AN INTEGRATED FRAMEWORK OF THE MARKET AREAS AND SUPPLY AREAS: COMBINING MARKET-AREA SYSTEMS WITH SUPPLY-AREA SYSTEMS

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An Integrated Framework of the Market Areas and Supply Areas: Combining Market-Area Systems with Supply-Area Systems

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Abstract: There is a potential theoretical problem in the established framework of market-area and supply-area analysis when both approaches are investigated in an integrated framework with respect to firm location. It is argued that the mechanism of the problem is illustrated by insufficient inclusions of the notion of internal and external economies. The alternative model formulates the spatial duality theory, which enables us to internalize these economic elements into the existing theoretical framework. The impact of the additional elements is summarized by comparative-static methods, examining the spatial equilibrium of market-area and supply-area competitions.

1. Introduction

In location theory, market-area analysis examines how products are distributed in an economic plain. Market areas have been analyzed in terms of spatial allocation of output, with demand conditions, technology and factor prices given. This was systematically formalized by Lösch (1954), applying the basic idea of the relationship between economies of mass production and transportation costs. Supply areas have been investigated with respect to spatial competition of inputs with the given structures of assembly cost, technology and the demand conditions of output. This approach explores how individual firms obtain their inputs, such as raw materials and labor, in order to achieve the optimal levels of production under given spatial economic conditions. Supply areas were initially investigated in a systematic way by Lösch (1938), although he did not extend this further in his analysis. As the major concern of each type of area analysis is solely spatial competition and formation, these approaches all assume firms to be located at the center of an area. From the standpoint of a producer, every plant has supply areas to obtain inputs from suppliers, and market areas to distribute output. As both types of area analysis assume the plant to be at the center of the area, this producer must logically be located at the center of the market area and supply area. However, this hypothesis cannot be applied in general, and is particularly unsuitable for manufacturing firms. This logical problem is caused by the fragmented approach of established location analysis. In the existing analysis of market areas, the sequence of production process and production inputs has been treated as a given constant factor for the purpose of theoretical simplification. This causes certain difficulty of theoretical linkage to the corresponding structure of the supply area. Likewise, in supply-area analysis, the sequence of production process and the condition of spatial competition of output have been treated as exogenous economic factors. This unhooks the methodological connection to its related structure of the market area. As a result, it is necessary to re-examine these neglected economic factors; namely, the formation of the production function in terms of areal notions and spatially constrained internal and external economies so that it enables us to integrate market-area and supply-area analysis in an input-output framework.

In terms of this framework, Koopmans and Beckmann (1957) investigated the problem of assigning plants to locations, considering a system of rents and profit maximization in the linear assignment problem. In their analysis, supply and demand were integrated in their analysis by means of linking supply nodes with transportation nodes and consumption nodes. However, the spatial structure of areas was not taken into account in this approach. Alternatively, the spatial CGE (Computable General Equilibrium) models solve this issue by means of dispersed spatial price equilibrium methods. For instance, Roy (1995) develops dispersed spatial input demand functions based on multiregional input-output production functions and this framework enables us to investigate discrete spatial patterns of market distribution. However, the theoretical interactions between firm location, market areas and supply areas cannot be solved, as there has been little attention to the notion of spatially unconstrained and constrained internal and external economies. Our analysis will attempt to integrate market areas and supply areas based on a Löschian framework with respect to firm location and these economies, applying a simple input-output framework which is known as duality theory in conventional economic theory. In earlier part of this paper, an alternative model under the condition of spatial monopoly of final-good producer and single supplier will be developed. In later part, the model results will be applied to more general spatial organization, which includes the case of intermediate goods and spatial competition of the market area and supply area.

2. Underlying Bases of the Alternative Model Framework

In this section, the spatial equilibrium under the condition of spatial monopoly will be examined. The core economic elements at this stage are the factor cost, production and cost functions. The factor cost is generated from the supply-area framework, and is directly related to the inputs of a product. This factor cost defines a cost function that represents the total cost of output. In order to derive the cost function, the spatial production function needs to be substituted into factor cost. The spatial production function is the combination of the conventional production function and additional spatial economic factors. The derived cost function is mapped onto the relevant spatial demand conditions and the optimal market-area radius and quantity of output are obtained. By applying spatial duality theory, the optimal amounts of inputs and supply-area radius are also derived from the optimal market-area conditions.

The examination begins with a simple case. Let us assume that 2 units of input x are required in order to produce an output q, 4 units of x in order to produce $2q$, and 9 units of x in order to produce $3q$, and so on. There are no other relevant costs for this production apart from factor price w. In these circumstances, the production function $q = f(x)$ becomes $q = \sqrt{x}$ as commonly approximated, and the square root of the technical transformation exists for the production process according to the input-output ratio. Now suppose also that the factor cost curve $C(x)$ is expressed as:

$$C(x) = wx + F \tag{1}$$

where w = unit factor price, x = amount of input and F = fixed cost. As $q = \sqrt{x}$, this expression can be re-expressed as $x = q^2$. x can then be substituted into equation (1) so that total the cost $C(q)$ is derived from the following equation:

$$C(q) = wq^2 + F \tag{2}$$

where the average cost $AC(q)$ is derived from the above equation by dividing it by q:

$$AC(q) = \frac{C(q)}{q} = wq + \frac{F}{q} \tag{3}$$

Under the condition of perfect competition, the optimal output level q^* is determined at the point where the average cost reaches a minimum. The value q^* will be solved by taking derivatives of $AC(q)$:

$$\frac{\partial AC(q)}{\partial q} = w - \frac{F}{q^2} = 0 \tag{4}$$

$$q^* = \sqrt{\frac{F}{w}} \tag{5}$$

This is the optimal production scale to satisfy the requirements of cost minimization and the equilibrium level under the condition of perfect competition. In order to find the corresponding optimal amount of input x^*, the production function $q = \sqrt{x}$ is substituted into the above equation and a unique solution is found from conditions of $w > 0$ and $F > 0$:

$$x^* = \frac{F}{w} \tag{6}$$

For a situation of monopoly, the optimal input level is not derived in a straightforward manner, and market demand conditions are required to be taken into account. Using the general assumption in conventional economic analysis, the average revenue curve AR is introduced as:

$$AR = a - bq \qquad (a, b > 0) \qquad (7)$$

Here, a is a positive constant value and b is a slope of this curve. Total revenue, TR, is the product of average revenue, AR and output level, q:

$$TR = AR \cdot q = (a - bq)q \qquad (8)$$

Marginal revenue, MR, is a partial derivative of total revenue, TR, with respect to q:

$$MR = \frac{\partial TR}{\partial q} = a - 2bq \qquad (9)$$

From equation (2), in similar fashion, marginal cost, MC, is a partial derivative of the cost function, $C(q)$, with respect to, q:

$$MC = \frac{\partial C(q)}{\partial q} = 2wq \qquad (10)$$

Under the condition of monopoly, the optimal output level, q_M^*, is a point at which marginal revenue, MR, equals marginal cost MC. Using equations (9) and (10),

$$(MR =)\ a - 2bq = 2wq \quad (= MC)$$

$$q_M^* = \frac{a}{2(w+b)} \qquad (11)$$

Substituting the production function, $q = \sqrt{x}$, into the above equation, the optimal input level x_M^* is specified as:

$$x_M^* = \left[\frac{a}{2(w+b)}\right]^2 \qquad (12)$$

From equations (11) and (12), it becomes clear that both optimal input and output levels are determined by parameters a, b and factor price w under the quadratic form of the production function. These results can be summarized as follows.

$$\frac{\partial q_M^*}{\partial a} > 0, \; \frac{\partial q_M^*}{\partial w} \text{ and } \frac{\partial q_M^*}{\partial b} < 0 \tag{13}$$

$$\frac{\partial x_M^*}{\partial a} > 0, \; \frac{\partial x_M^*}{\partial w} \text{ and } \frac{\partial x_M^*}{\partial b} < 0 \tag{14}$$

The above expressions indicate that the fixed cost has no effect on the derivation of equilibrium under the condition of monopoly, while equilibrium is expressed by the ratio of fixed cost and variable factor price under the condition of free-entry competition. Thus, under the condition of monopoly, it is clear that the corresponding input level is determined by the index of the technological transformation, factor price, the intercept of the vertical axis and the slope of the market demand curve.

3. Spatial Duality Theory and Additional Elements

The analysis will now be applied to the alternative spatial duality theory. The above investigation examines the relationship between the input and output of a product by applying duality theory. However, spatial aspects have not been included in the analysis and the approach will now refer to Lösch (1954) in order to introduce spatial economic interpretations. The derivation process of the relationship between quantity of output and market-area radius is illustrated from the $f.o.b.$ distribution freight rate and the individual conventional demand curve. This process enables not only the maximum market-area radius $U(p_1)$ under the price level p_1 to be found, but also the optimal market-area radius $u^*(p_1)$ under price p_1 to be specified once the individual conventional demand curve is replaced by the aggregate spatial demand curve. The individual conventional demand curve can be converted into the aggregate conventional demand curve by the horizontal summation of the individual demand curve, if all consumers have identical demand curves. In location analysis, by contrast, the conversion into the aggregate spatial demand curve cannot be achieved in such a straightforward manner, as not all consumers locate at the same site. However, the individual spatial demand curve will be identical, if consumption reveals homothetic preferences for commodities: the quantity demanded at any location, given a market price, will depend on distribution costs. Applying this method, the aggregate spatial demand curve can be derived from the horizontal summation of all individual spatial demand curves.

We will now solve spatial problems. Let us assume that an individual firm produces an output q which requires an input x and certain types of technology for processing. The transportation cost for distribution is expressed through a combination of market area radius u and the $f.o.b.$ transportation rate t. The individual consumer demand q_F is expressed as:

$$q_F = a - b(p + tu) \tag{15}$$

If the market area has a regular shape, the total sales, Q, are expressed with the maximum radius U and density of demand D as introduced by Mills and Lav (1964). For reasons of simplicity, our analysis applies a simplified circular market-area case. Following a generalized formulation in Mills and Lav, the total sales, Q, for a circular market area, we have:

$$Q = D\int_0^{2\pi}\int_0^U \left[(a - bp - btu)udud\theta\right] = D\pi U^2\left(a - bp - \frac{2}{3}btU\right) \tag{16}$$

As the symbol U expresses the maximum radius of the market area, consumer demand q_F in equation (15) becomes zero at U and price p is specified as follows:

$$a - bp - btU = 0$$

$$p = \frac{a}{b} - tU \tag{17}$$

In order to find total sales Q, substitute (17) into (16) to obtain:

$$Q = D\pi U^2\left(a - b\left(\frac{a}{b} - tU\right) - \frac{2}{3}btU\right) = \frac{1}{3}Dbt\pi U^3 \tag{18}$$

Total revenue TR is defined by $p \cdot Q$:

$$TR = p \cdot Q = \left(\frac{a}{b} - tU\right)\left(\frac{1}{3}Dbt\pi U^3\right) = \frac{1}{3}Dt\pi U^3(a - btU) \tag{19}$$

Marginal revenue MR is solved by the above equation of the partial derivative with respect to U:

$$MR = \frac{\partial TR}{\partial U} = \frac{1}{3}Dt\pi U^2(3a - 4btU) \tag{20}$$

For solving the spatial monopoly equilibrium, it is necessary to derive the marginal cost MC. In order to find the marginal cost MC, a spatial cost function, which is obtained from the spatial factor cost and production function, must be defined. In order to revise the structure of production function in a spatial context, the relationship between production and cost functions should initially be examined. This relationship is systematically analyzed by duality theory in Shepard (1953). Duality theory shows the following theoretical interactions: that the input is a function of its relevant production function and that the production function is a function of the cost function. As a result, input x is a function of total cost $C(q)$ through production function $q = f(x)$ and is expressed as follows:

$$C = C(q) \tag{21}$$

$$q = f(x) \tag{22}$$

As a result,

$$C = f(q, x) \tag{23}$$

In the analysis of market areas, the cost function becomes a function of the maximum market-area radius U, which is a function of market-area radius u. In addition, input x is a function of the supply-area radius s. By combining them with the form of the conventional production function, these relations are expressed in an integrated form:

$$C = f(U, u, q, x, s) \tag{24}$$

However, one argument has been left out in established spatial equilibrium analysis. The conventional production function solely refers to the internal dimensions of the firm. In addition, spatially unconstrained or constrained cases are not distinguished from each other.

Meade (1952) examines the inclusion of external economies in the conventional production function for a case in which there are two indirectly related economic organizations. The example he provides concerns apple-farmers who have their apples fertilized by bees, and beekeepers who are provided with food for the bees in the apple farm. The results show that the alternative production function contains not only functions of inputs for a single firm, but also functions of inputs and quantities of products relating to other firms. He argues that external economies are not included in the conventional production function, and introduces the following alternative production function between two indirectly related firms:

$$q_i = f_i(L_i, K_i, L_j, K_j, q_j) \qquad (i \neq j) \tag{25}$$

where f_i is not necessarily homogeneous in the first degree. This production function shows that a quantity of production is specified not only by the inputs and technical factors of a single firm, but also by the inputs, output levels and technical factors of other firms if external economies are present. For a single input case, expression (25) can be expressed more simply:

$$q_i = f_i(x_i, x_j, q_j) \qquad (i \neq j) \tag{26}$$

In our analysis, there are more than two firms. Moreover, the external economies are not brought solely by particular indirectly related firms, as examined in the case of the two specific firms mentioned in Meade (1952). Thus, x_j and q_j cannot be stated in a generalized form. As a result, the external economies will be simply expressed as A in this analysis:

$$q_i = f_i(x_i, A) \tag{27}$$

From this expression, it can be stated that spatially unconstrained and constrained external economies are not included in the notion of the conventional production function. In addition, these economies should be added in between market-area and supply-area analysis as the conventional production function is situated between the framework of input and output of processing. The necessity for including these economies is that the core focus of location theory is on the effect on a firm of the interaction from the economic activity of other firms and industries. This notion of externality cannot be removed from market-area and supply-area analysis, although these approaches have been excluding it for reasons of simplicity. As a result, the spatially unconstrained and constrained external economies A should be included in the integrated expression (24):

$$C = f(U, u, q, A, x, s) \tag{28}$$

Although the above system contains all of the relevant spatial economic factors within a single framework, this examination will initially suggest a bisected production function analysis. One part of the production function will be the conventional one while the other has an extended form.

The input-output relation in spatial analysis between output q and input x of a producer will assume that $q = f(x, A)$ and this can be expressed in a bisected way:

$$q = f(x, A) = \left[f^{con}(x), f^{ext}(x) \right] \tag{29}$$

where $q = f^{con}(x)$ represents the conventional production function and $q = f^{ext}(x)$ shows the extended production function. As this analysis examines an individual firm, agglomeration economies may not directly be contained in the relevant cost structure. However, the following interpretation should be considered. Let us assume that this firm produces beer in a market which it enjoys a monopoly. This firm would not normally have any agglomeration economies. However, it is possible to consider a case in which there are some other industries, such as the wine, whisky or soft drinks industry, with which they share bottles and storehouses within a region. In this case, the argument can be expanded to suggest that the analysis of a single firm can consider the relationship between its own operation and the relevant economies which are obtained from beyond their economic activity. In this way, certain types of localization economies, urbanization economies, activity-complex economies, or spatially constrained internal economies, can be observed in a single-firm investigation.

In order to combine the conventional production function and the extended production function, let us assume that these production functions have the following particular shapes, as an example:

$$\text{for } q = f^{con}(x): q = x^{0.45} \tag{30}$$

$$\text{for } q = f^{ext}(x): \quad q = x^{0.55} \tag{31}$$

The alternative production function $q = f(x)$ will be formed as:

$$[q = f(x)] = [q = f^{con}(x), q = f^{ext}(x)] \tag{32}$$

Equations (30) and (31) can be combined into the formulation (32) as follows:

$$[q = f(x)] = x^{\frac{0.45+0.55}{2}} = x^{0.5} \tag{33}$$

As a result, this can be generalized using coefficients ρ $(0 < \rho < 1)$ for $q = f^{con}(x)$ and ω $(0 < \omega < 1)$ for $q = f^{ext}(x)$ as:

$$[q = f(x)] = x^{\frac{\rho+\omega}{2}} \tag{34}$$

As examined in Solow (1955-1956), this can be easily shown, in particular, if the condition of Cobb-Douglas function is assumed. Spatial equilibrium of market areas and supply areas will now be examined using the formulation of the spatial production function (34). It is possible to draw this process as shown in figure 1. The curve $q = f^{con}(x)$ shows the conventional production function and this curve cannot have a locus beyond the 45° additional line by the general laws of economics, although the position of the production function is usually not explicitly stated in relevant literature. By contrast, the extended production function $q = f^{ext}(x)$ can be located above the 45° additional line in some areas. These areas represent the positive benefit of the external economies. Since these two elements are both situated between market area radius u and supply area radius s, these can be added vertically as the spatial production function $q = f(x)$.

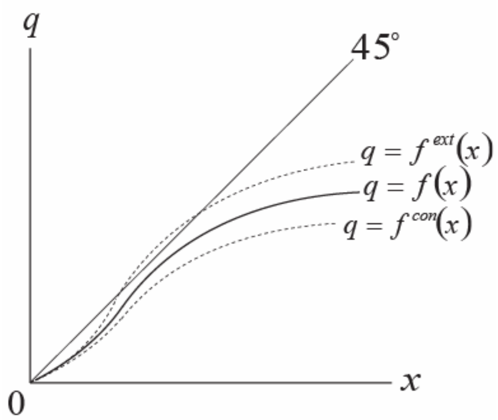

Figure 1. Derivation of the spatial production function

Under the full consideration of internal and external economies, the spatial production function $q = f(x)$ is derived by this procedure and can be stated as $q = (1/k)x^{\varphi}$, where φ $(0 < \varphi \le 1)$ denotes $(\rho + \omega)/2$ and k $(0 < k \le 1)$ represents an additional coefficient as a technological transformation from input into output. However, it should be noted that this is a technological type of economies and there is another part referred to as the pecuniary type, according to Meade (1952) and Scitovsky (1954). This latter type of economy can be contained in a part of the spatial factor cost as will now be shown.

4. Spatial Production Function and the Input-Output Framework

In order to transform duality theory into location theory with internal and external economies, the relationship between input x and output q must be reconsidered. For inputs, it is necessary to show how the alternative factor cost curve is formed in spatial analysis. First, the total assembly cost $C(x)$ can be expressed in an extended version of conventional factor cost (1):

$$C(x) = (1+\tau-\varepsilon)wx + (F + F_\tau - F_\varepsilon) \tag{35}$$

This extended equation is developed by adding four additional elements, τ, F_τ, ε and F_ε. The former two elements are assembly transportation rate and assembly transportation terminal cost. The latter two are explanatory variables of economies which cannot be fitted within the framework of the production function. The element ε represents this additional variable-cost factor and F_ε shows an additional fixed-cost factor. It is assumed that these factors contain transactions cost, communication cost and other relevant explanatory variables of the non-technological part of economies. These additional elements are related to the supply area of this producer. The variable factor ε is multiplied by distance s, while fixed factor F_ε does not rely on the amount of inputs and is kept constant. In our analysis, the notion of supply area is assumed that inputs are collected radially to the location of production. In other words, producer's necessary amount of inputs is converted into radius by means of the equation of a circle. In this way, the relationship between input x and supply area radius s can be expressed as:

$$x = \phi\pi s^2 \tag{36}$$

where ϕ is a constant. The relationship between factor cost and supply-area radius becomes:

$$C(s) = (1+\tau-\varepsilon)w\phi\pi s^2 + (F + F_\tau - F_\varepsilon) = \lambda s^2 + FC \tag{37}$$

where $\lambda(>0) = (1+\tau-\varepsilon) w\phi\pi$ and $FC(>0) = F + F_\tau - F_\varepsilon$. As a result, the above relationship can be illustrated in figure 2.

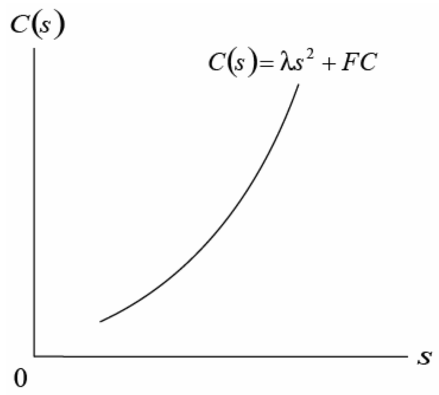

Figure 2. Factor cost curve and supply-area radius

This is a case where there is no spatial competition in the supply area and the supplier is in a spatial monopoly condition. If a number of suppliers appear in the long run, unused economic plain will be filled by new entrants and eventually spatial structure forms truncated circular or hexagonal shapes. In these situations, the curve in figure 2 becomes steeper due to the competition of the supply area.

For output, it is necessary to show how to convert the quantity of output into a market-area radius in spatial duality analysis. The relationship between the quantity of output q and circular market-area radius u can be expressed as:

$$u = \sqrt{\frac{q}{\mu\pi}} \tag{38}$$

This can be illustrated in figure 3.

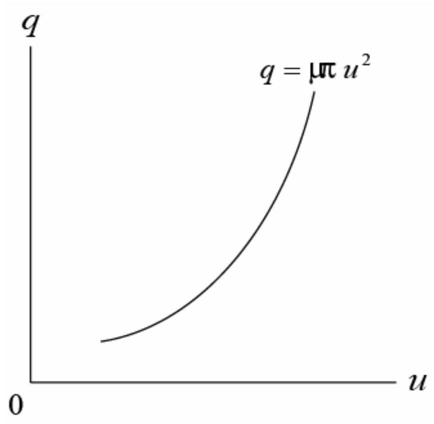

Figure 3. Market-area radius u and quantity of output q

By combining these interpretations with the spatial production function, an integrated framework of spatial analysis can be demonstrated in figure 4. In *Phase* (I) of the above figure, the spatial cost function $C(U)$ is derived from the spatial input-factor cost curve $C(x,s)$ in *Phase* (II) through *Phases* (III) and (IV). *Phase* (III) represents the spatial production function which is derived earlier in figure 1. *Phase* (IV) illustrates the relationship between market area radius u and quantity of output q as demonstrated in figure 3. The relevant spatial demand curve can also be added in *Phase* (I) in this figure. The spatial demand curve AR determines the marginal revenue curve MR. As examined earlier, spatial monopoly equilibrium is achieved at the point at which marginal revenue MR equals marginal cost MC. Marginal cost MC is derived from the spatial cost function $C(U)$, and average cost AC is also derived from this spatial cost function. The spatial equilibrium market price p_M under these conditions is where the spatial demand curve AR and the relevant average cost curve AC connect with each other. Moreover, this market-area radius satisfies $MR = MC$ and this is the optimal market-area radius u_M^*. Applying the integrated framework of market-area and supply-area analysis, the optimal amount of input x_M^* is derived through the spatial production function.

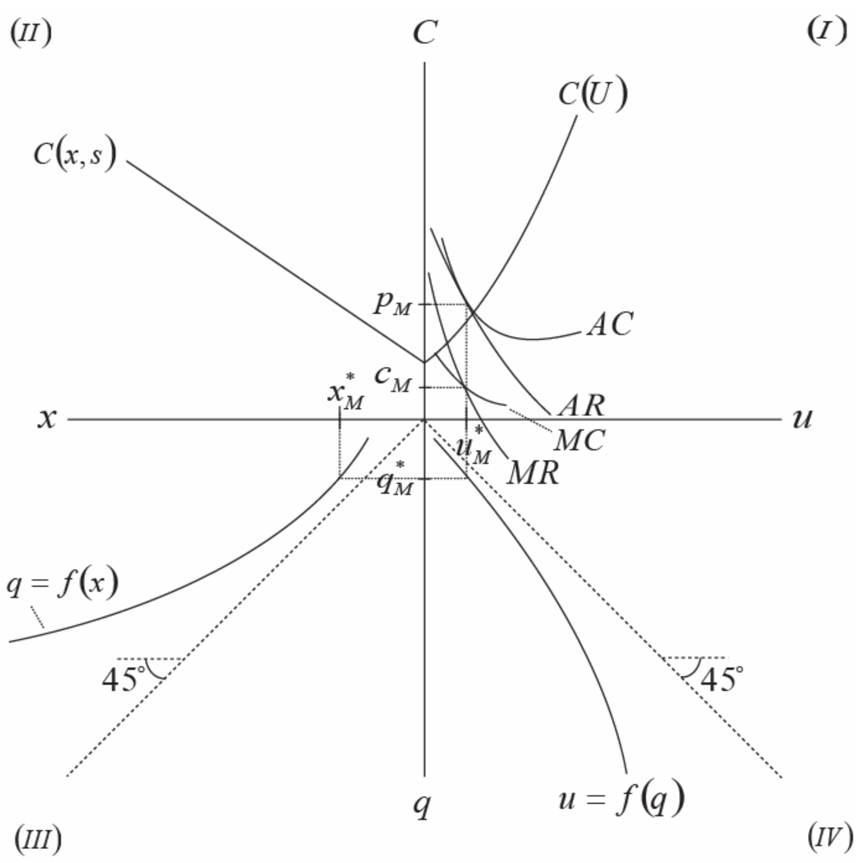

Figure 4. Integrated framework spatial analysis

In terms of agglomeration economies and spatially unconstrained internal and external economies, it is possible to observe the effect of these economies - with respect to the pecuniary type which appears in *Phase* (II) and the technological type which appears in *Phase* (III) - on the required amount of inputs for profit maximization and cost minimization within the firm in market-area and supply-area analysis. It can be projected that the firm requires less inputs and supply areas if either type of these economies is more readily available. By contrast, the firm requires more inputs and larger supply areas if either type of economy is less readily available. The relevant market area is observed under the condition of these economic factors and the given spatial demand curve. Regarding figure 3 that was interpreted earlier, the spatial production function is expressed as:

$$q = \frac{1}{k} x^{\varphi} \qquad (0 < \varphi \leq 1) \tag{39}$$

As commonly approximated, let us assume that $\varphi = 0.5$ and substitute the above equation which is solved with respect to x into equation (35):

$$C(q) = (1+\tau-\varepsilon)w(kq)^2 + (F + F_\tau - F_\varepsilon) \tag{40}$$

Applying the expression (38) to the above equation:

$$C(u) = (1+\tau-\varepsilon)wk^2\mu^2\pi^2u^4 + (F + F_\tau - F_\varepsilon) \tag{41}$$

Marginal cost MC is a partial derivative of the above equation with respect to u:

$$MC(u) = \frac{\partial C(u)}{\partial u} = 4u^3(1+\tau-\varepsilon)wk^2\mu^2\pi^2 \tag{42}$$

As demonstrated earlier, the results can be shown as follows:

$$u^* = \frac{3at}{4\left(bt^2 + 3k^2\mu^2\pi(1+\tau-\varepsilon)w\right)} \tag{43}$$

$$q^* = \frac{9a^2\mu\pi t^2}{16\left(bt^2 + 3k^2\mu^2\pi(1+\tau-\varepsilon)w\right)^2} \tag{44}$$

$$x^* = \frac{81a^4\mu^2\pi^2t^4}{256\left(bt^2 + 3k^2\mu^2\pi(1+\tau-\varepsilon)w\right)^4} \tag{45}$$

In addition, the optimal supply-area radius s^* can also be derived from the combination of the above expressions and equation (36):

$$s^* = 0.5625\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2 + 3k^2\mu^2\pi(1+\tau-\varepsilon)w\right)^4}\right)^{\frac{1}{2}} \tag{46}$$

In comparing the above results with the previously demonstrated aspatial results, which exclude the notion of spatial production function and external economies, it becomes clear that the index of pecuniary type of economies ε will have certain effects on the determination of the optimal quantity of output q^*, market-area radius u^* and amount of input x^*. The other additional spatial factor F_ε, by contrast, has no impact within the framework of the comparative-static method. The impact of the technological type of economies can also be observed as a variable k and as some of coefficients in the above equations.

5. Summary of Alternative Outcomes

At this stage of the analysis, it is possible to examine the relationship between market areas and supply areas through changes in particular relevant variables. Five significant cases can be shown from figure 4, two cases in *Phase* (II) and one case each in *Phases* (I), (III) and (IV).

Changes in Spatial Demand Conditions

This case is observed in the change of the spatial demand curve in *Phase* (I) in figure 4, which would occur when there is an increase or a decrease in population. The enlargement of the demand curve increases the optimal market-area radius and the optimal quantity of outputs in the short run, as the marginal cost curve cannot change its shape. The increase of the output level expands the amount of input and the relevant supply area. In the long run, furthermore, the cost curves can be moved to adjust to the modified demand curve, until the average cost curve touches the demand curve under the condition of spatial free-entry competition, as shown in Lösch (1954).

Changes in Assembly Transportation Rate and the Explanatory Variable-Cost Factor

This case shows a slope change of spatial factor cost curve in *Phase* (II) of the figure, which might be caused by exogenously generated improvement in the transportation network lowers transportation rate τ. This increases or decreases not only the formation of the spatial factor cost curve itself, but also the structure of spatial cost function in *Phase* (I) through *Phases* (III) and (IV). An increase or decrease in the spatial cost function changes the formation of marginal cost. As a result, the increase of assembly transportation rate, τ, or the decrease of explanatory variable-cost factor, ε, changes the shape of the marginal cost curve and reduces the size of the optimal market-area radius. This eventually reduces the size of the supply area in *Phase* (II) of the figure through *Phases* (IV) and (III).

Changes in Fixed Cost, Terminal Cost and the Explanatory Fixed-Cost Factor

In this case, the height of the spatial factor cost curve in *Phase* (II) changes in a parallel movement, which may occur when facilities for processing, distribution or exogenous public utilities are changed. An increase of fixed cost F, terminal cost F_τ or an decrease of explanatory fixed-cost factor F_ε not only changes the height of the spatial factor cost but also increases the level of the spatial cost function through *Phases* (III) and (IV). This increases the height of the marginal cost curve, and the optimal-market radius will be reduced. In addition, these changes eventually reduce the size of the supply area through *Phases* (IV) and (III) of figure 4.

Changes in Spatial Production Function

This is where the slope of the spatial production function is increased or decreased by the availability of more advanced processing technologies. The former case achieves lower spatial cost function $C(U)$ in *Phase* (I), lower average cost AC and marginal cost MC. As a result, the optimal market-area radius increases, but the size of the supply area is not necessarily increased. This can be achieved by a technological improvement. In the opposite case, a decreased level of technology changes the shape of the spatial production function and increases the spatial cost function $C(U)$ in *Phase* (I) through *Phase* (IV). This causes a reduction of the optimal market-area radius and the optimal quantity of output is reduced. Despite the reduction of the output level, the relevant amount of input and the supply area may increase in this case as the technology level requires more inputs than the previous level.

Changes in Shapes of the Market Area

This is a case in which the spatial configuration of the market area is changed, which might be caused by a change in the system of spatial structure. It affects the shape of the market-area spatial configuration curve in *Phase* (IV) of the figure. Figure 4 represents a circular case. The regular hexagonal case will be closer to the vertical q axis as the output level increases, and the truncated circular case is situated between these two cases. These shifts affect the structure of the spatial cost functions $C(U)$ in *Phase* (I) and the optimal market-area radius will be changed according to the condition of spatial competition. Furthermore, the optimal size of the supply area is also modified through changes in the optimal quantity of output and the amount of input.

6. Comparative Statics and Geometric Interpretations

We now demonstrate comparative-static analysis according to the results which were obtained earlier. We can observe the impact of a change in factor price w, distribution transportation rate t, assembly transportation rate τ, index of pecuniary type of economies ε, index of technological transformation k and index of spatial transformation μ on the optimal market-area radius u^*, quantity of output q^*, amount of input x^* and supply-area radius s^*. The impact of changes in the above stated variables, on the optimal market-area radius u^* is shown as follows.

$$\frac{\partial u^*}{\partial w}, \frac{\partial u^*}{\partial t}, \frac{\partial u^*}{\partial \tau}, \frac{\partial u^*}{\partial k}, \frac{\partial u^*}{\partial \mu} < 0 \text{ and } \frac{\partial u^*}{\partial \varepsilon} > 0 \quad (47)$$

The impact of a change in each variable on the optimal quantity of output q^* becomes:

$$\frac{\partial q^*}{\partial w}, \frac{\partial q^*}{\partial t}, \frac{\partial q^*}{\partial \tau}, \frac{\partial q^*}{\partial k}, \frac{\partial q^*}{\partial \mu} < 0 \text{ and } \frac{\partial q^*}{\partial \varepsilon} > 0 \tag{48}$$

The impact of a change in each variable on the optimal amount of input x^* is shown as follows:

$$\frac{\partial x^*}{\partial w}, \frac{\partial x^*}{\partial t}, \frac{\partial x^*}{\partial \tau}, \frac{\partial x^*}{\partial k}, \frac{\partial x^*}{\partial \mu} < 0 \text{ and } \frac{\partial x^*}{\partial \varepsilon} > 0 \tag{49}$$

The impact of a change in each variable on the optimal supply-area radius s^* becomes:

$$\frac{\partial s^*}{\partial w}, \frac{\partial s^*}{\partial t}, \frac{\partial s^*}{\partial \tau}, \frac{\partial s^*}{\partial \mu} < 0, \ \frac{\partial x^*}{\partial \varepsilon} > 0 \text{ and } \frac{\partial s^*}{\partial k} \gtreqless 0 \tag{50}$$

By the nature of comparative statics, the effects of changes in fixed costs on the optimal sizes of market area, output level, quantity of input and supply area cannot be observed from these results. In addition, an impact of change in technological transformation k on supply-area radius in equation (50) cannot be specified. However, it is not needed to determine the sign, as a change in the size of supply area is also affected by the extent of available economies. More details of these comparative-static results are provided in Appendix of the paper.

Although the comparative-static results enable us to observe the impact of change in individual spatial economic variable, this has a theoretical limitation on the analysis of firm location. As examined earlier in this paper, the impact of change in firm location will involve additional elements of spatially constrained internal and external economies. It is possible to reveal these impacts on the cost function in comparative statics. However, as long as transportation costs for distribution exist, the spatial consumer demand curve will be changed by the movement of firm location. It is assumed that comparative-static analysis cannot change multiply variables at the same time and this is one of the limitations of the static approach. In this circumstance, geometric interpretations can be suggested. Producers change their location of production when the alternative location achieves either less total cost or more total revenue. In general, if a producer chooses more cost advantageous firm location, this can mean that the plant is not locating at the center of the market area in order to avoid unnecessary diseconomies of agglomeration, for instance. This will cause an increase in distribution transportation cost and certain levels of revenue will be reduced through a change of the shape in spatial demand curve. By contrast, if the producer moves his location where the total revenue is maximized, it might cause reductions in the opportunities of spatially constrained internal and external economies. This notion can be referred to the trade-off interaction between agglomeration economies and transportation costs, which was initially introduced by Weber (1909). The increase or decrease in assembly and distribution transportation costs can be found within the series of spatial duality system. However, the increase or decrease levels of agglomeration economies cannot be specified, as these economies have three different dimensions in terms of scale, scope and complexity. In order to measure these economic impacts, it is necessary to find the aggregate levels of agglomeration economies. As demonstrated earlier, this paper categorizes these tripartite economies as pecuniary and technological types of economies for reasons of simplicity.

7. Further Aspects: The Effects of Market-Area Changes on the Spatial Structure of Supply Area and Vice Versa

A Change in the Number of Competitors

In the previous section, we examine the spatial monopoly case. If the economic plain of the monopolist has an increase in population due to migration in the long run, this may change the case in the nature of competition. In this circumstance, the number of competitor becomes important economic factors. As the size of the market increases, free entry will allow additional competitors to enter and they will all locate at the center of the market, unless there are spatially constrained internal and external economies, which affect firm location. Theoretically, they will locate at the center of the market unless there are spatially constrained internal and external economies, which affect firm location. A change in the number of competitors in a market area affects the shape of the spatial demand curve through a change in the conditions of market competition. An increase in the number of competitors in the market area results in a more restricted capacity of production levels for every individual firm if other economic conditions are assumed to remain constant. As a result, the number of relevant supply areas will be reduced. In this case, economies of large-quantity production and economies of scale are reduced and market price may increase, due to the cost increase, if the relevant average cost curve is downward sloping. For the reverse case, where the number of competitors supplying inputs increases, this may bring a decrease in the factor price due to market competition between suppliers. The decreased factor price also decreases the spatial cost function. This eventually increases the size of market areas if other economic conditions are assumed to remain constant.

A Change in Shapes

A change in the shape of market areas affects the formation of the spatial cost function as examined in the previous section. Regarding a change in the shape of supply areas, this may affect the structure of spatial factor cost. It should be noted that the following point is the most important difference between market areas and supply areas; while the shape of market areas concerns the maximization of revenue, the shape of supply areas concerns more the minimization of costs than the maximization of revenue. This can be illustrated by the fact that the circular shape of a supply area forms lower spatial cost function levels than the hexagonal supply area, which maximizes the level of total revenue of a producer. The truncated-circular case is situated between these two types.

A Change in Differentiated Product or Input Pattern

A change in the condition of the differentiated product affects the conditions of the spatial demand curve and the examination will follow spatial competition under the condition of product differentiation. In this case, the shape of the marginal revenue curve will be adjusted to the given marginal cost levels. This also modifies the size of the supply area through changes in output levels and relevant amount of input. By contrast, the presence of differentiated inputs solely affects the level of factor price. This will change the spatial factor cost level and the alternative marginal cost will be adjusted to the given marginal revenue. This determines the optimal market-area radius and the relevant supply-area size is also specified observing the optimal output and input levels.

A Change in External Trade Pattern

A change in the external trade pattern in the market area affects the structure of the $f.o.b.$ pricing system. An increase in external trade opportunities will increase the $f.o.b.$ price and this will change the marginal revenue level. As a result, the optimal market-area radius becomes smaller and the relevant size of the supply area also decreases through the reduction of optimal output levels and relevant amount of input. By contrast, a change in the external trade pattern in the supply area affects the structure of the spatial factor cost. An increase in external trade opportunities increases not only spatial factor cost, but also the level of the spatial cost function. This reduces the optimal market-area radius and eventually the size of the supply area becomes smaller through the reduction of output levels and relevant amount of input. In this case, it is important to specify the condition of output; whether firms are producing intermediate or final goods. If the product is intermediate goods, the intra-state or inter-state exchange spatial organization can be taken into account to the analysis. The issue of spatial substitution can be referred to the concept of hollowing out and the notion of fragmentation of production. As investigated in Parr *et al.,* (2002), the dramatic decrease in transportation costs affect certain influence of the change in trade patterns, in addition to decrease in communication and coordination costs. Jones and Kierzkowski (2005) introduce the notion of fragmentation of production to the theory of international trade with respect to economies of scale and agglomeration. They indicate that the interactions between intermediate goods and their production locations can be analyzed by the integrated framework of input and output, which are connected by the production function with the notion of Ethier (1982). In addition, it is also important to consider whether the area of distribution of products is to local or external. This is related to the notion of the market demand. According to Armington (1969), the growth of demand for a product is affected not only by an income effect, own-price effect but also by the effect of closely related products and of all other prices. In other words, consumers are assumed to have an ordered preference function for goods that is met from local or nonlocal markets based on price and a factor that assumes that similar goods produced in two locations are imperfect substitutes.

8. Concluding Comments

The comparative-static analysis is applied to the sizes of market areas and supply areas, output levels and relevant amount of input with respect to factor price, assembly and distribution transportation rates, and indices of internal and external economies. The results are basically consistent with the approaches of conventional aspatial economic conditions in duality theory, where the market area and supply area have certain relationships through the structure of internal and external economies in spatial production function and factor cost. Although the densities of demand and inputs are assumed to be constant for reasons of simplicity, it may be possible to observe these spatial factors as dependent variables in order to examine more general spatial structures. This analysis also may provide evidence showing the extent of the importance of the additional location factors, with respect to the spatial constraints and spatial enhancement forces of economies. However, it should be noted that some hypothetical scenarios require dynamic analysis between upstream and downstream linkages or between earlier and later stages of processing, which are beyond the scope of this paper. To speculate on the possible relevance and applications of this approach, the following extensions may be suggested. First, spatial extensions of Meade (1952) and Scitovsky (1954) enable us to apply the Cournot duopoly model (Cournot, 1838), which states that one's profit relies on not only one's own quantity of output but also another's quantity of output. If they choose the reasonable strategy for both firms by observing reaction functions of the other firm, a bargaining solution which is evolved in Nash (1950; 1953) should be taken into account. In addition, product differentiated spatial duopoly cases can be examined on the framework of the Cournot-Nash and Bertrand-Nash models between two firms or on the framework of multi-stage Stackelberg quantity-leadership game in oligopoly cases. Second, the decision-making mechanism between upstream and downstream linkages of firms and establishments can be analyzed by observing the negotiation process and dominant strategies in the theory of firm contracts. By the inclusions of multiple stages of production, the application of the separable production function in Leontief (1947) should also be considered. Hummels *et al*., (1995) examines an empirical study of the vertical specialization in terms of international trade and demonstrates an industry-level quantitative data analysis with input-output tables. These approaches are other possibilities to expand the established framework of location analysis, if the system of area is applied in a certain manner. Finally, numeric examples might also help to reveal the theoretical relevance between market areas, supply areas and firm location, which comparative-static method cannot deal with further extensions due to its restriction of the observation of the non-multiple variable.

Appendix

Partial derivatives of the optimal market-area radius u^**:*

$$\frac{\partial u^*}{\partial w} = -\frac{9ak^2\mu^2\pi t\left(1+\tau-\varepsilon\right)}{4\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} < 0 \tag{51}$$

$$\frac{\partial u^*}{\partial t} = -\frac{3abt^2}{2\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} + \frac{3a}{4\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)} \neq 0 \tag{52}$$

$$\frac{\partial u^*}{\partial \tau} = -\frac{9ak^2\mu^2\pi tw}{4\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} < 0 \tag{53}$$

$$\frac{\partial u^*}{\partial \varepsilon} = \frac{9ak^2\mu^2\pi tw}{4\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} > 0 \tag{54}$$

$$\frac{\partial u^*}{\partial k} = -\frac{9ak\mu^2\pi t\left(1+\tau-\varepsilon\right)w}{2\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} < 0 \tag{55}$$

$$\frac{\partial u^*}{\partial \mu} = -\frac{9ak^2\mu\pi t\left(1+\tau-\varepsilon\right)w}{2\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} < 0 \tag{56}$$

In the above results, the impact of a change in distribution transportation rate t has an indefinite sign (either $\partial u^* / \partial t > 0$ or $\partial u^* / \partial t < 0$). As previously examined in this analysis, this must be expressed as $\partial u^* / \partial t < 0$. Thus, the following additional sufficient condition will be provided:

$$\frac{3abt^2}{2\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} > \frac{3a}{4\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)} \tag{57}$$

Partial derivatives of the optimal output level q^**:*

$$\frac{\partial q^*}{\partial w} = -\frac{27a^2k^2\mu^3\pi^2t^2\left(1+\tau-\varepsilon\right)}{8\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} < 0 \tag{58}$$

$$\frac{\partial q^*}{\partial t} = -\frac{9a^2 b\mu\pi t^3}{4\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} + \frac{9a^2\mu\pi t}{8\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} \neq 0 \tag{59}$$

$$\frac{\partial q^*}{\partial \tau} = -\frac{27a^2k^2\mu^3\pi^2t^2w}{8\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} < 0 \tag{60}$$

$$\frac{\partial q^*}{\partial \varepsilon} = \frac{27a^2k^2\mu^3\pi^2t^2w}{8\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} > 0 \tag{61}$$

$$\frac{\partial q^*}{\partial k} = -\frac{27a^2k\mu^3\pi^2t^2\left(1+\tau-\varepsilon\right)w}{4\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} < 0 \tag{62}$$

$$\frac{\partial q^*}{\partial \mu} = -\frac{27a^2k^2\mu^2\pi^2t^2\left(1+\tau-\varepsilon\right)w}{4\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} + \frac{9a^2\pi t^2}{16\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} \neq 0 \tag{63}$$

In the above results, the impacts of a change in the distribution transportation rate t and a change in the index of spatial transformation μ have indefinite signs. The former case can be suggested to have the following additional sufficient condition, since $\partial q^* / \partial t < 0$.

$$\frac{9a^2 b\mu\pi t^3}{4\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} > \frac{9a^2\mu\pi t}{8\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} \tag{64}$$

The latter case can be treated in the same manner as the density of demand in this analysis. As a result, the sign must have $\partial q^* / \partial \mu < 0$. In this way, the additional sufficient condition will be:

$$\frac{27a^2k^2\mu^2\pi^2t^2\left(1+\tau-\varepsilon\right)w}{4\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^3} > \frac{9a^2\pi t^2}{16\left(bt^2 + 3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^2} \tag{65}$$

Partial derivatives of the optimal quantity of input x^**:*

$$\frac{\partial x^*}{\partial w} = -\frac{243a^4k^4\mu^4\pi^3t^4w}{64\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} < 0 \tag{66}$$

$$\frac{\partial x^*}{\partial t} = -\frac{81a^4bk^2\mu^2\pi^2t^5}{32\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} + \frac{81a^4k^2\mu^2\pi^2t^3}{64\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \neq 0 \tag{67}$$

$$\frac{\partial x^*}{\partial \tau} = -\frac{243a^4k^4\mu^4\pi^3t^4w}{64\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} < 0 \tag{68}$$

$$\frac{\partial x^*}{\partial \varepsilon} = \frac{243a^4k^4\mu^4\pi^3t^4w}{64\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > 0 \tag{69}$$

$$\frac{\partial x^*}{\partial k} = -\frac{243a^4k^3\mu^4\pi^3t^4\left(1+\tau-\varepsilon\right)w}{32\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} + \frac{81a^4k\mu^2\pi^2t^4}{128\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \neq 0 \tag{70}$$

$$\frac{\partial x^*}{\partial \mu} = -\frac{243a^4k^4\mu^3\pi^3t^4\left(1+\tau-\varepsilon\right)w}{32\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} + \frac{81a^4k^2\mu\pi^2t^4}{128\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \neq 0 \tag{71}$$

In the above results, the impacts of a change in distribution transportation rate t, a change in the index of technological formation k and a change in the index of spatial transformation μ have indefinite signs. As the first case should have the form $\partial x^* / \partial t < 0$ regarding the previous results, the additional sufficient condition is given as:

$$\frac{81a^4bk^2\mu^2\pi^2t^5}{32\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > \frac{81a^4k^2\mu^2\pi^2t^3}{64\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \tag{72}$$

The second case must be $\partial x^* / \partial k < 0$, regarding the previous results. Thus, the additional sufficient condition becomes:

$$\frac{243a^4k^4\mu^3\pi^3t^4\left(1+\tau-\varepsilon\right)w}{32\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > \frac{81a^4k^2\mu\pi^2t^4}{128\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \tag{73}$$

The third case should have the form $\partial x^* / \partial \mu < 0$ following from the previous results. As a result, the following additional sufficient condition is required:

$$\frac{243a^4k^4\mu^3\pi^3t^4\left(1+\tau-\varepsilon\right)w}{32\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > \frac{81a^4k^2\mu\pi^2t^4}{128\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \tag{74}$$

Partial derivatives of the optimal supply-area radius s^* *:*

$$\frac{\partial s^*}{\partial w} = -\frac{3.375a^4k^4\mu^4\pi^2t^4\left(1+\tau-\varepsilon\right)}{\phi\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)^{0.5}\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} < 0 \tag{75}$$

$$\frac{\partial s^*}{\partial t} = -\frac{0.28125\left(-\frac{8a^4bk^2\mu^2\pi t^5}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5}+\left(\frac{4a^4k^2\mu^2\pi t^3}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)\right)}{\phi\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)^{0.5}\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} \neq 0 \tag{76}$$

$$\frac{\partial s^*}{\partial \tau} = -\frac{3.375a^4k^4\mu^4\pi^2t^4w}{\phi\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)^{0.5}\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} < 0 \tag{77}$$

$$\frac{\partial s^*}{\partial \varepsilon} = \frac{3.375a^4k^4\mu^4\pi^2t^4\left(1+\tau-\varepsilon\right)}{\phi\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)^{0.5}\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > 0 \tag{78}$$

$$\frac{\partial s^*}{\partial k} = -\frac{0.28125\left(-\frac{24a^4k^3\mu^4\pi^2t^4\left(1+\tau-\varepsilon\right)w}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5}+\left(\frac{2a^4k\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)\right)}{\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)^{0.5}} \neq 0 \tag{79}$$

$$\frac{\partial s^*}{\partial \mu} = -\frac{0.28125\left(-\frac{24a^4k^4\mu^3\pi^2t^4\left(1+\tau-\varepsilon\right)w}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5}+\left(\frac{2a^4k^2\mu\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)\right)}{\left(\frac{a^4k^2\mu^2\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4}\right)^{0.5}} \neq 0 \tag{80}$$

In the above results, the impact of a change in the distribution transportation rate t, a change in the index of technological transformation k and a change in the spatial transformation μ have indefinite signs. For the first case, this should have $\partial s^* / \partial t < 0$, regarding the previous results. As a result, the additional sufficient condition will be:

$$\frac{8a^4bk^2\mu^2\pi t^5}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > \frac{4a^4k^2\mu^2\pi t^3}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \quad (81)$$

The second case cannot specify the signs, according to the investigation of the impact of a change in the index of technological formation k on the optimal supply-area radius in figure 4. As a result, it is not necessary to determine the sign in this case. The final case should have the form $\partial s^* / \partial \mu < 0$ regarding the previous results. As a result, the additional sufficient condition will be:

$$\frac{24a^4k^4\mu^3\pi^3t^4\left(1+\tau-\varepsilon\right)w}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^5} > \frac{2a^4k^2\mu\pi t^4}{\phi\left(bt^2+3k^2\mu^2\pi\left(1+\tau-\varepsilon\right)w\right)^4} \quad (82)$$

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