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TRANSBOUNDARY POLLUTION, R&D SPILLOVERS AND  
INTERNATIONAL TRADE  
by  
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# Transboundary pollution, R&D spillovers and international trade\*

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**Abstract :** We consider a symmetric three-stage game played by a pair of regulator-firm hierarchies to capture the scale and technology effects. Each firm produces one good sold on the market. The production process generates pollution characterized by a fixed emission/output ratio, and cross-borders. Firms can invest in R&D in order to lower their emission/output ratio, and this activity is characterized by positive R&D spillovers.

We show that R&D spillovers and the competition of firms on the common market help non-cooperating countries to internalize transboundary pollution more efficiently.

Opening markets to the international trade leads to more investment in R&D and more production. In most cases, pollution under common market is lower than under autarky, implying a greater social welfare. Nevertheless, in some other cases, pollution under common market is higher than under autarky implying that opening markets deteriorates social welfare.

*Keywords :* Transboundary pollution ; R&D spillovers ; Common market ; Social welfare.

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## 1.Introduction

This paper studies the impact of R&D decisions on transboundary pollution, and whether these investment possibilities encourage opening national markets to international trade.

The literature on trade and environment links free trade to environmental quality through three main effects :scale, composition, and technology effects.

The scale effect links environmental quality to the scale of production. Copeland and Taylor (1995) show that free trade may raise world pollution, and because pollution crosses borders, uncoordinated regulation of pollution at the national level does not eliminate all market failures, and consequently free trade does not necessarily raise welfare. Fernandez (2002) examines the effects of trade liberalization on transboundary water pollution between the United States and Mexico. She shows that trade liberalization give incentives to use wastewater as input to produce the traded cotton, thus reducing pollution. P echoux and Pouyet (2001) show that, under asymmetric information, international competition generated by the common market enables regulators to decrease the informational rents given up to firms, thereby reinforcing the need to open the markets to international competition.

The impact of trade through the composition effect is ambiguous even if certain dirty industries could relocate in other countries with more lenient regulations (see Copeland and Taylor (1995)).

The technology effect suggests a positive relation between trade and environmental quality because a higher income reduces the amount of pollution that the population is willing to supply, leading to the adoption of cleaner technologies. To our knowledge, there is no published theoretical work that has tried to capture this technological effect. Reppelin-Hill (1999) empirically shows that a cleaner technology (the electric arc furnace) is diffused more rapidly in countries having more open trade policies.

This paper differs from the existing literature by studying a three-stage game in which R&D is carried out to reduce the emission/output ratio, in the context of R&D spillovers and transboundary pollution. This model also captures the scale and

technology effects and tries to answer the question of whether opening markets to international trade reduces pollution and increases social welfare.

We consider a symmetric three-stage game played by two regulator-firm hierarchies. In the third stage, each firm produces one good sold on the market. The production process generates pollution characterized by a fixed emission/output ratio, and cross-borders. In the second stage, firms can invest in R&D in order to lower their emission/output ratio. As in D'Aspremont and Jacquemin (1988) where firms invest in R&D to lower their per-unit production cost, this innovation activity is characterized by positive R&D spillovers. In the first stage, regulators propose non-cooperatively their contracts which should be accepted by their respective firms while giving the socially optimal levels of pollution (or production) and R&D. We study the complete information context. Our objective is to assess the role of R&D spillovers and the opening of markets in the control of transboundary pollution, and to compare the equilibrium values under autarky and common market. We hope to contribute to the understanding of the interaction between the scale and technique effects.

We show that without R&D spillovers ( $\beta=0$ ), transboundary pollution is not internalized in the autarky regime. The higher R&D spillovers are, the higher is the proportion of transboundary pollution internalized by non-cooperating regulators. Moreover, opening markets to international trade helps countries to internalize transboundary pollution more efficiently through firms' competition on the common market.

Opening markets to international trade leads to more investment in R&D and more production. When the sensitivity of consumers to the environment is sufficiently low, pollution under common market is lower than under autarky, implying a greater social welfare. Nevertheless, when the sensitivity of consumers to the environment and the investment cost are high enough, pollution under common market is higher than under autarky; thus, the non-internalized transborder pollution is greater, even if markets opening enables to internalize a greater proportion of transborder pollution, which leads to a lower social welfare under common market.

The paper is organized as follows. Section 2 presents the basic model under autarky, resolves it and exhibits the role of the R&D spillovers for the internalization of transboundary pollution. Section 3 deals with the case when markets are opened to the international trade and shows how this contributes to internalize transborder pollution more efficiently. Section 4 compares the equilibrium under autarky and common market. Finally, section 5 concludes. All the proofs of the propositions are gathered in the appendix.

## 2. Autarky

We consider a three-stage game played by two regulator-firm hierarchies. In the first stage, each regulator offers to his firm a contract  $(q^i, x^i, T^i)$  where  $q^i$  is the level of production,  $x^i$  is the level of R&D, and  $T^i$  is a monetary transfer inducing the firm to accept this contract. Alternatively, the regulator may use an emission tax to induce the socially desired levels of pollution and production, and the monetary transfer may be the cost of R&D. We choose to resolve the problem in terms of the level of production because it is mathematically easier to resolve.

In the second stage, firms can invest in R&D in order to lower their emission/output ratio. The level  $x^i$  of R&D costs  $kx^{i2}$ ,  $k > 0$ .

In the third stage, firm  $i$  located in country  $i$  produces good  $i$  in quantity  $q^i$  sold in the domestic market with the following inverse demand function:  $p^i = a - 2q^i$ ,  $a > 0$ .

If we denote the marginal cost of production by  $\theta > 0$ , the profit of firm  $i$  is:  $\Pi^{ia} = p^i(q^i)q^i - \theta q^i - kx^{i2}$ .

The innovation activity carried out by the firms is characterized by positive externalities which imply that a proportion  $\beta$  of each firm's R&D level gratuitously spillovers to the other firm. Therefore, the direct external effect of firm  $j$ 's R&D level is to lower firm  $i$ 's emission/output ratio. This can be made possible by scientific communications, scientific exchanges or intelligence activities, which we assume have negligible costs. By normalizing the emission/output ratio to one without innovation, the emission of pollution of firm  $i$  is:

$$E^i = (1 - x^i - \beta x^j)q^i, 0 \leq \beta < 1, 0 \leq x^i + \beta x^j < 1$$

There are also negative externalities between countries through transborder pollution. Damages caused to country  $i$  are :  $D^i = \alpha E^i + \gamma E^j$ ,  $\alpha = \gamma > 0$ .

Since  $\alpha = \gamma$ , our problem can be interpreted as an international environmental problem because damages in one country are due to total pollution :  $D^i = \alpha(E^i + E^j)$ . It can also be interpreted as a pure transfrontier pollution problem where half of the pollution of firm  $i$  is exported to country  $j$ . To further clarify this last interpretation, let  $d = 2\alpha$ , then  $D^i = d \frac{1}{2} E^i + d \frac{1}{2} E^j$ . To explain how transfrontier pollution can be internalized more efficiently, we will work with  $\alpha$  and  $\gamma$  up to section 3, and then we will assimilate  $\gamma$  to  $\alpha$ .

The consumer surplus in country  $i$  engendered by the consumption of  $q^i$  is  $CS^{ia}(q^i) = q^{i2}$ .

The social welfare of a country is equal to the consumer surplus minus damages plus the profit of the firm :

$$S^{ia}(q^i, q^j, x^i, x^j, \beta) = CS^{ia}(q^i) - D^i(q^i, q^j, x^i, x^j, \beta) + \Pi^{ia}(q^i, x^i) \quad (1)$$

or written otherwise :

$$S^{ia} = q^{i2} - \alpha(1 - x^i - \beta x^j)q^i - \gamma(1 - x^j - \beta x^i)q^j + (a - 2q^i)q^i - \theta q^i - kx^{i2} \quad (2)$$

Expression (2) shows that in the third stage when regulator  $i$  chooses his production quantity  $q^i$ , the pollution coming from country  $j$  is not internalized. This is general for static models characterized by a linear damage function with respect to total pollution, or a separable one with respect to the pollution remaining at home and the one received from the other country.<sup>1</sup> However, in the second stage when regulator  $i$  chooses his level of R&D  $x^i$ , transboundary pollution is partially internalized if there is R&D spillovers ( $\beta \neq 0$ ). The higher the positive externality is, the more efficiently the negative externality is internalized.

The first order condition of the third stage is :

$$S_{q^i}^{ia} = CS_{q^i}^{ia} - D_{q^i}^i + \Pi_{q^i}^{ia} = 0 \quad (3)$$

The resolution of (3) gives :

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<sup>1</sup> If the damages are not linear nor separable, the transboundary pollution is partially non-internalized.

$$q^{ia} = \frac{1}{2} [a - \theta - \alpha(1 - x^i - \beta x^j)] \quad (4)$$

From (4), we have :

$$q_{x^i}^{ia} = \frac{\alpha}{2} \quad \text{and} \quad q_{x^j}^{ia} = \frac{\alpha\beta}{2} \quad (5)$$

Therefore, the quantity produced by a firm increases with its own R&D level, and with the R&D level of the other firm in case of positive spillovers, because they reduce its emission/output ratio.

Using equality (3), the first order condition of the second stage is reduced to :

$$S_{x^i}^{ia} = -q_{x^i}^{ja} D_{q^j}^i - D_{x^i}^i + \Pi_{x^i}^{ia} = 0 \quad (6)$$

The symmetric solution of (6), using (4) and (5), is :

$$x^{ia} = \frac{(\alpha + \beta\gamma)(a - \theta) - \alpha(\alpha + 2\beta\gamma)}{4k - (1 + \beta)\alpha(\alpha + 2\beta\gamma)} \quad (7)$$

Expression (7) confirms the fact that without R&D spillovers, transboundary pollution is completely non internalized. The higher  $\beta$  is, the more efficiently transboundary pollution is internalized. Part of this negative externality is internalized when a country chooses its level of R&D, because such a choice will affect the emission/output ratio of the firm of the other country in case of R&D spillovers, which will, in turn, affect the pollution received.

To insure that the above quantity is positive, we impose that :

$$a - \theta > \frac{\alpha(\alpha + 2\beta\gamma)}{\alpha + \beta\gamma} \quad (\text{C.1}) \quad , \quad \text{and} \quad k > \frac{1}{4}(1 + \beta)\alpha(\alpha + 2\beta\gamma) \quad (\text{C.2})$$

This last inequality guarantees the second order condition of equation (6).

$$\text{We also need that } (1 + \beta)x^{ia} < 1 \Leftrightarrow k > \frac{1}{4}(1 + \beta)(\alpha + \beta\gamma)(a - \theta) \quad (\text{C.3})$$

The symmetric production quantities are given by (4) :

$$q^{ia} = \frac{1}{2} [(1 + \beta)\alpha x^{ia} + (a - \theta) - \alpha] \quad (8)$$

Condition (C.1) guarantees that the symmetric production quantities are positive.

### 3.Common market

Firms produce the same good sold in both countries with the following inverse demand function :  $p = a - (q^i + q^j)$ .

The firms profits are :  $\Pi^{icm} = p(q^i, q^j)q^i - \theta q^i - kx^{i2}$ .

The total consumer surplus is equally divided between the two symmetric countries :  $CS^{icm} = \frac{1}{4}(q^i + q^j)^2$ .

The social welfare of country  $i$  is :

$$S^{icm}(q^i, q^j, x^i, x^j, \beta) = CS^{icm}(q^i, q^j) - D^i(q^i, q^j, x^i, x^j, \beta) + \Pi^{icm}(q^i, q^j, x^i) \quad (9)$$

The first order condition of the third stage is :

$$S_{q^i}^{icm} = CS_{q^i}^{icm} - D_{q^i}^i + \Pi_{q^i}^{icm} = 0 \quad (10)$$

The resolution of (10) is :

$$q^{icm} = \frac{1}{2} \left[ \frac{1}{2}(3 - \beta)\alpha x^i + \frac{1}{2}(3\beta - 1)\alpha x^j + (a - \theta) - \alpha \right] \quad (11)$$

From (11), we have :

$$q_{x^i}^{icm} = \frac{1}{4}(3 - \beta)\alpha \quad \text{and} \quad q_{x^j}^{icm} = \frac{1}{4}(3\beta - 1)\alpha \quad (12)$$

When a firm increases its level of R&D this has two opposite effects on its production. The first is positive and enables it to produce more because its emission/output ratio is lowered. The second is negative, because through R&D spillovers the rival firm has a lower emission ratio enabling it to produce more on the common market which forces the initial firm to reduce its production. The combination of these two effects always increases production ( $q_{x^i}^{icm} > 0$ ); however, such an increase is less important with higher R&D externalities ( $q_{x^i\beta}^{icm} < 0$ ).

When the rival firm increases its level of innovation, it affects the production of the firm both positively and negatively. Indeed, through  $\beta$ , the firm has a lower emission/output ratio which enables it to produce more. But since the rival firm has a lower pollution ratio it can produce more, forcing the firm to reduce its production. The first positive effect dominates when  $\beta$  is high enough ( $q_{x^j}^{icm} > 0 \Leftrightarrow \beta > 1/3$ ).

By using (10), the first order condition of the second stage is reduced to :



$$S_{x^i}^{icm} = q_{x^i}^{jcm} (CS_{q^j}^{icm} - D_{q^j}^i + \Pi_{q^j}^{icm}) - D_{x^i}^i + \Pi_{x^i}^{icm} = 0 \quad (13)$$

Using (11) and (12), the symmetric solution for (13) is :

$$x^{icm} = \frac{2(\alpha + \beta\gamma)(a - \theta) - \alpha[2\alpha + (5\beta - 1)\gamma]}{8k - (1 + \beta)\alpha[2\alpha + (5\beta - 1)\gamma]} \quad (14)$$

Under common market, transboundary pollution is internalized through two channels : R&D spillovers and competition of firms on the common market. Indeed, when country  $i$  chooses its level of innovation this affects the emission ratio of its firm and therefore its production and the production of the competing firm, which in turn affects the pollution received by country  $i$ . Therefore, opening the markets to international trade internalizes transboundary pollution more efficiently than under autarky, and when R&D possibilities are considered.

The second order condition for the second stage is verified iff :

$$k > \frac{1}{16}[(3\beta^2 - 6\beta + 7)\alpha^2 + 4\beta(3\beta - 1)\alpha\gamma] \quad (C.4)$$

The optimal solution given by (14) is positive when :

$$a - \theta > \frac{\alpha[2\alpha + (5\beta - 1)\gamma]}{2(\alpha + \beta\gamma)} \quad (C.5) \quad , \quad \text{and} \quad k > \frac{1}{8}(1 + \beta)\alpha[2\alpha + (5\beta - 1)\gamma] \quad (C.6)$$

$$\text{We also need that } (1 + \beta)x^{icm} < 1 \Leftrightarrow k > \frac{1}{4}(1 + \beta)(\alpha + \beta\gamma)(a - \theta) \quad (C.7)$$

The symmetric production quantities are given by (11) :

$$q^{icm} = \frac{1}{2}[(1 + \beta)\alpha x^{icm} + (a - \theta) - \alpha] \quad (15)$$

$$\text{The above production quantities are positive when } (a - \theta) > \alpha \quad (C.8)$$

In the remaining of the paper, to simplify our computations, we will take  $\gamma = \alpha > 0$ .

In the following propositions, we suppose that conditions (C.1) to (C.8) are verified. Notice that these conditions, when  $\gamma = \alpha$ , imply that  $\alpha$  is sufficiently low and  $k$  is sufficiently high.

#### 4.Common market versus autarky

In the previous sections we showed that opening markets to international trade internalizes transborder pollution more efficiently. This suggests that the level of

R&D and production are higher under common market. But what about emissions and social welfare ?

**Proposition 1.** *The optimal R&D level and production are higher under common market than under autarky.*

Opening markets to the international trade internalizes transboundary pollution more efficiently, which leads to a higher R&D level than under autarky. Consequently, the emission ratio is lower, enabling firms to produce more under common market.

**Proposition 2.** *When  $\alpha$  is sufficiently low, pollution under common market is lower than under autarky. However, when  $\alpha$  and  $k$  are high enough, pollution under common market is the highest.*

When we increase the R&D level, the emission/output ratio decreases and production increases, which, in general, lowers pollution implying that pollution under common market is lower than under autarky. However, when  $\alpha$  is important enough, the R&D level necessary to internalize pollution is important so that when it is too costly (i.e.  $k$  is high enough) the level of innovation provided is small while the emission ratio is high ; when markets are opened to the international trade, the emission/output ratio slightly decreases whereas production significantly increases, leading to an increase of pollution.

**Proposition 3.** *When  $\alpha$  is sufficiently low, the social welfare under common market is higher than under autarky. However, when  $\alpha$  and  $k$  are high enough, the social welfare under common market is the lowest.*

The results of proposition 3 are in concordance with those of propositions 1 and 2. Indeed, in general, opening markets to international trade increases production and innovation and decreases pollution, leading to an increase in social welfare. Nevertheless, when  $\alpha$  and  $k$  are sufficiently high, pollution under common market is

higher than under autarky ; this means that the non-internalized transborder pollution is greater, even if opening markets enables to internalize a greater proportion of transborder pollution, yielding to lower social welfare under common market. Since this last situation happens under very restrictive conditions<sup>2</sup>, we can, in general, say that opening the markets to international trade reduces pollution and improves the social welfare.

## 5. Conclusion

This model captures the scale and technology effects and tries to answer the question of whether opening markets to international trade, in case of transboundary pollution, reduces pollution and increases social welfare. It also studies the role of the positive R&D externality.

We consider a symmetric three-stage game played by a pair of regulator-firm hierarchies. Each firm produces one good sold on the market and can invest in R&D in order to lower its emission/output ratio. This research activity is characterized by positive R&D spillovers.

Under autarky, we show that without R&D spillovers ( $\beta=0$ ), transboundary pollution is completely non-internalized. The higher R&D spillover is, the higher the proportion of transboundary pollution internalized by non-cooperating regulators is. Moreover, opening markets to the international trade helps competing countries to internalize transborder pollution more efficiently through the competition of firms on the common market. Therefore, it is recommended for countries to voluntarily increase their positive R&D externality through scientific communications or exchanges. The issue of cooperation in R&D has not been examined in this paper, but we think that it would be beneficial for countries.

Opening markets to the international trade leads to both more investment in R&D and more production. When  $\alpha$  is sufficiently low, pollution under common market is lower than under autarky implying a greater social welfare when markets are opened. Nevertheless, when  $\alpha$  and  $k$  are high enough, pollution under common market is higher than under autarky implying that opening markets deteriorates

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<sup>2</sup> We recall that the optimal equilibrium conditions imply that  $\alpha$  is sufficiently low.

social welfare ; indeed, the non-internalized transborder pollution is greater, even if opening markets enables to internalize a greater proportion of transborder pollution, which leads to a lower social welfare under common market. Since this last situation happens under very restrictive conditions, we can say that, in general, opening markets to the international trade reduces pollution and improves the social welfare.

In this paper, we have assigned the same importance to consumer welfare and profit of the firm. A possible extension of this work is to give these latter different weights i.e. to suppose that there is positive cost of raising public funds. Another extension is to introduce asymmetric information between the regulators and their respective firms concerning their production costs or R&D activity.

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## Appendix

### A)Proof of Proposition 1

Using (7) and (14) for  $\gamma=\alpha$ , we obtain :

$$x^{icm} - x^{ia} = \frac{(1 - \beta)\alpha^2 [4k - (1 + \beta)^2 \alpha(a - \theta)]}{[8k - (1 + \beta)(1 + 5\beta)\alpha^2][4k - (1 + \beta)(1 + 2\beta)\alpha^2]}$$

From (C.3), we have  $x^{icm} - x^{ia} > 0$ .

Since  $x^{icm} > x^{ia}$ , from (8) and (15), we also have  $q^{icm} > q^{ia}$ .

### B) Proof of Proposition 2

To compare the emission levels under autarky and common market, we rewrite the optimal pollution level of a firm as  $E^i = (1 - (1 + \beta)x^i)q^i(x^i)$ , where  $q^i(x^i)$  are given

by (8) and (15). Thus,  $E_{x^i}^i = \frac{1}{2}(1 + \beta)[-2(1 + \beta)\alpha x^i + 2\alpha - (a - \theta)]$ .

$$i) E_{x^i}^i < 0 \Leftrightarrow x^i > x_1 = \frac{2\alpha - (a - \theta)}{2(1 + \beta)\alpha}$$

The above inequality is verified for any  $x^i > 0$  when  $(a - \theta) > 2\alpha$ .

Therefore, if  $(a - \theta) > 2\alpha$ , then  $E_{x^i}^i < 0 \quad \forall x^i > 0$ , and since  $x^{icm} > x^{ia}$ , this imply that

$$E^{icm} < E^{ia}.$$

$$ii) E_{x^i}^i > 0 \Leftrightarrow x^i < x_1$$

Therefore, if  $(a - \theta) < 2\alpha$ , we have  $E_{x^i}^i > 0 \quad \forall x^i < x_1$ .

To compare  $E^{icm}$  and  $E^{ia}$ , we need that  $x^{icm} < x_1$ .

$$x^{icm} - x_1 = \frac{8k[(a - \theta) - 2\alpha] + (1 + \beta)(3 - \beta)\alpha^2(a - \theta)}{2(1 + \beta)\alpha[8k - (1 + \beta)(1 + 5\beta)\alpha^2]} < 0 \Leftrightarrow k > k_1 = \frac{(1 + \beta)(3 - \beta)\alpha^2(a - \theta)}{8(2\alpha - (a - \theta))}$$

Therefore, if  $(a - \theta) < 2\alpha$  and  $k > k_1$ , then  $E_{x^i}^i > 0 \quad \forall x^i < x_1$ , implying that  $E^{icm} > E^{ia}$ .

### C) Proof of Proposition 3

By using expressions (2) and (9) for the symmetric case, the equilibrium social welfare of a country can be written as :

$$S^i = -q^{i2}(x^i) - 2\alpha(1 - (1 + \beta)x^i)q^i(x^i) + (a - \theta)q^i(x^i) - kx^{i2}$$

where  $q^i$  and  $x^i$  could be the equilibrium values under autarky or common market.

By using expressions (8) and (15), we can show that :

$$S_{x^i}^i = \left[ \frac{3}{2}(1 + \beta)^2\alpha^2 - 2k \right] x^i + (1 + \beta)\alpha(a - \theta) - \frac{3}{2}(1 + \beta)\alpha^2$$

• Suppose that  $k < \frac{3}{4}(1 + \beta)^2\alpha^2$

The above inequality is not in contradiction with (C.2), (C.3), (C.4), (C.6) and (C.7).

$$S_{x^i}^i > 0 \Leftrightarrow x^i > x_2 = (1 + \beta)\alpha \frac{2(a - \theta) - 3\alpha}{4k - 3(1 + \beta)^2 \alpha^2}$$

Therefore, if  $(a - \theta) > \frac{3}{2}\alpha$  and  $k < \frac{3}{4}(1 + \beta)^2 \alpha^2$ , then  $S_{x^i}^i > 0 \quad \forall x^i > 0$ , implying that  $S^{icm} > S^{ia}$ .

• Suppose that  $k > \frac{3}{4}(1 + \beta)^2 \alpha^2$

$$S_{x^i}^i > 0 \Leftrightarrow x^i < x_2$$

If  $(a - \theta) > \frac{3}{2}\alpha$ , then  $x_2 > 0$  and we need to have  $x^{icm} < x_2$ .

$$x^{icm} - x_2 = \alpha \frac{4k[-2(1 + \beta)(a - \theta) + (5 + \beta)\alpha] - 4(1 + \beta)(1 - \beta^2)\alpha^2(a - \theta)}{[8k - (1 + \beta)(1 + 5\beta)\alpha^2][4k - 3(1 + \beta)^2 \alpha^2]}$$

If  $(a - \theta) > \frac{5 + \beta}{2(1 + \beta)}\alpha$ , then  $x^{icm} < x_2$ .

Therefore, if  $(a - \theta) > \frac{5 + \beta}{2(1 + \beta)}\alpha$  and  $k > \frac{3}{4}(1 + \beta)^2 \alpha^2$ , then  $S_{x^i}^i > 0 \quad \forall x^i < x_2$ , implying

that  $S^{icm} > S^{ia}$ .

Since  $\frac{5 + \beta}{2(1 + \beta)}\alpha > \frac{3}{2}\alpha$ , then if  $(a - \theta) > \frac{5 + \beta}{2(1 + \beta)}\alpha$ , we have  $S_{x^i}^i > 0 \quad \forall x^i < x_2$ ,

implying that  $S^{icm} > S^{ia}$ .

Lastly,  $S_{x^i}^i < 0 \Leftrightarrow x^i > x_2$ : true for any  $x^i > 0$  when  $(a - \theta) < \frac{3}{2}\alpha$ .

Therefore, if  $(a - \theta) < \frac{3}{2}\alpha$  and  $k > \frac{3}{4}(1 + \beta)^2 \alpha^2$ , then  $S_{x^i}^i < 0 \quad \forall x^i > 0$ , implying that

$S^{icm} < S^{ia}$ .