.8502 | 0.3876 | 28.2210 |
| Output (B) | 7.9929 | 3.2557 | 4.8748 | 6.0022 | 2.9510 | 165.0061 |
| Investment-Output ratio (A / B) | 0.1615 | 0.1223 | 0.1359 | 0.1416 | 0.1313 | 0.1710 |

Table 11: Steady-state results- Time share of educational investment and average human capital stock

| | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|
| Time share in education (%) | 13.18 | 10.42 | 10.55 | 11.42 | 10.97 | 13.55 |
| Avg. human capital stock | 2.27 | 1.77 | 1.78 | 1.94 | 1.85 | 2.39 |
| Gross State Product / Annual Employment: 1998 thru 2007[1)] | 80.52 | 67.77 | 74.88 | 68.94 | 65.06 | 78.66 |

Note: 1) Unit: thousand dollars chained with 2000 price level. Source: Bureau of Labor Statistics and Bureau of Economic Analysis

Table 12: Steady state results- Prices

| | | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|---|
| Goods price | Production | 0.9783 | 0.7619 | 0.7611 | 0.8816 | 0.8316 | 1.0000 |
| | Consumption | 0.9720 | 0.8085 | 0.8057 | 0.9010 | 0.8608 | 0.9963 |
| | Investment | 0.9701 | 0.7841 | 0.8011 | 0.8892 | 0.8457 | 0.9968 |
| Rental return (physical capital) | | 0.0857 | 0.0648 | 0.0662 | 0.0723 | 0.0690 | 0.0888 |
| Wage rate | | 1.5363 | 0.9494 | 0.9717 | 1.2228 | 1.1041 | 1.6090 |

Table 13: Growth of population size (age 15+) from 2007 to 2030

| Age | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|
| 15-64 | 0.9665 | 0.9873 | 0.9474 | 0.9105 | 0.9750 | 1.1163 |
| 65+ | 1.5595 | 1.5774 | 1.6320 | 1.5261 | 1.7803 | 1.9415 |
| 15+ | 1.0571 | 1.0796 | 1.0535 | 1.0136 | 1.1057 | 1.2464 |
| Dependency ratio | 18.04% → 29.10% (+11.1%p) | 18.54% → 29.63% (+11.1%p) | 18.33% → 31.57% (+13.2%p) | 20.11% → 33.70% (+13.6%p) | 19.39% → 35.40% (+16.1%p) | 18.70% → 32.53% (+13.8%p) |

Source: Census Bureau's projection

Table 14: Steady state result- Per-capita output under the alternative age-cohort structures

| | | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|---|
| Per-capita output | 2007: A | 7.9932 | 6.6194 | 6.0017 | 6.5813 | 6.5866 | 8.2374 |
| | 2030: B | 7.3336 | 6.1256 | 5.6248 | 6.4631 | 6.1252 | 6.6928 |
| | B/A | 0.9175 | 0.9254 | 0.9372 | 0.9820 | 0.9299 | 0.8125 |

Table 15: Per-capita output from the dynamic simulation results

| | | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|---|
| Per-capita output | 2007: A | 7.6775 | 5.7624 | 5.3164 | 6.1117 | 5.9618 | 7.9339 |
| | 2030: B | 7.8491 | 7.0798 | 6.3597 | 6.4586 | 6.5759 | 8.2189 |
| | B/A | 1.0224 | 1.2286 | 1.1962 | 1.0568 | 1.1030 | 1.0359 |

Table 16: Average physical capital per effective labor and human capital stock per labor

| | | 2007 (A) | 2010 | 2020 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|
| Educational investment (fraction of endowment time) | IL | 0.1204 | 0.1205 | 0.1218 | 0.1233 | 1.0241 |
| | IN | 0.0967 | 0.1007 | 0.1069 | 0.1104 | 1.1417 |
| | MI | 0.0980 | 0.1015 | 0.1073 | 0.1105 | 1.1276 |
| | OH | 0.1073 | 0.1089 | 0.1117 | 0.1139 | 1.0615 |
| | WI | 0.1022 | 0.1049 | 0.1090 | 0.1110 | 1.0861 |
| | ROUS | 0.1263 | 0.1262 | 0.1269 | 0.1281 | 1.0143 |
| Human capital stock (ROUS in 2007 = 1) | IL | 0.9485 | 0.9611 | 0.9971 | 1.0565 | 1.1139 |
| | IN | 0.7393 | 0.7258 | 0.7733 | 0.8618 | 1.1657 |
| | MI | 0.7448 | 0.7335 | 0.7800 | 0.8676 | 1.1649 |
| | OH | 0.8104 | 0.8093 | 0.8414 | 0.9062 | 1.1182 |
| | WI | 0.7725 | 0.7657 | 0.8049 | 0.8803 | 1.1395 |
| | ROUS | 1.0000 | 1.0236 | 1.0757 | 1.1384 | 1.1384 |
| Physical capital/effective labor (ROUS in 2007 =1) | IL | 0.9394 | 0.9251 | 0.9514 | 0.9451 | 1.0061 |
| | IN | 0.5702 | 0.6696 | 0.8387 | 0.8709 | 1.5272 |
| | MI | 0.5955 | 0.6834 | 0.8389 | 0.8782 | 1.4747 |
| | OH | 0.7198 | 0.7487 | 0.8326 | 0.8523 | 1.1842 |
| | WI | 0.6451 | 0.7075 | 0.8337 | 0.8622 | 1.3365 |
| | ROUS | 1.0000 | 0.9806 | 1.0214 | 1.0454 | 1.0454 |

Table 17: Preference parameter of ROUS

| | IL | IN | MI | OH | WI | ROUS |
|---|---|---|---|---|---|---|
| Intermediate input | .0067 | .0025 | .0033 | .0055 | .0024 | .4210 |
| Consumption | .0091 | .0025 | .0046 | .0074 | .0031 | .9733 |
| Investment | .0029 | .0023 | .0057 | .0057 | .0010 | .9824 |

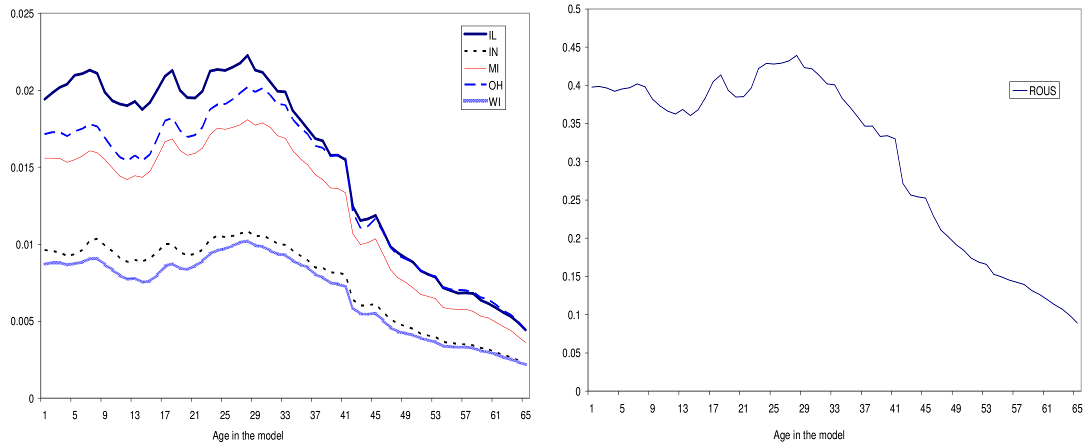


Figure 1: Age-cohort population structure

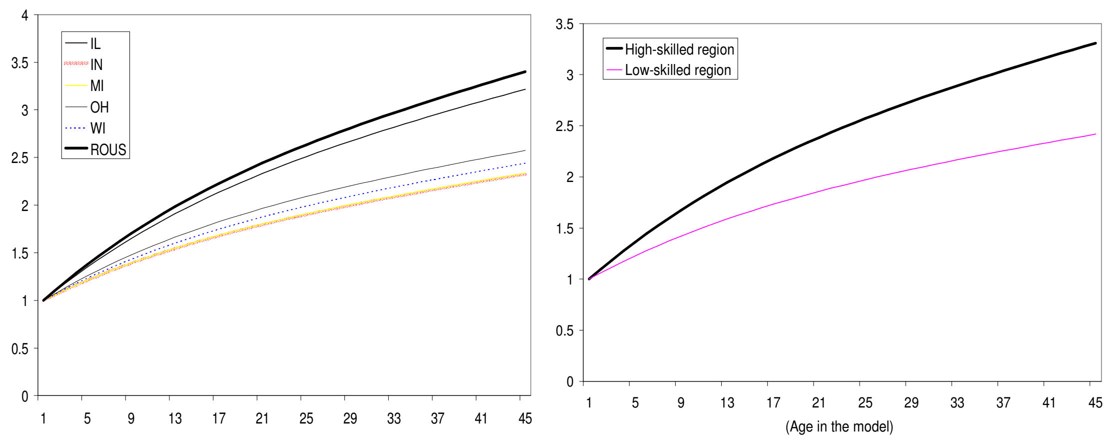


Figure 2: Steady state results- Age profile of human capital stock

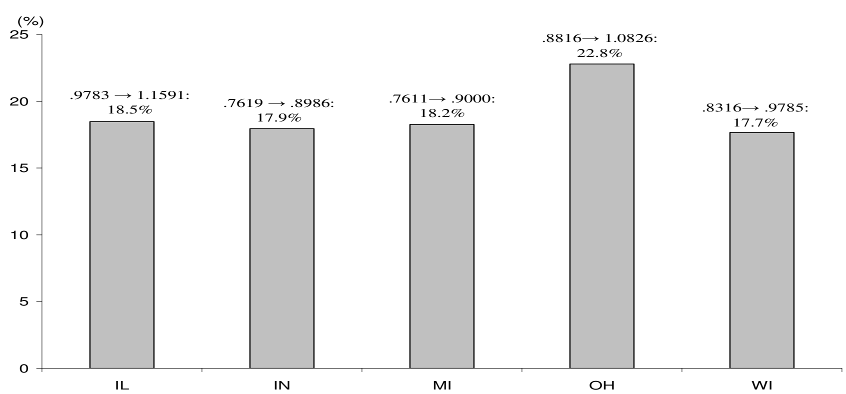


Figure 3: Percentage growth of (relative) regional output prices from 2007 to 2030 according to the steady-state simulation

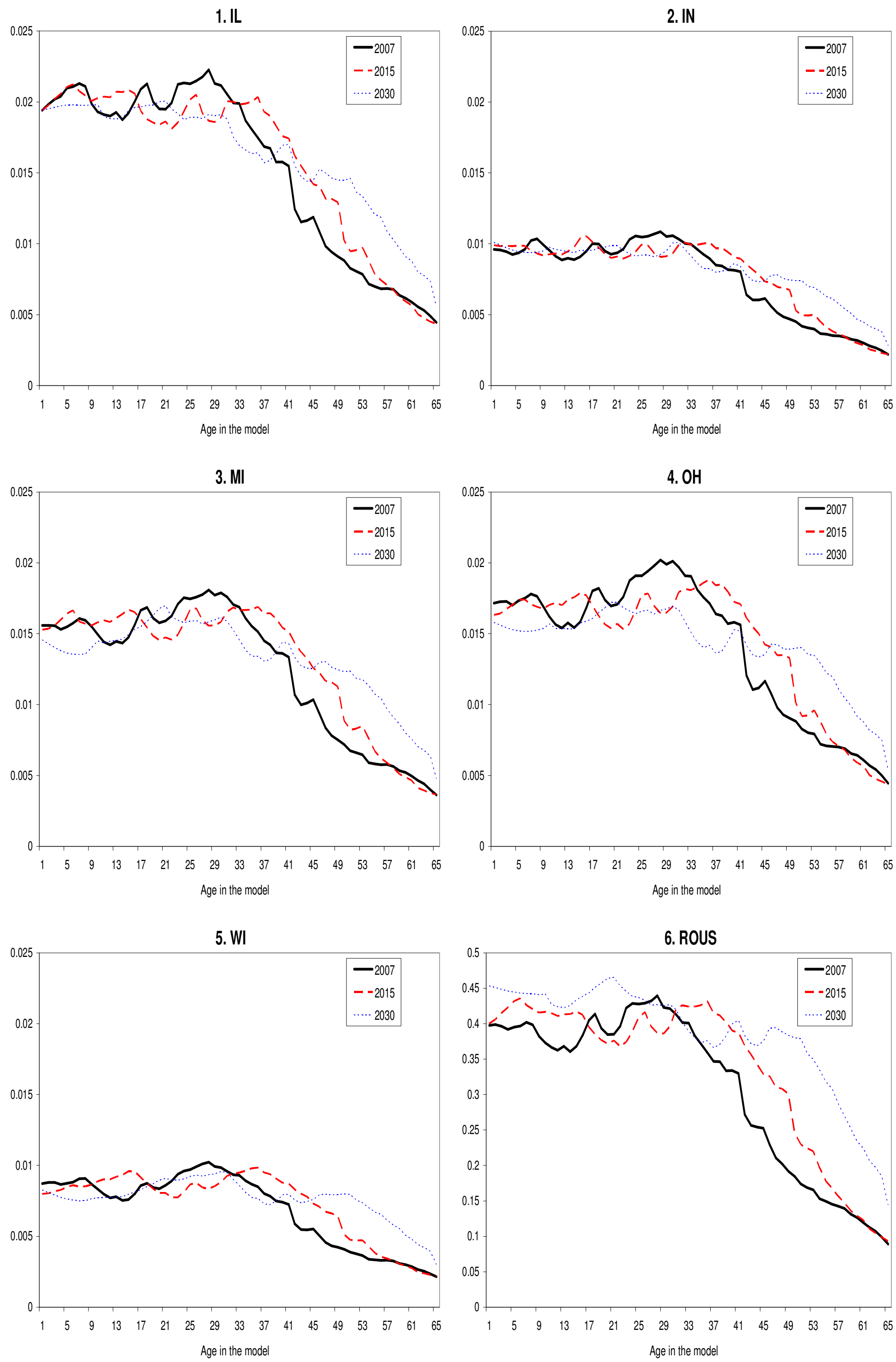


Figure 4: Age-cohort population structures of the each region (to be incorporated into the simulation)

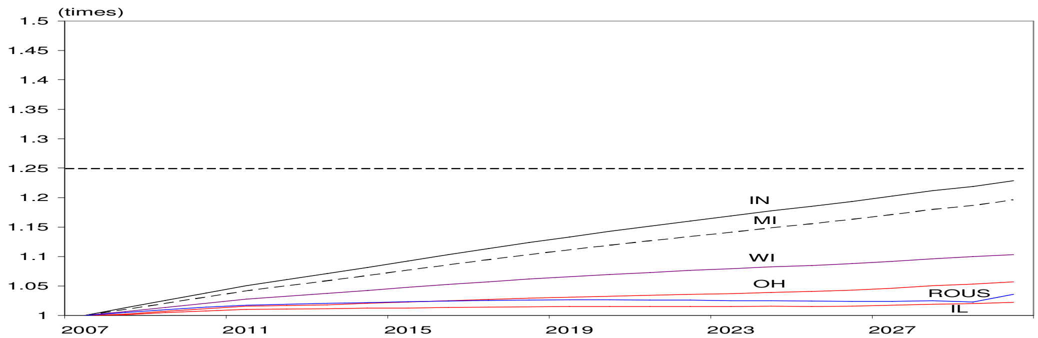


Figure 5: Growth of per-capita output in each region (2007=1)

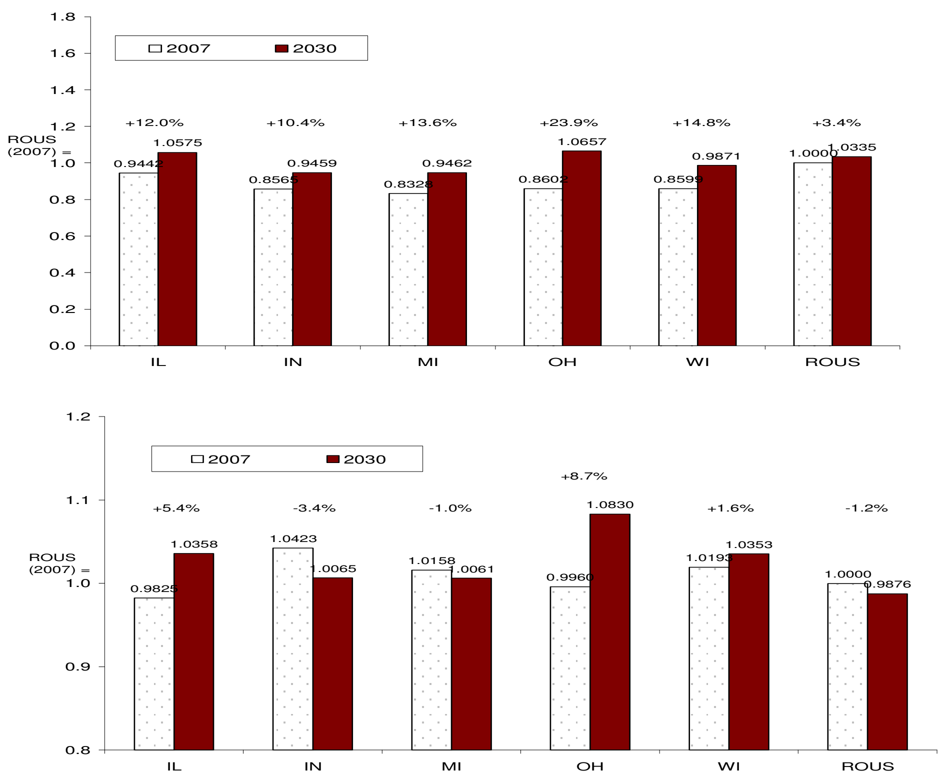


Figure 6: Change of wage rate (upper) and output price (lower) from 2007 thru 2030 from the dynamic simulation

**Appendix: Key variables from dynamic simulation results**

1 Per-capita output

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 7.6775 | 7.7364 | 7.7744 | 7.7934 | 7.7942 | 7.8491 | 1.0224 |
| IN | 5.7624 | 5.9814 | 6.2942 | 6.5838 | 6.8300 | 7.0798 | 1.2286 |
| MI | 5.3164 | 5.4831 | 5.7250 | 5.9501 | 6.1436 | 6.3597 | 1.1962 |
| OH | 6.1117 | 6.1815 | 6.2491 | 6.3104 | 6.3594 | 6.4586 | 1.0568 |
| WI | 5.9618 | 6.0851 | 6.2454 | 6.3742 | 6.4676 | 6.5759 | 1.1030 |
| ROUS | 7.9339 | 8.0432 | 8.1170 | 8.1429 | 8.1242 | 8.2189 | 1.0359 |

2 Fraction of time-spending in educational investment (average per worker)

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 0.1204 | 0.1205 | 0.1210 | 0.1218 | 0.1229 | 0.1233 | 1.0241 |
| IN | 0.0967 | 0.1007 | 0.1045 | 0.1069 | 0.1091 | 0.1104 | 1.1417 |
| MI | 0.0980 | 0.1015 | 0.1050 | 0.1073 | 0.1093 | 0.1105 | 1.1276 |
| OH | 0.1073 | 0.1089 | 0.1105 | 0.1117 | 0.1132 | 0.1139 | 1.0615 |
| WI | 0.1022 | 0.1049 | 0.1076 | 0.1090 | 0.1104 | 0.1110 | 1.0861 |
| ROUS | 0.1263 | 0.1262 | 0.1265 | 0.1269 | 0.1277 | 0.1281 | 1.0143 |

3 Human capital stock (average per worker)

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 2.2669 | 2.2971 | 2.3336 | 2.3830 | 2.4479 | 2.5250 | 1.1139 |
| IN | 1.7669 | 1.7347 | 1.7633 | 1.8481 | 1.9524 | 2.0597 | 1.1657 |
| MI | 1.7800 | 1.7530 | 1.7824 | 1.8641 | 1.9662 | 2.0736 | 1.1649 |
| OH | 1.9369 | 1.9342 | 1.9561 | 2.0110 | 2.0837 | 2.1659 | 1.1182 |
| WI | 1.8463 | 1.8300 | 1.8556 | 1.9237 | 2.0108 | 2.1039 | 1.1395 |
| ROUS | 2.3900 | 2.4465 | 2.5074 | 2.5709 | 2.6428 | 2.7207 | 1.1384 |

4 Rental return to physical capital

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 0.0932 | 0.0946 | 0.0941 | 0.0931 | 0.0931 | 0.0968 | 1.0386 |
| IN | 0.1301 | 0.1177 | 0.1058 | 0.0996 | 0.0965 | 0.0982 | 0.7548 |
| MI | 0.1222 | 0.1121 | 0.1022 | 0.0972 | 0.095 | 0.0976 | 0.7987 |
| OH | 0.0985 | 0.0968 | 0.0938 | 0.0924 | 0.0926 | 0.0979 | 0.9939 |
| WI | 0.1102 | 0.1039 | 0.0968 | 0.093 | 0.0917 | 0.0952 | 0.8639 |
| ROUS | 0.0955 | 0.0965 | 0.0948 | 0.0924 | 0.0909 | 0.0903 | 0.9455 |

5 Wage rate (before tax)

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 1.4393 | 1.4475 | 1.4796 | 1.5168 | 1.5477 | 1.612 | 1.1200 |
| IN | 1.3056 | 1.3643 | 1.4151 | 1.4248 | 1.4176 | 1.4419 | 1.1044 |
| MI | 1.2695 | 1.3234 | 1.3736 | 1.3944 | 1.4032 | 1.4423 | 1.1361 |
| OH | 1.3112 | 1.3485 | 1.4156 | 1.4747 | 1.5273 | 1.6244 | 1.2389 |
| WI | 1.3108 | 1.3475 | 1.3954 | 1.4276 | 1.4489 | 1.5046 | 1.1478 |
| ROUS | 1.5243 | 1.5138 | 1.5306 | 1.5556 | 1.5711 | 1.5754 | 1.0335 |

6 Output price

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 0.9632 | 0.9707 | 0.9797 | 0.9885 | 0.9991 | 1.0155 | 1.0543 |
| IN | 1.0219 | 1.0164 | 1.0060 | 0.9944 | 0.9864 | 0.9868 | 0.9657 |
| MI | 0.9959 | 0.9927 | 0.9859 | 0.9801 | 0.9787 | 0.9864 | 0.9905 |
| OH | 0.9765 | 0.9853 | 0.9995 | 1.0140 | 1.0320 | 1.0618 | 1.0874 |
| WI | 0.9993 | 0.9978 | 0.9963 | 0.9968 | 1.0011 | 1.0150 | 1.0157 |
| ROUS | 0.9804 | 0.9814 | 0.9820 | 0.9822 | 0.9832 | 0.9682 | 0.9876 |

7 Consumption price

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 0.9672 | 0.9731 | 0.9800 | 0.9867 | 0.9951 | 1.0053 | 1.0394 |
| IN | 1.0103 | 1.0070 | 1.0003 | 0.9926 | 0.9880 | 0.9878 | 0.9777 |
| MI | 0.9912 | 0.9895 | 0.9852 | 0.9815 | 0.9815 | 0.9864 | 0.9952 |
| OH | 0.9786 | 0.9851 | 0.9950 | 1.0050 | 1.0179 | 1.0358 | 1.0585 |
| WI | 0.9947 | 0.9940 | 0.9931 | 0.9936 | 0.9974 | 1.0060 | 1.0114 |
| ROUS | 0.9805 | 0.9815 | 0.9822 | 0.9825 | 0.9838 | 0.9695 | 0.9888 |

8 Investment price

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| IL | 0.9861 | 0.9916 | 0.9988 | 1.0060 | 1.0142 | 1.0434 | 1.0581 |
| IN | 1.0363 | 1.0307 | 1.0215 | 1.0114 | 1.0039 | 1.0196 | 0.9839 |
| MI | 1.0117 | 1.0087 | 1.0035 | 0.9993 | 0.9982 | 1.0188 | 1.0070 |
| OH | 0.9979 | 1.0035 | 1.0132 | 1.0234 | 1.0357 | 1.0723 | 1.0746 |
| WI | 1.0180 | 1.0157 | 1.0138 | 1.0140 | 1.0168 | 1.0439 | 1.0254 |
| ROUS | 1.0001 | 1.0002 | 1.0002 | 1.0002 | 1.0003 | 1.0009 | 1.0008 |

9 Social security tax rate

| | 2007 (A) | 2010 | 2015 | 2020 | 2025 | 2030 (B) | B/A |
|---|---|---|---|---|---|---|---|
| US | 0.1184 | 0.1195 | 0.1309 | 0.1457 | 0.1601 | 0.1657 | 1.3995 |