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INTRODUCTION TO INPUT-OUTPUT STRUCTURAL Q-ANALYSIS

by

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REAL 00-T-1 February, 2000

Introduction to Input-output Structural Q-Analysis.

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Abstract. The topological principles of the well-known Atkin Q-analysis are applied to the analysis of interconnectedness of sectors in input-output systems. The operational methodology is presented in detail and supported by empirical application to the Israeli input-output system for the period 1967-1988. The rank-size ordering of the elements of the Leontief inverse, the slicing procedure and a new graphical tools - the accumulation diagram and the complication curve - are introduced, providing a new explanation of economic complexity through the process of structural economic complication.

1. Introduction.

The purpose of this paper is to propose a new method of decomposition analysis of the economic interdependencies of sectors in an input-output system. Our attention is directed to the application and further elaboration of the ideas of combinatorial topology to the analysis of economic structure of input-output system, using methodology originally proposed by Atkin (1974, 1981) for the analysis of the structure of human interactions. The central concern is the complication of structure or deepening of economic complexity through structural changes generated by simultaneous change in technology and in the composition of final demand: the regional fundamental economic structures appear in the form of hierarchies of interacting economic subsystems. These structural changes are often complex and difficult to extract: hence, new tools for illustration, interpretation and visualization provide the potential for greater insights into the nature of these changes.

An important component of the modern process of industrialization is the change in the nature of interdependence in production characterized by the essential interdependence found in input-output and social accounting tables. Analysis of the *evolution* of interindustry relations has now become, once more, a major point of interest for economic analysts. The traditional approach, proposed by Chenery and others in the 1950s (Chenery, 1953; Chenery and Watanabe, 1958;

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Chenery and Clark, 1959) was extended further in various subsequent studies (see Carter, 1970; Long Jr., 1970; Ohkawa and Rosovsky, 1973; Song, 1977; Matthews *et al.*, 1982; Harrigan *et. al.*, 1980; Deutsch and Syrquin, 1989 among others). In this regard, three sets of studies of the Israeli economy within the input-output framework should be especially mentioned. An important early study was conducted by Bruno (1962) and complemented by later analyses by Weksler *et al.,* (1978) and Freeman *et al,* (1982). At the end of the 1980s, the major contributions rely on the work of Syrquin (1986, 1988) and Deutsch and Syrquin (1989) in which the processes of economic development and changes in the structure of production in the Israeli economy were compared with similar changes in the production structure of more then 30 other countries.

In section 2, the paper will present and interpret the methodology of structural Q-analysis based on a *slicing* procedure of the ordered set of the elements of the Leontief inverse. Further, the chains of structural complication and rank-size ordering procedure will be introduced. In section 3, structural Q-analysis will be applied for the analysis of a set of 6-sector aggregated Israeli input-output tables for the period 1967-1988. The main purpose of this paper is to illustrate some new approaches of Q-analysis to enhancing an understanding of the economic structural changes caused by simultaneous technological changes reflected in a set of input-output tables. In addition, the methodology may be seen to have important relationships with popular notions of backward and forward linkages.

2. Methodology of Structural Q-analysis.

The following methodological description of the procedure of Q-analysis is taken from the Atkin studies (Atkin, 1974, 1981; see also, Sonis and Hewings, 1998; Sonis, 1988, and Sonis, Hewings and Bronstein, 1994).

2.1. Slicing procedure.

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Consider the Leontief inverse matrix $B=\left[b_{ij}\right]$ of some input-output system and let $(i_1, j_1),(i_2, j_2),...,(i_m, j_m)$ be a fixed set of pairs of economic sectors entering the input-output system. Let $b_{i_1 j_1}, b_{i_2 j_2}, \ldots, b_{i_m j_m}$ be the corresponding components of the matrix *B*. The *slicing procedure* results in the construction of a new matrix B_s whose only non-zero components are $b_{i_1 j_1}, b_{i_2 j_2}, \ldots, b_{i_m j_m}$ while all other components are zeros. This slicing procedure, referred to as a *variable filter* approach is the basic element of minimal flow analysis (see, Holub *et al*., 1985; Schnabl ,1994).

The matrix I_S with the unit entries in the place of non-zero components of the matrix B_s is called the *incidence matrix* associated with the slicing procedure. Obviously, 2^{n^2} different slicing procedures exist for each *nxn* matrix *B*. The simplest slicing procedure consists of the choice of the slicing parameter μ, and the exclusion from the matrix *B* of all components b_{ij} such that $b_{ij} < \mu$. The choice of a definite slicing parameter depends on the investigator's preferences about the economic structure of the interaction matrix (see also section 3).

2.2. Simplicial families for forward linkages.

Initially, Q-analysis will be applied to forward linkages; backward linkages will be considered separately. Consider a slicing procedure defined with the help of the set of components $b_{i_1 j_1}, b_{i_2 j_2}, \ldots, b_{i_m j_m}$. This procedure defines the sliced matrix B_s and the corresponding incidence matrix I_S. The set $j_1, j_2, ..., j_m$ of the corresponding economic sectors serves as a set of vertices of a many-dimensional polyhedron generating the *partial forward linkages backcloth*.

The procedure for the construction and partition of this polyhedron into a set of simplexes can be defined in the following way: for each fixed economic sector, $i_k, k=1,2,...,m,$ consider the set of all different economic sectors $j_{r_0}, j_{r_1}, ... j_{r_{qk}}$ corresponding to the non-zero $b_{i_k j_{r_0}}, b_{i_k j_{r_1}}, ..., b_{i_k j_{r_{qk}}}$ associated with the inputs of the sector i_k into the economic sectors $j_{r_0}, j_{r_1}, ... j_{r_{qk}}$ The *simplex*, $S^f_{q_k}(i_k) = S_{i_k}$ associated with the sector i_k, is a minimal convex polyhedron in q_k-dimensional space with *q+1* vertices $j_{r_0}, j_{r_1}, ... j_{r_{qk}}$. (Each such simplex represents part of some economic *filière,* - the notion elaborated by Torre, 1993).

The set of simplexes, $S^f_{q_1}(i_1), S^f_{q_2}(i_2), ..., S^f_{q_m}(i_m)$, associated with all economic sectors i_k, *k=1,2,...,m*, is called *the forward linkages simplicial family*, generating the polyhedron with vertices $j_1, j_2, ..., j_m$, and its partition - *the simplicial complex K(S).*

2.3. q-nearness and q-connectedness.

Two simplexes $S(i_k)$ and $S(i_s)$ are *q-near* in the simplicial family *iff* they share at least *q+1* vertices. Thus, two sectors i_k and i_s are *q-near iff* there are at least *q+1* economic sectors accepting the inputs from the sectors i_k and i_s. If all vertices of a simplex $S(i_s)$ are the vertices of a simplex $S(i_k)$ then the simplex $S(i_s)$ is a face of the simplex $S(i_k)$.

Two simplexes $S(i_k)$ and $S(i_s)$ are *q-connected* by a chain of simplexes of length *r iff* there is a sequence of *r* pair-wise *q-near* simplexes $S(i_k), S(i_p), ..., S(i_q), S(i_s),$. The relationship *of q-connectedness* generates the partition of the simplicial family *K(S)* into *q-connected* components. The enumeration of all *q-connected* components for each dimension $q \geq 0$ is the essence of the Q-analysis of the simplicial family.

2.4 Procedure and meaning of the forward linkage Q-analysis.

Following Atkin (1974, 1981) the operational basis for Q-analysis is given by *a shared face matrix SF* of the form:

$$SF = I_S I_S^T - U \tag{1}$$

where I_S is the incidence matrix corresponding to the chosen slicing procedure, I_S^T is its transpose and *U* is the matrix with unit entries. The components of the matrix *SF* give the amounts of mutual vertices for each pair of sectors i_k, i_s, $k, s = 1, 2, ..., m$. In other words, the components of the shared face matrix *SF* are the dimensions of the maximal mutual faces for each pair of simplices $S(i_k)$, and $S(i_s)$.

The Atkin operational algorithm for Q-analysis includes the following iterative steps for each dimension q, $q = 0,1,...,N$, where N is the maximal dimension of simplices from the simplicial complex:

(i) Identify the economic sectors and their corresponding simplices whose dimensions are equal to or larger than q; these dimensions are on the main diagonal of the shared face matrix *SF*.

(ii) Identify all distinct *q-connected* components - *q-chains* - of the set of simplices constructed in the previous step: two *q-dimensional* simplices $S_q(i_k)$ and $S_q(i_s)$ belong to the same *q-chain* if the corresponding rows i_k and i_s of the shared face matrix *SF* include at least one column with entries larger than or equal to q; the number of distinct *q-chains* is denoted as Q_q. The vector

$$Q = \{Q_N, Q_{N-1}, ..., Q_o\} \tag{2}$$

is called the *structural vector* of the simplicial complex *K(S)* and the maximal *q-value* N is a dimension of this complex.

Further, the *eccentricity* value of each simplex from the complex *K(S)* can be introduced with the help of the formula:

$$Ecc\left(S_q(i_k)\right) = \frac{\hat{q} - \tilde{q}}{\hat{q} - 1} \tag{3}$$

where $\hat{q}$ is the diagonal element of the i_k^{th} row of the matrix *SF*, and $\tilde{q}$ is the largest non-diagonal value of the entries of the i_k^{th} row of *SF*. It is obvious that the simplex derived from the simplicial complex is eccentric *iff* it is not a face of another simplex from the complex. The eccentricity of a simplex is equal to infinity *iff* this simplex is disconnected from all other simplices.

2.5. The conjugate simplicial complex and the backward linkages input-output Q-analysis.

So far, a one-sided perspective (forward linkages Q-analysis) of the interaction matrix has been presented. The same fixed slicing procedure can be used for the generation of the scheme of backward linkages Q-analysis. Consider the transpose interaction matrix F^T and the corresponding conjugate slicing matrix F_S^T with the corresponding conjugate incidence matrix

I_S^T. This matrix can be used for the construction of the conjugate simplicial complex $K\left(S^T\right)$. As a result, the set of simplices can be constructed corresponding to the set of economic sectors $j_1, j_2, ..., j_m$: each simplex $S(j_k)$ represents the set of economic sectors sending the inputs into the sector j_k.

Moreover, the backward linkages input-output Q-analysis can be performed analogously with the help of the *conjugate shared face matrix*:

$$S^T F = I_S^T I_S - U \qquad (4)$$

2.6. Chains of structural complication of simplicial families and the rank-size ordering.

Consider two slicing procedures S_1 and S_2 and their corresponding simplicial families associated with the simplicial complexes $K(S_1)$ and $K(S_2)$. The simplicial complex $K(S_2)$ is called the *structural complication* of the simplicial complex $K(S_1)$ and noted $K(S_1) \prec K(S_2)$ if each simplex $S_p^{/}(i_k)$ from $K(S_1)$ is a face of some simplex $S_q^{\prime\prime}(i_k)$ from $K(S_2)$. This means that the incidence matrix I_{S_2} includes all non-zero (unit) components from the incidence matrix I_{S_1}. The set of *m* simplicial complexes $K(S_1), K(S_2), ..., K(S_m)$ is called *the chain of structural complication* if for each pair of complexes $K(S_s)$ and $K(S_r)$, one of them is the structural complication of the other. Obviously, the chain of structural complication is defined with the help of the set of corresponding incidence matrices such that, for each pair of incidence matrices, one of them includes all the units from the other. This means also that the chain of structural complication is generated by a sequence that extends sets of the components of the interaction matrix *B*.

One of the important methods of the generation of the chains of structural complication will now be illustrated, namely, the *rank-size ordering method*. The rank-size ordering method is based on the construction of the sequence of all components of the interaction matrix $B = \left[b_{ij}\right]$ ordered by size in such a way that the largest components is at the top of the decreasing-by-size sequence of components. Thus, it is possible to consider only the qualitative rank-size sequence in place of the absolute value of each component. For example, consider the sequence of slicing procedures and the corresponding set of sliced matrices, the first of which includes only the largest components of the interaction matrix *B*, while the second matrix includes the two largest components, the third matrix includes three largest components, and so forth. In such a way, one obtains the chain of structural complication associated with the relative size on the matrix components. The Q-analysis of each element of the chain of structural complication reveals some hidden features of the inter-sectoral interactions.

2.7. Complication curves.

The geometrical form of the structural complication, associated with pictures of simplices, is not possible in multi-dimensional spaces. Therefore, use is made instead of complication curves, elaborated for the needs of migration analysis (see, Sonis, 1988; Portugali and Sonis, 1991). The complication curve represents a simplex with the help of a planar graph: the vertices of the simplex associated with i^{th} economic sector are represented through points on the plane, and the oriented arcs connect the point of the sector i with all other points (see explanation for figure 7). The complication curve provides a convenient graphical representation of the multi-dimensional simplices, and also a simple picture of the mutual faces for each pair of simplices. Furthermore, complication curves reveal a straightforward picture of the process of structural complication of input-output systems. Figures 5 and 6 depict the simultaneous gradual enlargement of each backward and forward linkage simplices for the Leontief inverse matrix. It is important to stress that the process of structural complication can be described with the help of complication curves, depicting the process of sequential enlargement of simplices in the process of structural complication (see figure 7). The complication curve describes the sequential addition of economic sectors to the simplex according to the rank-size step of structural complication. Moreover, the structural complication leads to the new approach of explanation of economic complexity through the process of complication.

3. Example: Hierarchical Rank-Size analysis of Israeli input-output System, 1968-1988.

3.1. Israeli input-output System.

In this paper the theoretical findings will be illustrated using Israeli input-output tables. Although the first Israeli input-output tables date from 1950, the systematic development of tables using a consistent framework and methodology begun in the 1960s. The development of the Israeli input-output system to a great extent reflects the tendencies and specific features of the economic development of Israel (see Bar-Eliezer, 1993). All the input-output tables of Israel are industry by industry tables; they feature up to 192 sectors, that, for illustrative purposes were aggregated into the following six categories: Agriculture (**AGR**) - 36 sectors; Manufacturing (**MNF**) - 104; Construction (**CNS**) - 14; Electricity and Water (**ELT**) - 2; Transportation and Communication (**TRS**) - 12; Commerce, Business, Services and Other (**SRV**)- 24. The essential role of import (almost one third of all resources) was reflected in the breakdown of imports into complementary and competitive components.

The benchmark input-output tables are published in Israel each five years. Further, in connection with a high level of inflation (especially in the 1980s), estimated intermediate tables are constructed. The basic tables were constructed for 1968/69,1972/73,1978/79, 1982/83 and 1987/88 together with estimated tables for 1981, 1985 and 1990 in prices of 1968, 1972, 1981, 1985 and 1990. In this paper, use will be made of tables from the first time period 1977 and for the second time period 1982, both in the prices of 1968.

<insert table 1 here>

3.2. Temporal changes in direct inputs, measured by information Kullback-Leibler distance.

Each industrial sector is represented in table 1 by the vector-column at time t (=1977) and t+1(= 1982) marked as

$$\overline{a}_j^t = \begin{pmatrix} a_{1j}^t \\ a_{2j}^t \\ \vdots \\ a_{nj}^t \end{pmatrix}; \ \overline{a}_j^{t+1} = \begin{pmatrix} a_{1j}^{t+1} \\ a_{2j}^{t+1} \\ \vdots \\ a_{nj}^{t+1} \end{pmatrix}' j = 1, 2, \ldots, n \tag{5}$$

To measure the changes in the vector-column of direct inputs during the five-year period 1977-1982, the Kullback-Leibler information distance measure will be used (see Cover and Tomas, 1991, p. 18) in the form:

$$d_j^{t,t+1} = \frac{\sum_{i=1}^{n} a_{ij}^{t} \log a_{ij}^{t+1}}{\sum_{i=1}^{n} a_{ij}^{t} \log a_{ij}^{t}} - 1 \tag{6}$$

In above definition, the usual conventions are adopted (based on continuity arguments) that 0 log 0 = 0 and for $a \neq 0$ $a \log 0 = -\infty$. As is well known in information theory, $d_j^{t,t+1} \geq 0$ and $d_j^{t,t+1} = 0$ if and only if $a_{ij}^{t} = a_{ij}^{t+1}$ for each *i*.

<*insert table 2 here*>

Table 2 (a) represents the ranking of the economic branches with respect to the five-year changes in direct inputs measured by information distance, defined in (6). According to this table, the largest direct input changes occurred in the Electricity and Water (*ELT*) sector, indicated by the highest information distance 0. 024484. Next follows Commerce, Business, Services and Other (*SRV*), Manufacturing (*MNF*), Construction (**CNS**), Transportation and Communication (*TRS*) and with the smallest direct inputs changes occurring in Agriculture (*AGR*) where the information distance was 0.008949.

3.3. Input Accumulation diagram.

The addition of the row of value added to matrices A_I and A_{II} converts each vector-column reflecting the relative inputs into each individual industry into probabilistic vectors and allows the application of the rank-size ordering procedure and the construction of the *input accumulation diagram* describing the structure of inputs into each industry. Table 2 (b) represents the probabilistic vector-columns of the *ELT* for 1977 and 1982 and table 2 (c) their rank-size ordering used for the construction of the *ELT* input accumulation diagram in figures 1 and 2, that also include the graphical representation of the vector of inputs of the Electricity and Water sector in 1977and 1982. Figures 1 and 2 taken together represent the changes in the inputs into Electricity and Water sector during a five-year period. These changes do not involve ranking by absolute volume, only the changes in the *relative* volumes of inputs. In all probability, it would be usual to find changes in the rankings of both absolute and relative volumes in more disaggregated input matrices.

<insert figures 1 and 2 here>

3.4. Israeli Backward and Forward Linkages.

The corresponding Leontief inverse for the first time period $B_I = (I - A_I)^{-1}$ and its row and column multipliers are presented in table 3 Moreover, in the same table, there are forward and backward linkages of the economic sectors calculated with the help of Rasmussen (1956) indices presenting:

1. *Power of dispersion for the backward linkages:*

$$BL_j = \frac{1}{n}\sum_i b_{ij} / \frac{1}{n^2}\sum_{ij} b_{ij} = nB_{\bullet J} / V \qquad (7)$$

and

2. *Sensitivity of dispersion for forward linkages:*

$$FL_i = \frac{1}{n}\sum_j b_{ij} / \frac{1}{n^2}\sum_{ij} b_{ij} = nB_{i\bullet} / V \qquad (8)$$

where $B_{\bullet j} = \sum_i b_{ij}$ is the column multiplier of sector *j*, $B_{i\bullet} = \sum_j b_{ij}$ is the row multiplier of sector *i*, and $V = \sum_{ij} b_{ij}$ is the sum of all components of the Leontief inverse.

<insert tables 3 and 4 here>

Table 3 presents also the rank-size hierarchy of backward linkages of economic branches: *MNF, SRV, TRS, ELT, AGR, CNS* (see Table 4(a)); and the forward linkages hierarchy : *ELT, AGR, SRV, TRS, CNS, MNF* (see Table 4(b))**.** Thus, *MNF* (Manufacturing sector) is receiving the largest amount of inputs from other sectors and sending the smallest amount of inputs into other sectors.

3.5. Dynamics of linkages and Key sectors.

Using the value of backward and forward linkages, BL_i, FL_i, as coordinates of points representing the corresponding sector *I*, figure 3 illustrates the distribution of sectors in 1977 on the space of backward and forward linkages. This space is divided into four quadrants: key sectors, backward- and forward-linkage oriented sectors and weakly oriented sectors.

As usually defined in conventional key sector theory, the sector *i* is considered a *key sector* if its coordinates $BL_i > 1$ and $FL_i > 1$; sector *i* is considered as *a backward linkage oriented sector* if $BL_i > 1$ and $FL_i < 1$; sector *i* is defined as *a forward linkage oriented sector* if $BL_i < 1$ and $FL_i > 1$; sector *i* with both backward and forward linkages less than one, is considered as a *weakly linked sector*. As may be seen from table 4, in 1977 Commerce, Business and Services (*SRV*) and Manufacturing (*MNF*) were backward linkage oriented sectors; Electricity and Water (*ELT*) was the only forward linkage oriented sector; while Agriculture *(AGR)* and Transportation and Communication (*TRS*) were weakly oriented sectors. The results in table 4 are presented graphically in the space of backward and forward linkages in figure 3.

<insert figure 3 here>

The dynamics of linkages and key sectors from 1968 through 1988 are presented in table 5 and figure 4.

<insert figure 4 and table 5 here>

Figure 4 reveals that only *MNF* sector was a key sector in 1968 and 1988, *MNF* and *SRV* were the backward linkages oriented sectors, while *ELT, AGR* and *CNS* were forward linkage oriented sectors through almost two decades, and the *TRS* sector was weakly oriented sector all the time. From table 5 it is possible to derive the rank-size hierarchical dynamics of forward and backward linkages separately (see table 6).

<insert table 6 here>

Table 6 shows that the hierarchy of forward linkages is more stable then the hierarchy of backward linkages. In the hierarchy of forward linkages *MNF* and *SRV* are always on the top of hierarchy, *CNS* is always on the bottom; in the hierarchy of backward linkages *AGR* is always near the top, *ELT* moved down in 1988 while all other sectors changed their place in the hierarchy. This change in both hierarchies signified the qualitative change in the economic relationships between the sectors during time.

3.6. Inner hierarchical structure of column and row multipliers of the Leontief inverse, Israel 1977: complication curve analysis.

For this analysis, use will be made of the rank-size ordering of the elements of the Leontief inverse and the methodology of the accumulation diagram, presented in the section 3.3. In table 7 the rank size ordering is presented of the components of the Leontief inverse, for 1977, derived from the table 3.

<insert table 7 here>

For the backward and forward linkages of each sector, *i.e.*, for each row and column of the Leontief inverse, the complication curve can be constructed in the following way (*cf.* section 2.7). The horizontal axes of this diagram reveal the ranks of elements from the row or column; the vertical axes show the cumulative number and order of the sectors entering into the row or column (see figures 5 and 6).

3.7. Accumulation indices for backward and forward linkages.

The accumulation of the indirect inputs in row and column multipliers, *i.e.*, within the inner structure of corresponding linkages, can be measured by the following *accumulation indices* i_{BL_i}, i_{FL_j} , presenting the area under the complication curve, normalized by maximum value of this area. In constructing these indices, the ordering of each row and column of the matrix of the rank-size hierarchy of the components of Leontief inverse from table 4 needs to be accessed: $r_1 < r_2 < \ldots < r_n$ and define $r_{n+1} = n^2$. Then

$$i_{BL_i}, i_{FL_j} = \frac{\sum_{s=1}^{n} (r_{s+1} - r_s) s}{n^3 - \frac{n^2}{2}} \tag{9}$$

It is easy to show that

$$1/(2n-1) \le i_{BL_i}, i_{FL_j} \le 1 \tag{10}$$

<insert table 8 here>

According to (10), all accumulation indices lie in the range: $0.0909 \le i_{BL_i},\ i_{FL_j} \le 1$.

4. Structural Q-analysis of Israeli input-output system, 1977-1982.

4.1 Simplicial complexes and q-chains of structural interdependencies.

The Q-analysis procedure is applicable to any matrix: for example, the following matrices connected with input-output analysis can be used, such as the matrix of direct inputs *A*, the matrix of increments *E*, the old and new Leontief Inverses *B* and *B(E),* the fields of influence of different orders, and the total intensity matrices. In this paper, the analysis will be directed to the structure of the Leontief inverse matrix *B* for 1977 (see table 3). The six largest elements of this matrix are on its main diagonal with the largest direct input, 1.4184, in the cell *(MNF,MNF)*. The rank-size entries of the matrix *B* are shown in table 4, along with the qualitative rank-size cross-structure of this matrix.

The proposed Q-analysis procedures provide new insights into pair-wise sector interaction, insights that cannot be provided by conventional methods of input-output analysis. As an initial slicing procedure, consider, as an example of the Q-analysis methodology, the removal of 50% of the smallest components of the Leontief inverse matrix *B*. Therefore, the associated incidence matrix will now have this form:

$$I_S = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{11}$$

First, consider the backward linkages Q-analysis; each column of the incidence matrix generates the corresponding simplex. For example, the backward linkages simplex, generated by the first economic sector *AGR* (the first column of the matrix I_S) is $S_3^b(AGR) = \{AGR, MNF, ELT, SRV\}$. This means that the first sector *AGR* obtains inputs from the sectors *AGR, MNF, ELT* and *SRV*.

Geometrically, this simplex is the three-dimensional tetrahedron with the vertices *AGR, MNF, ELT, SRV*. The following backward linkages simplicial family corresponds to the economic sectors:

$$
\begin{aligned}
&S_3^b(AGR) = \{AGR, MNF, ELT, SRV\}\\
&S_2^b(MNF) = \{AGR, MNF, SRV\}\\
&S_2^b(CNS) = \{MNF, CNS, SRV\}\\
&S_2^b(ELT) = \{MNF, ELT, SRV\}\\
&S_2^b(TRS) = \{MNF, TRS, SRV\}\\
&S_1^b(SRV) = \{MNF, SRV\}
\end{aligned}
\tag{12}
$$

This simplicial family includes one three-dimensional simplex - a pyramid:

$$S_3^b(AGR) = \{AGR, MNF, ELT, SRV\}$$

four two-dimensional simplices, the triangles that are the faces of the pyramid:

$$
\begin{aligned}
&S_2^b(MNF) = \{AGR, MNF, SRV\}, S_2^b(CNS) = \{MNF, CNS, SRV\},\\
&S_2^b(ELT) = \{MNF, ELT, SRV\}, S_2^b(TRS) = \{MNF, TRS, SRV\}
\end{aligned}
$$

and one one-dimensional simplex $S_1^b(SRV) = \{MNF, SRV\}$ - a mutual side of all triangles.

The analytical and geometrical representation of all other simplices and the backward linkage simplicial complex are described in figure 7 and the explanation that follows this figure.

<insert figure 7 here>

Hence, using the methodology described in section 2.4, the following conjugate shared face matrix may be obtained:

$$
S^T F = \begin{bmatrix}
3 & 2 & 1 & 2 & 1 & 1\\
2 & 2 & 1 & 1 & 1 & 1\\
1 & 1 & 2 & 1 & 1 & 1\\
2 & 1 & 1 & 2 & 1 & 1\\
1 & 1 & 1 & 1 & 2 & 1\\
1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\tag{13}
$$

from which matrix the following set of *q-chains* of different dimensions may be generated:

$$\begin{aligned} &q=3: \quad \left\{S^b(AGR)\right\} \\ &q=2: \quad \left\{S^b(AGR), S^b(MNF), S^b(ELT)\right\};\left\{S^b(TRS)\right\} \\ &q=1 \quad \left\{S^b(AGR), S^b(MNF), S^b(CNS), S^b(ELT), S^b(TRS), S^b(SRV)\right\} \end{aligned} \tag{14}$$

with the following structural vector

$$Q=\begin{Bmatrix} 3 & & 1 \\ 1 & 2 & 1 \end{Bmatrix} \tag{15}$$

The interpretation of the *3-chain* is that sector *AGR* is the sector with the largest amount of essential backward linkages inputs from four sectors.

The *2-chain* presented suggests that the sectors *AGR, MNF*, *ELT* and *SVR* obtain inputs from at least three sectors in such a way that the pairs of sectors, *AGR* and *MNF,* and *MNF* and *ELT,* have mutual backward linkages with same triads of sectors. The other component of the 2-chain, *SRV* also obtains inputs from three sectors, but the corresponding set of sectors is different from these, connected to other component of the 2-chain. The *1-chain* implies that each sector is connected with some other sector by the inputs from the same pair of sectors.

Turning attention to the forward linkages Q-analysis, a quite different situation is revealed. The following forward linkages simplicial family corresponds to the economic sectors:

$$\begin{aligned} &S_1^f(AGR)=\{AGR, MNF\} \\ &S_5^f(MNF)=\{AGR, MNF, CNS, ELT, TRS, SRV\} \\ &S_0^f(CNS)=\{CNS\} \\ &S_1^f(ELT)=\{AGR, ELT\} \\ &S_0^f(TRS)=\{TRS\} \\ &S_5^f(SRV)=\{AGR, MNF, CNS, ELT, TRS, SRV\} \end{aligned} \tag{16}$$

The dimension of this simplicial family is equal to 5, and the largest 5-dimensional simplices are

$$S_5^f(AGR)=S_5^f(SRV)=\{AGR, MNF, CNS, ELT, TRS, SRV\} \tag{17}$$

and thus includes all economic sectors, i.e., *MNF* and *SRV* are sending inputs to all economic sectors.

The procedure of the forward linkages Q-analysis, described in the section 2.4, yields the following conjugate shared face matrix:

$$SF = \begin{bmatrix} 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 5 & 0 & 1 & 0 & 5 \\ -1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 1 \\ -1 & 0 & -1 & -1 & 0 & 0 \\ 1 & 5 & 0 & 1 & 0 & 5 \end{bmatrix} \tag{18}$$

This matrix generates the following set of q-chains of the different dimensions:

$$\begin{aligned}
&q = 5: \quad \left\{S^f(MNF) = S^f(MNF)\right\} \\
&q = 4: \quad \left\{S^f(MNF) = S^f(SRV)\right\} \\
&q = 3 \quad \left\{S^f(MNF) = S^f(SRV)\right\} \\
&q = 2 \quad \left\{S^f(MNF) = S^f(SRV)\right\} \\
&q = 1 \quad \left\{S^f(AGR), S^f(MNF), S^f(ELT), S^f(SRV)\right\} \\
&q = 0 \quad \left\{S^f(AGR), S^f(MNF), S^f(CNS), S^f(ELT), S^f(TRS), S^f(SRV)\right\}
\end{aligned} \tag{19}$$

with the following structural vector;

$$Q = \begin{Bmatrix} 5 & & & & & 0 \\ 1, & 1, & 1, & 1, & 1, & 1 \end{Bmatrix} \tag{20}$$

This Q-analysis highlights the case in which *MNF* and *SRV* sends inputs to all six sectors. The 1-chain describes the situation in which different pairs of sectors from the set of sectors *AGR, MNF ,ELT,* and *SRV* send inputs into at least two of the same sectors while the 0-chain includes all sectors since *MNF* and *SRV* send inputs to all sectors.

The comparison between the results of backward and forward linkage Q-analyses suggests that there is strong asymmetry between the forward and backward linkages structures. Of course, this asymmetry would be readily apparent in analysis conducted with much more desegregated sectoral detail.

4.2. Structural complication of the Israeli input-output system.

The structural complication through the rank-size ordering of the components of the Leontief inverse associated with the following chain of incidence matrices, constructed with the help of slicing procedure including consequently 1, 2, 3, etc... elements of rank-size ordering of the components of the Leontief inverse matrix (see table 4);

$$m = 1 \qquad \begin{bmatrix} 0&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{bmatrix} \qquad m = 6 \qquad \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{bmatrix}$$

$$m = 8 \qquad \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&1&1&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{bmatrix} \qquad m = 12 \qquad \begin{bmatrix} 1&0&0&0&0&0\\ 1&1&1&1&1&1\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&1&1&1&1 \end{bmatrix}$$

$$m = 18 \qquad \begin{bmatrix} 1&1&0&0&0&0\\ 1&1&1&1&1&1\\ 0&0&1&0&0&0\\ 1&0&0&1&0&0\\ 0&0&0&0&1&0\\ 1&1&1&1&1&1 \end{bmatrix} \qquad m = 21 \qquad \begin{bmatrix} 1&1&0&1&0&0\\ 1&1&1&1&1&1\\ 0&0&1&0&0&0\\ 1&0&0&1&0&0\\ 0&0&1&0&1&1\\ 1&1&1&1&1&1 \end{bmatrix}$$

$$m = 30 \qquad \begin{bmatrix} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 0&0&1&0&0&1\\ 1&1&0&1&0&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1 \end{bmatrix} \qquad m = 36 \qquad \begin{bmatrix} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1 \end{bmatrix}$$

The geometrical form of the structural complication, associated with pictures of simplices, is not possible in multi-dimensional spaces; therefore, complication curves will be used since they are able to represent the simplices in the form of planar graphs. The vertices of the simplex associated with the i^{th} economic sector are shown as points on the plane, while connections between points are represents as arcs (see figure 7). The complication curves provide a convenient graphical representation of the multi-dimensional simplices, and also a simple picture of the mutual faces for each pair of simplices. Furthermore, complication curves facilitate an appreciation of the process of structural complication of an input-output system. Figure 7 depicts the simultaneous gradual enlargement of each of the backward linkages simplices for the Leontief inverse. It is clearly visible that there is a relatively rapid increase in the complexity of the simplicial family connected with simultaneous enlargement of the backward linkages of two leading sectors, *AGR* and *ELT*, and after that, *MNF* and *SRV*. This process of structural complication can be summarized by reference to a polyhedral presentation of complication curves, depicting the process of sequential enlargement of simplices in the process of structural complication (see figure 7). The complication curve describes the sequential addition of economic sectors to the simplex accordingly to the rank-size step of structural complication. The complication curves for the forward linkage simplices highlight the different behavior of economic sectors: for example, there is a very rapid enlargement of the simplices corresponding to the economic sectors *MNF* and *SRV,* defining the fine structure of q-chains. This structural complication of q-chains is presented in the Appendix.

4.3. Structural Change in the Israeli Economy 1977-1982: Interpretation

The period covered by this analysis saw the Israeli economy move from a period of significant capital investment to one of extensive growth and then to a period of decline. By the end of the 1970s, the economy was one characterized by significant, say might claim, over-consumption of capital in the primary and secondary sectors with an over-consumption of labor in the services sector. These findings reflected the small size of the domestic market and the weak competitive position of Israeli exports. As a reaction to this situation, new tendencies appeared. The center of gravity of the economy moved toward scientifically based productive sectors with more attractive comparative advantages; these sectors included chemicals (based on the Dead Sea complex), organic chemicals and pharmaceuticals and high technology-based electronics and computer technology.

The complication curves presented in figures 5 and 6 substantiates these findings - with a strong, backward-oriented manufacturing sector and a service-based set of sectors with strong forward linkages. However, at this level of aggregation, the major interactions were between pairs of sectors rather than for a specific sector as a whole. In essence, the Israeli economy was characterized as one based on some very strong *pair-wise* interactions but not with the dominating *sector-specific* tendencies that one finds in say regional economies of the same size (see Hewings, 1982, for example). The analysis suggests that for many economies, attention should be focused more on the chains of linkages rather than on evaluation of the more macro effects of linkages mechanisms (such as the column or row multipliers). Q-analysis is able to exploit the analytical characteristics of chains of different size and complexity and the degree to which sectors share the same partners in a backward and forward sense. Complementing this appreciation of structure is the introduction of complication curves that provide an assessment of the different behaviors of economic sectors. At the high level of aggregation presented here, much of the important, detailed differences between sectors are diminished but the analysis provides the opportunity for a classification of sectors that is more useful than standard linkage analysis since the visualization provides for rapid assessment of differences and similarities.

4. Conclusion.

The objective of this paper is two fold. The analysis of economic interactions between the sectors reflected in the complex system of backward and forward linkages requires the elaboration of a new tool for such an analysis; therefore, the ideas of combinatorial topology in the form of Atkin's Q-analysis were used, and their graph-theoretical representation was presented. This new method is especially important for the analysis of the structural complication in input-output systems.

Secondly, these theoretical methods were used for the analysis of the Israeli input-output system, 1977-1982. The hierarchical analysis of the chains of economic dependencies in input-output scheme of Israel revealed the important dissimilarities between the backward and forward linkages influences of economic changes in Israeli economy.

The methodology can thus provide some important, complementary insights into the structure of economies and to furthering an understanding of the changes that have taken place in cases in which consistent input-output or social accounting matrices are available for more than one time period. While the methodology has been illustrated with very aggregated tables, it will probably prove to be most valuable in applications involving very large matrix systems. However, this methodology, like many others that have been used to interpret economic structure and structural change, should be used to supplement other approaches that have proven to be of value (see Sonis *et al*., 1994).

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Appendix

Structural complication of q-chains for the backward and forward linkages simplicial families.

Backward Linkages

$m = 1.$

$\textit{Simplicial family}: \ S_0^b(MNF) = \{MNF\}$

$\textit{Q-chain}: \ q = 0: \ \{S^b(MNF)\}$

$\textit{Structural vector}: \ Q = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$

$m = 6.$

$\textit{Simplicial family}: S_0^b(AGR) = \{AGR\}$

$S_0^b(MNF) = \{MNF\}$

$S_0^b(CNS) = \{CNS\}$

$S_0^b(ELT) = \{ELT\}$

$S_0^b(TRS) = \{TRS\}$

$S_0^b(SRV) = \{SRV\}$

$\textit{Q-chain}: \ q = 0: \ \{S^b(AGR)\};\{S^b(MNF)\};\{S^b(CNS)\};\{S^b(ELT)\};\{S^b(TRS)\};\{S^b(SRV)\}$

$\textit{Structural vector}: \ Q = \begin{Bmatrix} 0 \\ 6 \end{Bmatrix}$

$m = 8.$

$\textit{Simplicial family}: S_0^b(AGR) = \{AGR\}$

$S_0^b(MNF) = \{MNF\}$

$S_1^b(CNS) = \{MNF, CNS\}$

$S_1^b(ELT) = \{MNF, ELT\}$

$S_0^b(TRS) = \{TRS\}$

$S_0^b(SRV) = \{SRV\}$

$\textit{Q-chain}: \ q = 1: \{S^b(CNS)\};\{S^b(ELT)\}$

$q{=}0: \{S^b(AGR)\};\{S^b(MNF), S^b(CNS), S^b(ELT)\};\{S^b(TRS)\};\{S^b(SRV)\}$

$\textit{Structural vector}: \ Q = \begin{Bmatrix} 1 & 0 \\ 2 & 4 \end{Bmatrix}$

$m = 12.$

$$
\begin{aligned}
&\textit{Simplicial family}: S_1^b(AGR) = \{AGR, MNF\}\\
&S_0^b(MNF) = \{MNF\}\\
&S_1^b(CNS) = \{MNF, CNS\}\\
&S_1^b(ELT) = \{MNF, ELT\}\\
&S_2^b(TRS) = \{MNF, TRS, SRV\}\\
&S_1^b(SRV) = \{MNF, SRV\}
\end{aligned}
$$

$$
\begin{aligned}
\textit{Q-chain}:\quad & q = 2: \{S^b(TRS)\}\\
& q = 1: \{S^b(AGR)\}; \{S^b(CNS)\}; \{S^b(ELT)\}; \{S^b(TRS), S^b(SRV)\}\\
& q = 0: \{S^b(AGR), S^b(MNF), S^b(CNS), S^b(ELT), S^b(TRS), S^b(SRV)\}
\end{aligned}
$$

$$\textit{Structural vector}: Q = \begin{Bmatrix} 2 & & 1 \\ 1 & 4 & 1 \end{Bmatrix}$$

$m = 18.$

$$
\begin{aligned}
&\textit{Simplicial family}: S_3^b(AGR) = \{AGR, MNF, ELT, SRV\}\\
&S_2^b(MNF) = \{AGR, MNF, SRV\}\\
&S_2^b(CNS) = \{MNF, CNS, SRV\}\\
&S_2^b(ELT) = \{MNF, ELT, SRV\}\\
&S_2^b(TRS) = \{MNF, TRS, SRV\}\\
&S_1^b(SRV) = \{MNF, SRV\}
\end{aligned}
$$

$$
\begin{aligned}
Q-chain:\ & q = 3: \{S^b(AGR)\}\\
& q = 2: \{S^b(AGR), S^b(MNF), S^b(ELT)\}; \{S^b(TRS)\}\\
& q = 1: \{S^b(AGR), S^b(MNF), S^b(CNS), S^b(ELT), S^b(TRS), S^b(SRV)\}\\
& q = 0: \{S^b(AGR)\}; \{S^b(MNF)\}; \{S^b(CNS)\}; \{S^b(ELT)\}; \{S^b(TRS)\}; \{S^b(SRV)\}
\end{aligned}
$$

$$\textit{Structural vector}:\quad Q = \begin{Bmatrix} 3 & & 1 \\ 1, & 2, & 1 \end{Bmatrix}$$

$m = 21.$

$$\begin{aligned}
&\textit{Simplicial family}: S_3^b(AGR) = \{AGR, MNF, ELT, SRV\}\\
&S_2^b(MNF) = \{AGR, MNF, SRV\}\\
&S_3^b(CNS) = \{MNF, CNS, TRS, SRV\}\\
&S_3^b(ELT) = \{AGR, MNF, ELT, SRV\}\\
&S_2^b(TRS) = \{MNF, TRS, SRV\}\\
&S_2^b(SRV) = \{MNF, TRS, SRV\}
\end{aligned}$$

$$\textit{Q-chain}: \; q = 3: \{S^b(AGR), S^b(ELT)\}; \{S^b(CNS)\}$$

$$q = 2: \{S^b(AGR), S^b(MNF), S^b(CNS), S^b(ELT), S^b(TRS), S^b(SRV)\}$$

$$\textit{Structural vector}: \; Q = \begin{Bmatrix} 3 & 2 \\ 2 & 1 \end{Bmatrix}$$

$m = 30.$

$$\begin{aligned}
&\textit{Simplicial family}: S_4^b(AGR) = \{AGR, MNF, ELT, TRS, SRV\}\\
&S_4^b(MNF) = \{AGR, MNF, ELT, TRS, SRV\}\\
&S_4^b(CNS) = \{AGR, MNF, CNS, TRS, SRV\}\\
&S_4^b(ELT) = \{AGR, MNF, ELT, TRS, SRV\}\\
&S_3^b(TRS) = \{AGR, MNF, TRS, SRV\}\\
&S_5^b(SRV) = \{AGR, MNF, CNS, ELT, TRS, SRV\}
\end{aligned}$$

$$\textit{Q-chain}: \; q = 5: \; \{S^b(AGR), S^b(MNF), S^b(TRS), S^b(SRV)\}$$

$$q = 3: \{S^b(AGR), S^b(MNF), S^b(ELT), S^b(TRS), S^b(SRV)\}$$

$$q = 1: \{S^b(AGR), S^b(MNF), S^b(CNS), S^b(ELT), S^b(TRS), S^b(SRV)\}$$

$$\textit{Structural vector}: \; Q = \begin{Bmatrix} 5 & & & & 1 \\ 1 & 0 & 1 & 0 & 1 \end{Bmatrix}$$

Forward Linkages

$m = 1.$

$$\textit{Simplicial family}: \; S_0^f(MNF) = \{MNF\}$$

$$\textit{Q-chain}: \; q = 0: \; \{S^f(MNF)\}$$

$$\textit{Structural vector}: \; Q = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

$m = 6.$

$Simplicial\ family : S_0^b\left(AGR\right) = \left\{AGR\right\}$

$S_0^b\left(MNF\right) = \left\{MNF\right\}$

$S_0^b\left(CNS\right) = \left\{CNS\right\}$

$S_0^b\left(ELT\right) = \left\{ELT\right\}$

$S_0^b\left(TRS\right) = \left\{TRS\right\}$

$S_0^b\left(SRV\right) = \left\{SRV\right\}$

$Q\text{-}chain:\ q = 0:\ \left\{S^b\left(AGR\right)\right\};\left\{S^b\left(MNF\right)\right\};\left\{S^b\left(CNS\right)\right\};\left\{S^b\left(ELT\right)\right\};\left\{S^b\left(TRS\right)\right\};\left\{S^b\left(SRV\right)\right\}$

$Structural\ vector:\ Q = \begin{Bmatrix} 0 \\ 6 \end{Bmatrix}$

$m = 8.$

$Simplicial\ family : S_0^f\left(AGR\right) = \left\{AGR\right\}$

$S_2^f\left(MNF\right) = \left\{MNF, CNS, ELT\right\}$

$S_0^f\left(CNS\right) = \left\{CNS\right\}$

$S_0^f\left(ELT\right) = \left\{ELT\right\}$

$S_0^f\left(TRS\right) = \left\{TRS\right\}$

$S_0^f\left(SRV\right) = \left\{SRV\right\}$

$Q\text{-}chain:\ q = 2:\left\{S^f\left(MNF\right)\right\}$

$q{=}0{:}\left\{S^f\left(AGR\right)\right\};\left\{S^f\left(MNF\right), S^f\left(CNS\right), S^f\left(ELT\right)\right\};\left\{S^f\left(TRS\right)\right\};\left\{S^f\left(SRV\right)\right\}$

$Structural\ vector:\ Q = \begin{Bmatrix} 2 & & 0 \\ 1 & 0 & 4 \end{Bmatrix}$

$m = 12.$

Simplicial family : $S_0^f\left(AGR\right) = \left\{AGR\right\}$

$S_5^f\left(MNF\right) = \left\{AGR, MNF, CNS, ELT, TRS, SRV\right\}$

$S_0^f\left(CNS\right) = \left\{CNS\right\}$

$S_0^f\left(ELT\right) = \left\{ELT\right\}$

$S_0^f\left(TRS\right) = \left\{TRS\right\}$

$S_1^f\left(SRV\right) = \left\{TRS, SRV\right\}$

Q-chain : $q = 5 : \left\{S^f\left(MNF\right)\right\}$

$q = 1 : \left\{S^f\left(MNF\right), S^f\left(SRV\right)\right\}$

$q = 0 : \left\{S^f\left(AGR\right), S^f\left(MNF\right), S^f\left(CNS\right), S^f\left(ELT\right)\right\}; \left\{S^f\left(TRS\right), S^f\left(SRV\right)\right\}$

Structural vector : $Q = \begin{Bmatrix} 5 & & & & & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{Bmatrix}$

$m = 18.$

Simplicial family : $S_1^f\left(AGR\right) = \left\{AGR, MNF\right\}$

$S_5^f\left(MNF\right) = \left\{AGR, MNF, CNS, ELT, TRS, SRV\right\}$

$S_0^f\left(CNS\right) = \left\{CNS\right\}$

$S_1^f\left(ELT\right) = \left\{AGR, ELT\right\}$

$S_0^f\left(TRS\right) = \left\{TRS\right\}$

$S_5^f\left(SRV\right) = \left\{AGR, MNF, CNS, ELT, TRS, SRV\right\}$

Q-chain : $q = 5 : \left\{S^f\left(MNF\right) = S^f\left(MNF\right)\right\}$

$q = 4 : \left\{S^f\left(MNF\right) = S^f\left(SRV\right)\right\}$

$q = 3 : \left\{S^f\left(MNF\right) = S^f\left(SRV\right)\right\}$

$q = 2 : \left\{S^f\left(MNF\right) = S^f\left(SRV\right)\right\}$

$q = 1 : \left\{S^f\left(AGR\right), S^f\left(MNF\right), S^f\left(ELT\right), S^f\left(SRV\right)\right\}$

$q = 0 : \left\{S^f\left(AGR\right), S^f\left(MNF\right), S^f\left(CNS\right), S^f\left(ELT\right), S^f\left(TRS\right), S^f\left(SRV\right)\right\}$

Structural vector : $Q = \begin{Bmatrix} 5 & & & & & 0 \\ 1, & 1, & 1, & 1, & 1, & 1 \end{Bmatrix}$

$m = 21.$

Simplicial family : $S_2^f(AGR) = \{AGR, MNF, ELT\}$

$S_5^f(MNF) = \{AGR, MNF, CNS, ELT, TRS, SRV\}$

$S_0^f(CNS) = \{CNS\}$

$S_1^f(ELT) = \{AGR, ELT\}$

$S_2^f(TRS) = \{CNS, TRS, SRV\}$

$S_5^f(SRV) = \{AGR, MNF, CNS, ELT, TRS, SRV\}$

Q-chain : $q = 5 : \{S^f(MNF), S^f(SRV)\}$

$q = 2 : \{S^f(AGR), S^f(MNF), S^f(TRS), S^b(SRV)\}$

$q = 1 : \{S^f(AGR), S^f(MNF), S^f(ELT), S^f(TRS), S^f(SRV)\}$

$q = 0 : \{S^f(AGR), S^f(MNF), S^f(CNS), S^f(ELT), S^f(TRS), S^f(SRV)\}$

Structural vector : $Q = \begin{Bmatrix} 5 & & & & & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{Bmatrix}$

$m = 30$

Simplicial family : $S_5^f(AGR) = \{AGR, MNF, CNS, ELT, TRS.SRV\}$

$S_5^f(MNF) = \{AGR, MNF, CNS, ELT, TRS, SRV\}$

$S_1^f(CNS) = \{CNS, SRV\}$

$S_3^f(ELT) = \{AGR, MNF, ELT, SRV\}$

$S_5^f(TRS) = \{AGR, MNF, CNS, ELT, TRS, SRV\}$

$S_{\%}^f(SRV) = \{AGR, MNF, CNS, ELT, TRS, SRV\}$

Q-chain : $q = 5 : \{S^f(SRV)\}$

$q = 4 : \{S^f(AGR), S^f(MNF), S^f(CNS), S^f(ELT), S^f(SRV)\}$

$q = 3 : \{S^f(AGR), S^f(MNF), S^f(CNS), S^f(ELT), S^f(TRS), S^f(SRV)\}$

Structural vector : $Q = \begin{Bmatrix} 5 & & 3 \\ 1 & 1 & 1 \end{Bmatrix}$

Table 1. Matrices of regional direct inputs.

(a) First time period(1977): A_I

	AGR	MNF	CNS	ELT	TRS	SRV
(1) AGR	.060	.063	.002	.000	.000	.0005
(2) MNF	.300	.250	.364	.522	.142	.152
(3) CNS	.0003	.003	.0004	.007	.003	.021
(4) ELT	.072	.012	.002	.139	.005	.006
(5) TRS	.011	.018	.034	.006	.191	.033
(6) SRV	.032	.040	.065	.048	.118	.246
Value Added	.525	.614	.532	.2788	.5428	.532

(b) Second time period (1982): A_{II}

	AGR	MNF	CNS	ELT	TRS	SRV
(1) AGR	.091	.058	.001	.000	.000	.0002
(2) MNF	.267	.198	.311	.435	.099	.115
(3) CNS	.000	.003	.001	.005	.003	.023
(4) ELT	.064	.012	.002	.118	.005	.006
(5) TRS	.011	.018	.030	.019	.024	.026
(6) SRV	.048	.068	.097	.062	.117	.192
Value Added	.521	.642	.558	.360	.541	.638

Table 2. Descriptors of Change, 1977-1982

(a) Ranking of sectors with respect of five years information distance.
(b) Vector-columns of the direct inputs into Electricity and Water (ELT) branch in 1977 and 1982;
(c) Rank-size hierarchical ordering of the direct inputs into Electricity and Water sector in 1977 and 1982.

(a)		(b)			(c)			
Ranking of Sectors	1997-1982 distance	*Sectors*	1977	1982	*Ranking of sectors*	1977	1982	Difference
ELT	.024484	*AGR*	0	0	*MNF*	.5222	.43507	- .08513
SRV	.017454	*MNF*	.5222	.43507	*V. A*	.2783	.36025	+.08195
MNF	.012477	*CNS*	.00744	.00546	*ELT*	.13886	.1182	- .02066
CNS	.011599	*ELT*	.13886	.11820	*SRV*	.04767	.06184	+.01417
TRS	.010545	*TRS*	.00553	.01918	*CNS*	.00744	.01918	+.01174
AGR	.008949	*SRV*	.04767	.06184	*TRS*	.00553	.00546	-.00007
		V.A.	.27830	.36025	*AGR*	0	0	0

Table 3. Leontief Inverse, Column and Row Multipliers and Sectoral Backward and Forward Linkages, Israel input-output System, 1977.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*Row Multipliers*	*Forward Linkages*	*Rank Size Hierarchy of Forward Linkages*
Sectors	*AGR*	*MNF*	*CNS*	*ELT*	*TRS*	*SRV*	*Sectors*	*AGR*	
(1) AGR	1.0998	.0951	.0392	.0594	.0204	.0224	1.3363	.7123	V
(2) MNF	.5354	1.4184	.5504	.8845	.3021	.3212	4.0120	2.1386	I
(3) CNS	.0045	.0069	1.0052	.0146	.0094	.0294	1.0700	.5703	VI
(4) ELT	.1003	.0280	.0147	1.1794	.0142	.0166	1.3532	.7213	IV
(5) TRS	.0313	.0374	.0611	.0345	1.2519	.0644	1.4806	.7893	III
(6) SRV	.0863	.0879	.1284	.1310	.2140	1.3584	2.0060	1.1068	II
Column Multipliers	1.8576	1.6737	1.7990	2.3034	1.8120	1.8124	*V* = 11.2581		
Backward Linkages	.9902	.8922	.9590	1.2278	.9658	.9661			
Rank size Hierarchy of Backward Linkages	II	VI	V	I	IV	III			

Table 4. Backward and Forward linkages hierarchy, 1977.

(a *)* Forward linkages hierarchy		*(*b*)* Backward linkages hierarchy	
Sectors	*FL*	*Sectors*	*BL*
(2)MNF	2.1386	*(4)ELT*	1.2278
(6)SRV	1.1068	*(1)AGR*	.9902
(5)TRS	.7893	*(6)SRV*	.9661
(4)ELT	.7213	*(5)TRS*	.9658
(1)AGR	.7123	*(3)CNS*	.9590
(3)CNS	.5703	*(2)MNF*	.8922

Table 5 . Dynamics of backward and forward linkages, 1968-1988.

Sectors		*1968*	*1972*	*1977*	*1982*	*1988*	
(1)AGR	*BL*	0.8480	1.1247	0.9902	1.0779	1.1633	
	FL	0.8479	0.8172	0.7123	0.7140	0.7721	
(2) MNF	*BL*	1.0058	0.9808	0.8922	0.9572	1.0085	
	FL	1.7342	1.6733	2.1428	2.0883	1.6055	
(3) CNS	*BL*	1.0594	1.0161	1.1495	0.9851	1.0778	
	FL	0.6486	0.6882	0.5303	0.5563	0.6824	
(4) ELT	*BL*	1.1329	1.0902	1.2278	1.1383	0.9169	
	FL	0.8409	0.8340	0.7213	0.7182	0.8104	
(5) TRS	*BL*	0.8415	0.8220	0.9658	0.9801	0.8547	
	FL	0.7859	0.8012	0.7893	0.7800	0.8050	
(6) SRV	*BL*	0.8882	0.9662	0.9661	0.8614	0.9787	
	FL	1.1419	1.1941	1.1068	1.1431	1.3410	

Table 6. Dynamics of the changes of sector ordering in linkages hierarchy, 1968-1988.

(*a* **)** *Forward linkages hierarchy*

Years	*1968*	*1972*	*1977*	*1982*	*1988*
	MNF	*MNF*	*MNF*	*MNF*	*MNF*
	SRV	*SRV*	*SRV*	*SRV*	*SRV*
	AGR	*ELT*	*TRS*	*TRS*	*ELT*
	ELT	*AGR*	*ELT*	*ELT*	*TRS*
	TRS	*TRS*	*AGR*	*AGR*	*AGR*
	CNS	*CNS*	*CNS*	*CNS*	*CNS*

(b) Backward linkages hierarchy

Years	*1968*	*1972*	*1977*	*1982*	*1988*
	ELT	*AGR*	*ELT*	*ELT*	*AGR*
	AGR	*ELT*	*AGR*	*AGR*	*CNS*
	CNS	*CNS*	*SRV*	*CNS*	*MNF*
	MNF	*MNF*	*TRS*	*TRS*	*SRV*
	SRV	*SRV*	*CNS*	*MNF*	*ELT*
	TRS	*TRS*	*MNF*	*SRV*	*TRS*

Table 7. Rank-Size hierarchy of the components of Leontief Inverse, 1977.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*Rank-Size Hierarch of Row Multipliers*
Sectors	*AGR*	*MNF*	*CNS*	*ELT*	*TRS*	*SRV*	
(1) AGR	**5**	**16**	22	21	29	28	V
(2) MNF	**9**	**1**	**8**	**7**	**11**	**10**	I
(3) CNS	36	35	**6**	32	34	26	VI
(4) ELT	**15**	27	31	**4**	33	30	IV
(5) TRS	25	23	20	24	**3**	19	III
(6) SRV	**18**	**17**	**14**	**13**	**12**	**2**	II
Rank-Size Hierarchy of Column Multipliers	II	VI	V	I	IV	III	

Note: 18 (50%) highest ranks are indicated in bold.

Table 8. Backward and Forward linkages hierarchy and the corresponding accumulation indices, 1977.

(*a*) *Forward linkages Index of accumulation*

Sectors	FL	i_{FL}
(2)MNF	2.1386	0.8586
(6)SRV	1.1068	0.7071
(5)TRS	0.7893	0.5152
(4)ELT	0.7213	0.3838
(1)AGR	0.7123	0.4798
(3)CNS	0.5703	0.2374

(*b*) *Backward linkages Index of accumulation*

Sectors	*BL*	i_{BL}
(4)ELT	1.2278	0.5808
(1)AGR	0.9902	0.5455
(6)SRV	0.9661	0.5101
(5)TRS	0.9658	0.4747
(3)CNS	0.9590	0.5808
(2)MNF	0.8922	0.4899

Figure 1: Accumulation diagram, ELT, 1977

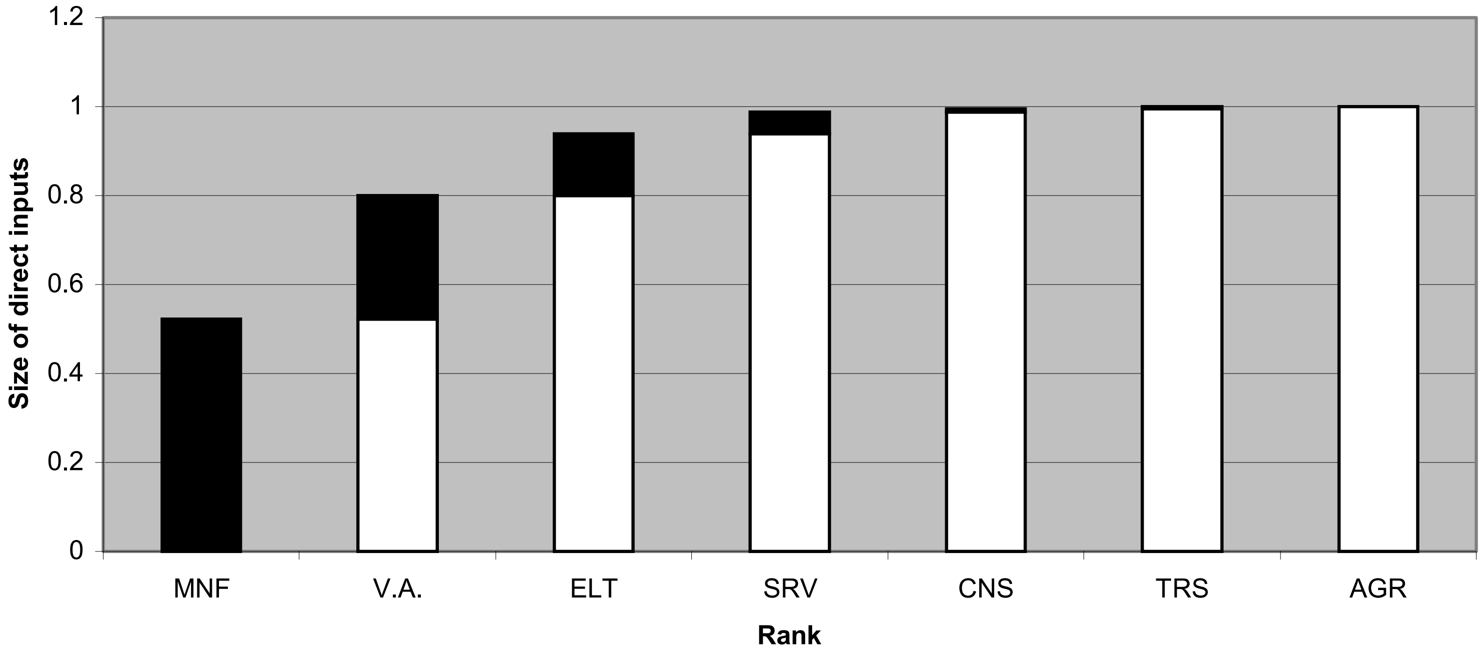

Figure 2: Accumulation diagram, ELT, 1982

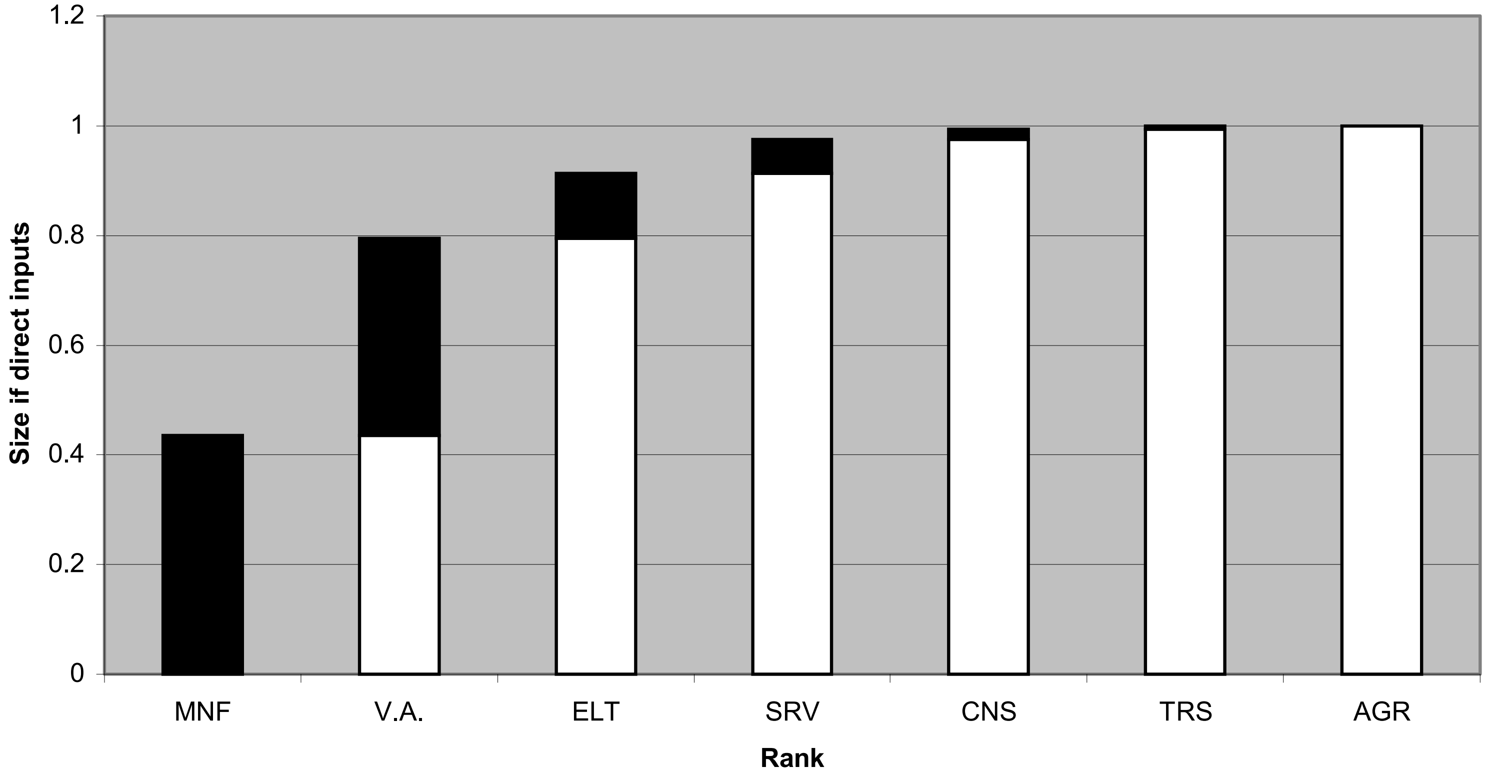

Figure 3 Backward and Forward Linkages, 1977

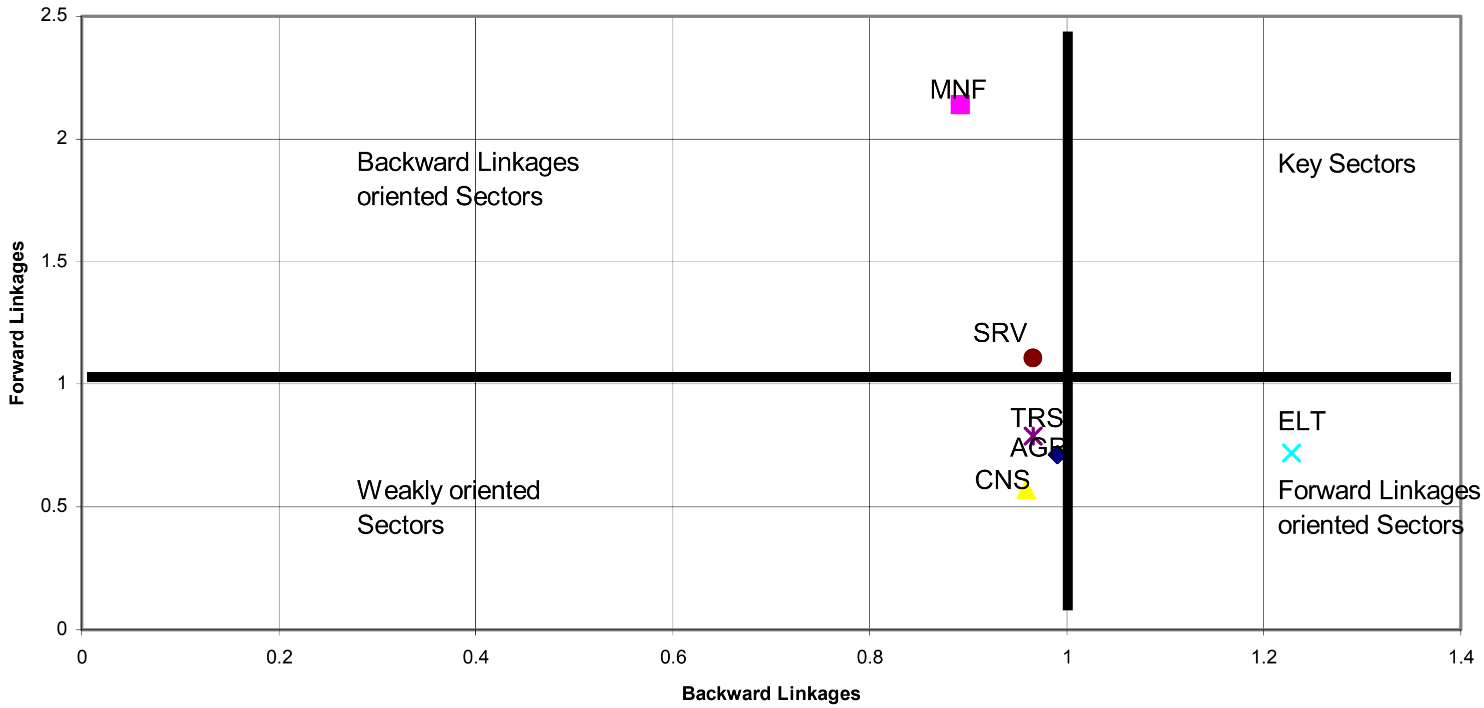

Figure 4 Dynamics of Linkages and Key Sectors, 1968-88

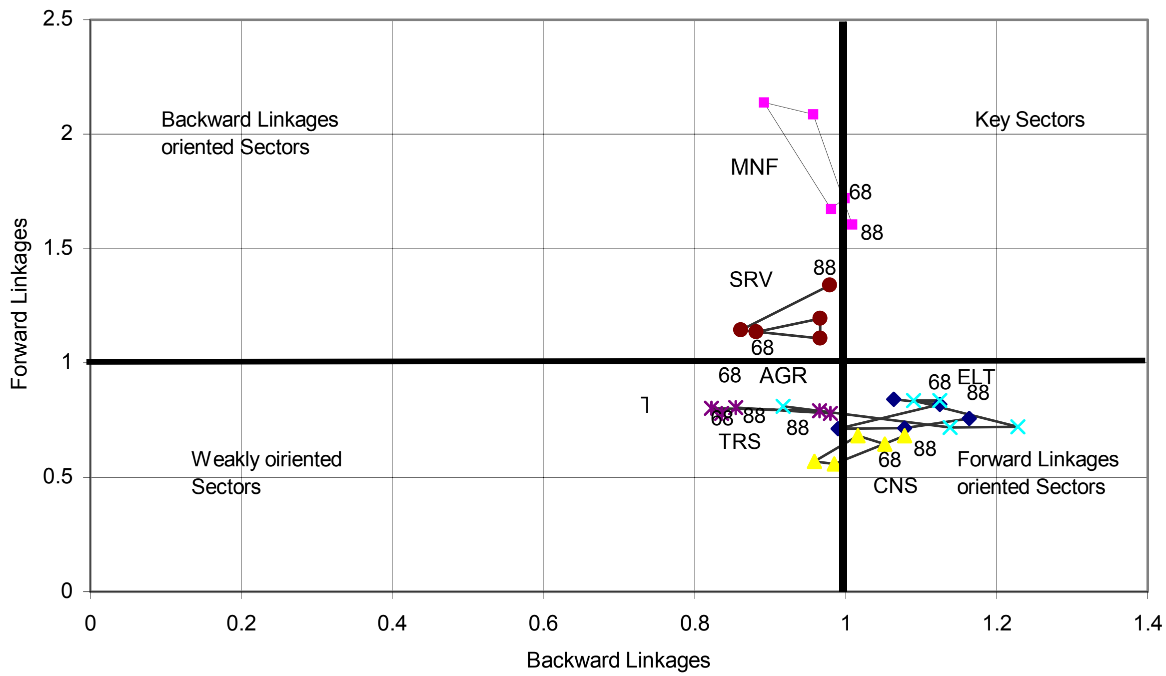

Figure 5 Accumulation curves for forward linkages.

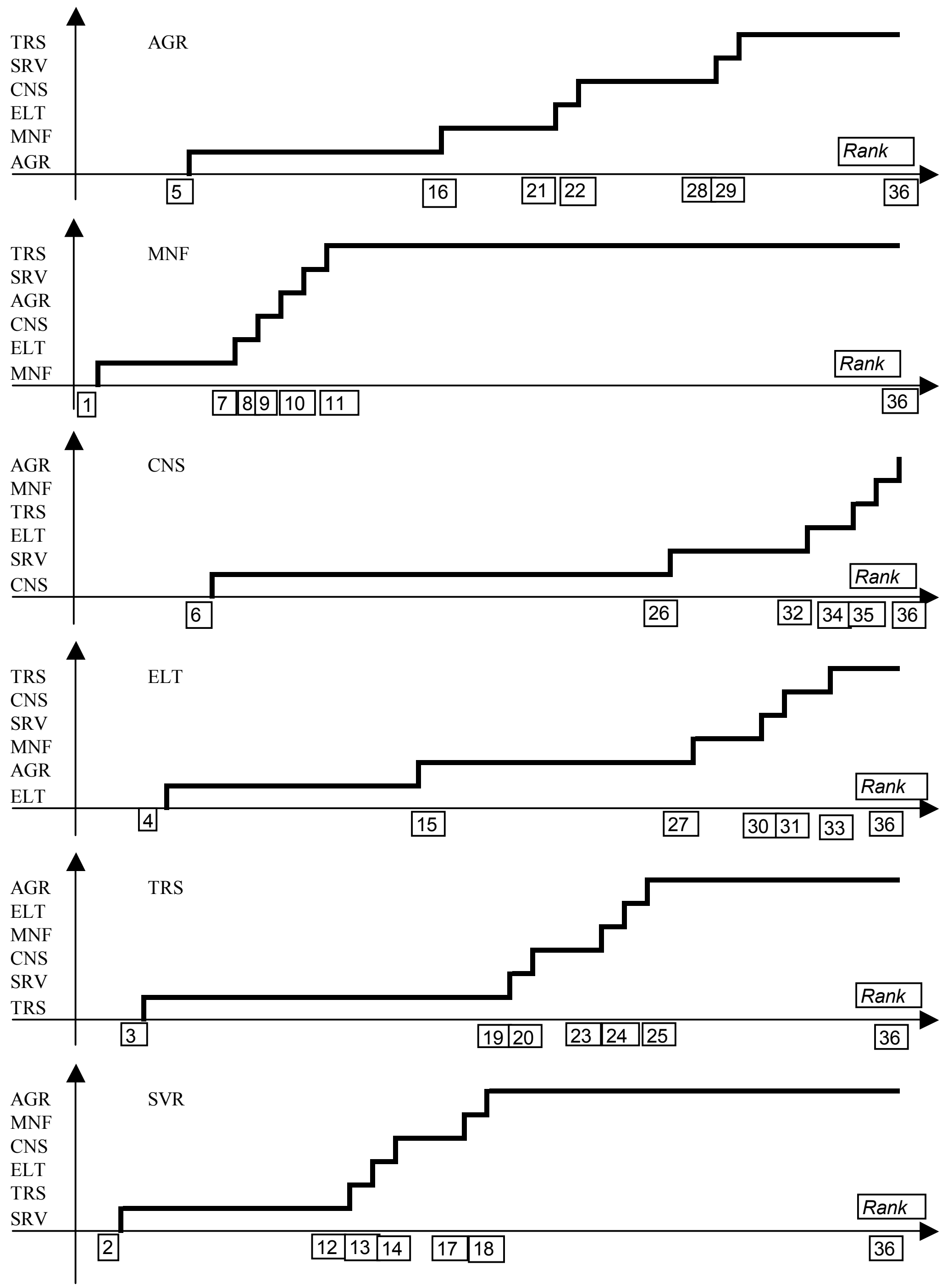

Figure 6: Accumulation curves for backward linkages.

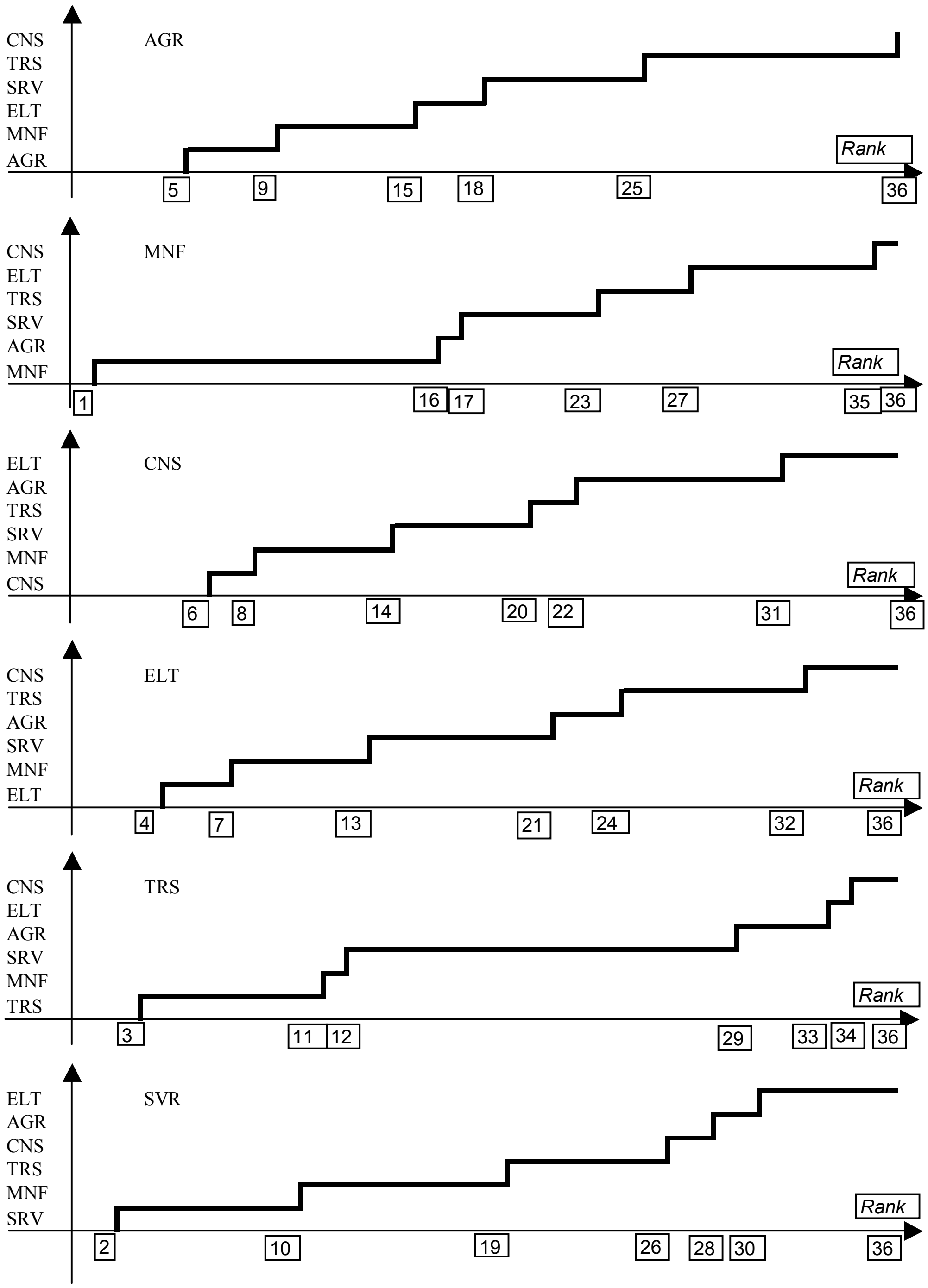

Figure 7 Visualization of the Simplices of Structural Complication